An Ordered Multiple Discrete-Continuous Extreme Value (OMDCEV) Modelling Framework for Episode-level Activity Participation and Time-Use Analysis

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ABSTRACT

This paper formulates a novel, ordered multiple discrete continuous extreme value (OMDCEV) modeling framework to analyze multiple discrete continuous (MDC) choices at a disaggregate level, including the number of instances different choice alternatives are chosen and the amount of consumption at each instance of choice. The model is formulated for disaggregate, episode-level analysis of individuals' activity participation and time allocation. In this model, choice alternatives are defined as episodes of different activity-types. Further, the formulation also ensures a logical consistency that a higher-numbered episode of an activity-type does not occur without the occurrence of all lower-numbered episodes of that activity-type. Accommodating such conditions via explicit constraints in the utility maximization framework is difficult, for it leads to mixed integer constraints. To circumvent this problem, we exploit the properties of MDC choice models with additive utility functions to condition the utility maximization problem on a non-increasing ordering of the baseline marginal utility parameters of different episodes of an activity-type. The model results in a conditional likelihood function, where the likelihood arising from the optimality conditions of the utility maximization problem is conditioned on the ordering of baseline marginal utilities. Combining this strategy with independent and identically distributed (IID) Gumbel stochastic terms in the utility functions results in a closed form likelihood expression that is not much more difficult compared to that of the traditional MDCEV model. Simulation experiments conducted to verify the proposed formulation demonstrate that the model parameters can be retrieved accurately and precisely using the maximum likelihood technique. The proposed framework is applied for an empirical analysis of episode-level activity participation and time allocation behavior of non-working adults in Los Angeles, California. The empirical OMDCEV model provided better fit and better predictive accuracy (in both estimation and validation datasets) than a traditional MDCEV model applied at an episode-level.

Keywords: *multiple discrete-continuous choice models, conditional likelihood, time use, episode-level activity generation.*

1. INTRODUCTION

Many consumer decisions involve discrete choice alternatives that are not perfect substitutes in that the consumer can potentially choose multiple alternatives as opposed to a single alternative. In addition, consumers also decide how much to consume for each chosen alternative. Such choice situations, labeled multiple discrete continuous (MDC) choice situations (Dubé, 2004; Bhat, 2005) have been recognized and analyzed in many different empirical contexts, including individual activity participation and time-use, household vehicle ownership and utilization, and consumer brand choice and purchase quantity.

The primary approach to modelling MDC choices is based on random utility maximization (RUM), where consumer choices arise from maximizing an increasing and continuously differentiable non-linear utility function of consumption, subject to budget constraints. The non-linear utility function is specified such that it accommodates decreasing marginal utility while also allowing zero optimal consumptions. Doing so allows imperfect substitutes in that choice of multiple alternatives (as opposed to a single alternative) might lead to optimal utility. At the same time, the utility structure allows zero consumptions so that not all goods need to be in the optimal consumption bundle. This approach may be traced back to the seminal work of Wales and Woodland (1983) (also see Hanemann, 1978). A notable formulation of this approach is the Multiple Discrete Continuous Extreme Value (MDCEV) model of Bhat (2005) that employs log extreme value distributions in the utility formulation to derive compact, closed form likelihood expression that subsumes the multinomial logit model as a special case. Further, a box-cox utility formulation of Bhat (2008) subsumes most other additively separable utility forms used in the literature, such as those in Kim *et al.* (2002) and von Haefen and Phanuef (2005), as special cases.

The basic random utility maximization approach to modeling MDC choices has been extended in many ways, such as to incorporate: (a) flexible stochastic specifications (Pinjari and Bhat, 2009; Pinjari, 2011; Bhat *et al.*, 2013; Wang *et al.*, 2017) (b) non-additively separable and other flexible utility profiles (Lavín and Hanemann, 2008; Bhat *et al.*, 2015; Bhat, 2018), and (c) and multiple linear budgets (Satomura *et al.*, 2011; Castro *et al.*, 2012). These modeling frameworks have been applied in a wide variety of empirical contexts including activity participation and time-use analysis (Bhat, 2005; Habib and Miller, 2009; Chikaraishi *et al.*, 2010; Pinjari *et al.*, 2016; Calastri *et al.*, 2017; Enam *et al.*, 2018), household-level mode choice and expenditure (Rajagopalan and

Srinivasan, 2008), and vehicle ownership and usage (You *et al.*, 2014; Bhat *et al.*, 2009; Jäggi and Axhausen, 2013).

1.1 Research Gap

An important limitation of most RUM-based MDC models, despite the advances made in the field, is the inability to analyze consumption considering a disaggregate categorization of alternatives that includes the number of instances of consumption of an alternative, along with the amount of consumption at each instance of choice of that alternative. In most MDC model formulations thus far, the analysis, be it time allocation across activities, expenditure allocation across goods, or any other resource allocation, has always pertained to aggregate allocation across all instances of consumption of each choice alternative. However, the number of times an alternative is chosen and the amount of consumption at each instance of choice is not considered in the analysis. For example, in time-use applications of most MDC models, individuals' daily time allocation across different types of activities is modelled while ignoring the number of episodes of each activity-type. That is, the current MDC modeling frameworks for time-use analysis do not consider in the model structure, time allocation at an activity-episode level. Therefore, such MDC modeling frameworks cannot be used to analyze or forecast the specific activity-episodes one participates in on a day, but only the total time allocated across all episodes of each activity on the day.[1] For many applications, however, it is useful to model and forecast time allocations at an activity episode level (instead of aggregate, activity[2] level). For example, it is more useful to model eating out activity generation and time allocation at an episode level instead of modeling all the time allocated to eating out in a given time period such as a day or a week. Doing so helps in understanding the factors influencing the generation and duration of activity-episodes (rather than an aggregate effect on time allocation to all episodes of an activity in a day). Further, knowledge of all activity-episodes generated in a day along with their duration helps in avoiding additional steps to model/forecast the episode-level activity generation and durations.

[1] Similarly, in the context of household vehicle fleet composition and usage, the current MDC frameworks do not acknowledge the possibility that a household can own multiple vehicles of a same type, and model the total mileage accrued on all household vehicles of a type rather than allocating mileage to each separate vehicle (You *et al.*, 2014).

[2] Note that the term 'activity' is sometimes used in this paper to refer to 'activity-type', especially when we refer to activity level time allocation.

1.2 Existing Approaches

To be sure, the above shortcoming has been addressed to some extent in different ways. For example, in modeling individual activity participation and time-use, one may define choice alternatives as combinations of activity-type and discrete time-of-day categories (Rajagopalan *et al.*, 2009). This allows one to model the occurrence and duration of a same activity-type across different time-of-day periods but does not account for multiple occurrences/episodes of a given activity-type within the same time-of-day period. A similar approach models frequency of occurrence of different activities based on location of occurrence of that activity and is used to analyze children's after school out-of-home activity patterns (Paleti *et al.*, 2011). The choice alternatives are defined as a combination of activity-type and location of occurrence. However, both these approaches do not help in understanding and predicting the frequency (or number of episodes) of an activity within a given time-of-day or spatial location. Another approach has been to define choice alternatives as combination of activity-type and with whom one takes part in the activity. Again, the model would be agnostic to the number of times a joint activity is undertaken with a same accompanying person.

In another line of research, Garikapati *et al.* (2014) integrate a multivariate count data model system with an MDCEV model of household vehicle ownership and usage. Specifically, the MDCEV component is used to model the ownership of and mileage accrual on a broadly categorized portfolio of vehicles whereas the multivariate count system is used to model the number of vehicles owned of each type. This framework, however, does not model the mileage on each vehicle within a given vehicle type.

1.3 Contribution of this Paper – A Novel OMDCEV Model

In view of the issues discussed above, we propose a new modeling framework to analyze MDC choices at a disaggregate level, including the number of instances different alternatives are chosen and the amount of consumption at each instance of choice. The model is formulated for disaggregate, episode-level analysis of individuals' time-use, where all the episodes of each activity-type an individual undertakes in a day are analyzed along with the duration of each episode. To do so, the model formulation starts with defining choice alternatives as episodes of different activity-types (*i.e.*, activity-type and episode number forms a choice alternative), so that time-allocation can be modeled at an episode level. As importantly, the formulation also ensures that a higher-numbered episode of an activity-type does not occur without the occurrence of all lower-numbered episodes of that activity-type (*i.e.*, j^{th} episode does of an activity-type does not occur without the $(j\text{-}1)^{th}$ episode occurring). Accommodating such conditions via explicit constraints in the utility maximization framework is difficult, for it leads to mixed integer programs that cannot be solved analytically. To circumvent this problem, we exploit the properties of MDC choice models with additive utility functions to condition on the baseline utility parameters. Specifically, instead of explicit constraints on time allocations of different episodes of an activity-type, we impose non-increasing ordering conditions on the baseline marginal utility functions of all episodes of that activity-type; with that of the first episode being first in the order, followed by the second episode, and so on. The reason for imposing such ordering conditions is to ensure a logical ordering of episodes. That is, when used in models with additively separable utility forms, these ordering conditions ensure that a j^{th} episode of an activity-type does not occur without the occurrence of the $(j\text{-}1)^{th}$ episode.

Although conceptually simple, the methodological difficulty of imposing ordering conditions across baseline marginal utility parameters of different episodes of an activity is that the parameters are typically specified as unbounded stochastic distributions. And it is difficult to ensure ordering across different unbounded distributions. To address this issue, we develop a conditional likelihood function, where the likelihood of the random utility maximization model is conditioned on the above discussed ordering of baseline marginal utilities. As a result, as demonstrated later in the paper, the model structure is parsimonious and leads to closed-form likelihood expressions under the assumption of IID type-1 extreme value distributions typically used for MDCEV models. In fact, it so happens that the resulting likelihood function is a ratio of the likelihood of the traditional MDCEV model to that of the rank-ordered logit functions for the ordering of baseline marginal utility functions of the chosen episodes of each activity-type. Such a likelihood expression is not any more difficult to understand and code for estimation purposes than that of the traditional MDCEV model. Further, it is straight forward to extend the model to include mixing distributions for correlations among choice alternatives and/or random coefficients. Besides, the proposed modeling framework is not only useful for modeling episode-level activity participation and time-use but also applicable to other MDC choices such as vehicle ownership and usage where it is of interest to analyze the number of instances an alternative type is chosen, along with the amount of consumption at each instance of choice of that alternative.

To be sure, in a recent conference presentation, Palma *et al.* (2019) propose a model of time-use by defining choice alternatives as disaggregate, activity-episodes (instead of aggregate, activity-types) to model the generation of episodes of different types of activities along with their duration. To accommodate logical ordering of occurrence of episodes of any activity-type (*i.e.*, for a higher-numbered episode not to occur without the occurrence of a lower-numbered episode) they allow the deterministic component of the baseline marginal utility of a higher-numbered episode of that activity-type to be lower than that of the lower-numbered episodes of that same activity-type by introducing a negative penalty term for the higher-numbered episode in the utility profile. Doing so simplifies the model structure to one that is similar to the original MDCEV formulation of Bhat (2008), except that the choice alternatives are at an activity-episode level, instead of at the activity-level. While this is a helpful empirical strategy to be able to forecast time allocation at the disaggregate level of activity-episodes, the formulation does not ensure that j^{th} episode of an activity-type does not occur without the occurrence of $(j\text{-}1)^{th}$ episode of the same activity-type unless the stochastic terms in the baseline utility of the j^{th} and $(j\text{-}1)^{th}$ activity-episode are either same or appropriately truncated. In our paper, we propose a variant of the MDCEV model that allows the stochastic distributions of the baseline marginal utilities for different episodes of an activity-type to be different from each other but are in a non-increasing order from the first episode to the last episode (more later).

The remainder of the paper is organized in the following manner. Section 2 discusses the proposed model structure along with the derivation of the closed-form likelihood expression for the proposed model for time use analysis at an activity-episode level. We also present a procedure to apply the model to forecast activity participation and time allocation at an episode level. Section 3 presents a simulation analysis of the proposed model, where synthetic data generated from the proposed model is used to evaluate the accuracy and precision with which one can retrieve the model parameters. In addition, the simulations are used to demonstrate the proposed model's property that it does not predict episodes of an activity without a logical ordering. Section 4 presents an empirical application of the proposed model for analyzing individuals' disaggregate, episode-level activity participation and time allocation across different activity-types. The empirical analysis, carried out using data from the South California region, which involves model estimation and interpretation as well as a k-fold validation exercise on set-aside samples, demonstrates that the proposed model provides better performance than a model that ignores logical ordering of choice alternatives. The fifth section concludes the paper with discussion and avenues for future research. In addition, an appendix is presented with important derivations for the likelihood expression of the proposed model.

2. METHODOLOGY

In this section, beginning with the notation preliminaries, we first outline an MDC choice model with activity-episodes as choice alternatives and then enhance it to ensure a logical ordering of episodes. We also outline a procedure to apply the proposed model for forecasting and simulation.

2.1. Notation and Preliminaries

Let k be an index for activity-type, with $\mathbb{K}$ denoting the set of activity-types ($\mathbb{K} = \{1,2,3 \dots. K\}$) a person can take part in and K denoting total number of activities (or activity-types). Let the first of these activities ($k = 1$) be the essential Hicksian outside good that includes time allocation to all activities other than those of interest for our analysis. Time allocation to the remaining activities ($k \in \{2,3 \dots K\}$) is of interest to the analyst. Let us not divide the first activity into multiple episodes. That is, all the time allocated to the first activity is assumed to be in a single episode.[3] However, let each other k^{th} ($k \in \{2,3 \dots K\}$) activity have a maximum number of J_k episodes (and let $j_k \in \mathbb{J}_{\mathbb{k}} : \mathbb{J}_{\mathbb{k}} = \{1,2, \dots J_k\}$). Note that the terms activity and episode are <u>not</u> used interchangeably. Each choice alterative is defined by an activity and episode combination. Specifically, (k, j_k) is an index for the ${j_k}^{th}$ episode of activity k, for all $j_k \in \mathbb{J}_{\mathbb{k}}$ and $k \in \mathbb{K}$.

Let m be an index for the chosen activities, with $\mathbb{M}$ denoting the set of chosen activities, and M denoting the total number of chosen activities. Let m' be the be an index for non-chosen activities, with $\mathbb{M}'$ denoting the set of non-chosen activities. Without loss of generality, let us assume that the first M of the K activities are the chosen activities (*i.e.*, $\mathbb{M} = \{1,2,3 \dots. M\}$) and that the remaining are the non-chosen activities (*i.e.*, $\mathbb{M}' = \{M+1, M+2, \dots K\}$).

For each chosen activity m, let i_m ($i_m \in \mathbb{I}_{\mathbb{m}}$: $\mathbb{I}_{\mathbb{m}} = \{1,2,3 \dots I_m\}$) be the index for chosen episodes while ${i'}_m$ (${i'}_m \in \mathbb{I}'_{\mathbb{m}}$; $\mathbb{I}'_{\mathbb{m}} = \{I_m + 1, I_m + 2, \dots J_m\}$) be the index for non-chosen

[3] In this specification, the budget allocation for the essential Hicksian outside good (*i.e.*, alternative 1) is assumed to occur at an aggregate level (that is, no multiple activity-episodes for the outside good). In notation, it would mean that $J_1 = 1$. Extension of this formulation to accommodate multiple occurrences for the outside good is straight forward.

episodes. Similarly, for each non-chosen activity m', all episodes are represented by $i'_{m'}$ ($i'_{m'} \in \mathbb{I}'_{\mathbb{m}'} : \mathbb{I}'_{\mathbb{m}'} = \{1,2,3 \ldots J_{m'}\}$). Note that none of the episodes of non-chosen activities is chosen.

For the reader's convenience, all the above defined indices are described in Table 1 below. Other notation used in the model formulation is described later during the formulation.

Table 1. Indices used in the model formulation

Notation	Description
$k \in \mathbb{K}: \mathbb{K} = \{1,2,3 \ldots . K\}$	k is an index for activity-type. $\mathbb{K}$ denotes the set of activity-types a person can take part in. K denotes total number of activities. Note: For brevity, we use the term activity interchangeably with activity-type.
$j_k \in \mathbb{J}_{\mathbb{k}}: \mathbb{J}_{\mathbb{k}} = \{1,2, \ldots J_k\}$	j_k is an index for episode number of activities of type k. $\mathbb{J}_{\mathbb{k}}$ denotes the set of episodes one can participate in activities of type k. J_k denotes the maximum number of episodes possible for activity k .
(k, j_k) $\forall$ $j_k \in \mathbb{J}_{\mathbb{k}}$ and $k \in \mathbb{K}$	(k, j_k) is an index for the ${j_k}^{th}$ episode of activity k.
$m \in \mathbb{M}: \mathbb{M} = \{1,2,3 \ldots . M\}$	m is an index for activity-types that are allocated some time (*i.e.*, chosen activities). $\mathbb{M}$ denotes the set of chosen activities. M denotes total number of chosen activities. Without loss of generality, one can assume that the first M of the K activities are chosen.
$m' \in \mathbb{M}': \mathbb{M}' = \{M+1, M+2, \ldots K\}$	m' is an index for activity-types that are not allocated any time. (*i.e.*, non-chosen activities). $\mathbb{M}'$ is the set of non-chosen activities.
$i_m \in \mathbb{I}_{\mathbb{m}}: \mathbb{I}_{\mathbb{m}} = \{1,2,3 \ldots I_m\}$	$\mathbb{I}_{\mathbb{m}}$ is the set of chosen episodes (*i.e.*, episodes with positive time allocation) of a chosen activity m. I_m is the number of chosen episodes of a chosen activity m. i_m denotes the episode number of the chosen episode for a chosen activity m. Therefore, the index (m, i_m) is used to denote the ${i_m}^{th}$ episode of a chosen activity m.
$i'_m \in \mathbb{I}'_{\mathbb{m}}: \mathbb{I}'_{\mathbb{m}} = \{I_m+1, I_m+2, \ldots J_m\}$	$\mathbb{I}'_{\mathbb{m}}$ is the set of non-chosen episodes (*i.e.*, episodes with zero time allocation) of a chosen activity m. i'_m denotes the episode number of the non-chosen episodes of a chosen activity m (*i.e.*, non-chosen episodes of chosen activities). Therefore, the index (m, i'_m) is used to denote the ${i'_m}^{th}$ episode of a chosen activity m.
$i'_{m'} \in \mathbb{I}'_{\mathbb{m}'} ; \mathbb{I}'_{\mathbb{m}'} = \{1,2,3 \ldots J_{m'}\}$	$\mathbb{I}'_{\mathbb{m}'}$ is the set of episodes of a non-chosen activity m'; all these episodes would have zero time allocation. $i'_{m'}$ denotes episode numbers for non-chosen activities ($m' = \mathrm{M}+1, \mathrm{M}+2, \mathrm{M}+3 \ldots \mathrm{K}$). Therefore, the index $(m', i'_{m'})$ is used to denote the ${i'_{m'}}^{th}$ episode of a non-chosen activity m'.

2.2 Activity Episode-level Model Formulation

The proposed model assumes, similar to most time-use models in the literature, that individuals make their activity participation and time allocation decisions to maximize the utility derived from their time allocation subject to a time budget constraint. However, the choice alternatives in the model are disaggregate, activity-episodes, as opposed to activities. Therefore, we specify an additively separable non-linear utility function that is a sum of utility derived from the time spent in each episode. Specifically, we employ a linear expenditure system (LES) version of Bhat's (2008) MDCEV utility form, as below:

$$U(\boldsymbol{t}_q) = \psi_{q1} \ln(t_{q1}) + \sum_{k=2}^{K} \sum_{j_k=1}^{J_k} \psi_{qk,j_k} \gamma_{qk} \ln\left(\frac{t_{qk,j_k}}{\gamma_{qk}} + 1\right) \tag{1}$$

In the above expression, $U(\boldsymbol{t}_q)$ is the total utility that an induvial q accrues from spending a non-negative amount of time in each of the possible J_k episodes of each of the K activities. $\boldsymbol{t}_q$ is a vector of time allocations $t_{qk,j_k} \forall\, j_k \in \mathbb{J}_{\mathbb{k}}$ and $k \in \mathbb{K}$, where t_{qk,j_k} is the time allocation by individual q to ${j_k}^{th}$ episode of the k^{th} activity. The right hand side of Equation (1) is a valid utility function if the parameters of the utility function are such that $\psi_{q1} > 0$, $\psi_{qk,j_k} > 0$, and $\gamma_{qk} > 0 \;\forall\, j_k \in \mathbb{J}_{\mathbb{k}}$ and $k \in \mathbb{K}/\{1\}$. Each of these parameters is discussed next.

ψ_{q1} is the utility parameter of the essential outside activity. This parameter (in conjunction with other parameters in the model) influences how much time is allotted to the outside activity vis-à-vis all other activities ($k = 2,3, \dots K$) of interest to the analyst. ψ_{qk,j_k} is the baseline marginal utility parameter for ${j_k}^{th}$ episode of the k^{th} activity, Mathematically, it is the marginal utility of ${j_k}^{th}$ episode of the k^{th} activity at zero time allocation to the episode. This parameter influences the discrete decision of whether to allocate time to the corresponding episode as well as influences the amount of time allotted to the episode. That is, *ceteris paribus*, activity-episodes with a greater value of baseline marginal utility are more likely to be chosen as well as allotted more time than other activity-episodes. Further, to accommodate heterogeneity in individuals' preferences for different activity-episodes, the baseline marginal utility parameter may be parameterized as $\psi_{qk,j_k} = \exp(\beta' z_{qk,j_k} + \varepsilon_{qk,j_k})$, where z_{qk,j_k} are the individual q's attributes influencing ψ_{qk,j_k} and ε_{qk,j_k} is a random term to recognize unobserved factors that influence individual q's preferences. Similarly, ψ_{q1} is expressed as $\psi_{q1} = \exp(\varepsilon_{q1})$. [4]

[4] Note that observed variables do not enter the expression for ψ_{q1} for identification purposes arising from an exogenously available time budget constraint that influences the time allocation decisions. That is, the time allocation to outside activity need not be estimated if the available time budget is known and the time allocation to all other activity-episodes are estimated.

γ_{qk} is the satiation parameter for any episode of activity k and influences the amount of time an individual q allocates to episodes of that activity. *Ceteris paribus*, activity-episodes with higher γ_{qk} values have lower rate of satiation, *i.e.*, greater time allocation than those with lower γ_{qk} value. It is worth noting here that the satiation parameter (γ_{qk}) is assumed to be same across different episodes of an activity-type (k), while it can be different across different activity-types. While this assumption can be argued to the extent that the satiation effects across different episodes of an activity may not be same, this is an innocuous assumption since each episode is of a same activity-type. Besides, differences in baseline utilities (ψ_{qk,j_k}) across different episodes of an activity help in allowing for different durations across episodes. Moreover, as discussed later in Section 2.3.4, having a same satiation parameter (γ_{qk}) across different episodes of an activity helps with maintaining tractability of the model and its estimation.

Next, as with most MDC models in the literature, it is assumed that individuals are subject to an exogenously given time budget T_q, such that,

$$\sum_{k=1}^{K}\sum_{j=1}^{J_k} t_{qk,j_k} = T_q \tag{2}$$

In addition, the model formulation ought to recognize non-negativity constraints on time allocations: $t_{qk,j_k} \geq 0, \forall\, j_k \in \mathbb{J}_{\mathbb{k}}$ and $k \in \mathbb{K}/\{1\},$ while the utility formulation ensures $t_{q1} > 0$.

The individual's utility maximization problem may be solved by setting up the Lagrangian and deriving the Karush-Kuhn-Tucker (KKT) conditions of optimality. The Lagrangian for the above problem may be written as:

$$L_q = \psi_{q1}\, ln\big(t_{q1}\big) + \sum_{k=2}^{K}\sum_{j_k=1}^{J_k} \psi_{qk,j_k}\gamma_{qk}\, ln\left(\frac{t_{qk,j_k}}{\gamma_{qk}} + 1\right) - \lambda_q\left(\sum_{k=1}^{K}\sum_{j=1}^{J_k} t_{qk,j_k} - T_q\right) \tag{3}$$

where, λ_q is the Lagrange multiplier for the binding budget constraint. The optimal time allocations satisfy the following KKT conditions of optimality for all non-essential choice alternatives:

$$\frac{\psi_{qk,j_k}}{\left(\frac{t^*_{qk,j_k}}{\gamma_{qk}} + 1\right)} = \lambda_q \;\; if\; t^*_{qk,j_k} > 0\; \forall\, j_k \in \mathbb{J}_{\mathbb{k}} \text{ and } k \in \{2,3,\ldots,K\} \tag{4}$$

and,

$$\frac{\psi_{qk,j_k}}{\left(\frac{t^*_{qk,j_k}}{\gamma_{qk}}+1\right)} < \lambda_q \;\; if \; t^*_{qk,j_k} = 0 \; \forall \; j_k \in \mathbb{J}_{\mathbb{k}} \text{ and } k \in \{2,3,\dots,K\} \tag{5}$$

Next, alternative 1, being the essential Hicksian outside good, is always allocated some non-zero consumption value. Thus, on account of $t^*_{q1} > 0$, the optimality condition for the outside good is:

$$\frac{\psi_{q1}}{t^*_{q1}} = \lambda_q \tag{6}$$

One may substitute Equation (6) into Equations (4) and (5) then take logarithms to rewrite the KKT conditions as below:

$$\begin{aligned} V_{qk,j_k} + \varepsilon_{qk,j_k} &= V_{q1} + \varepsilon_{q1} \; if \; t^*_{qk,j_k} > 0 \; \forall \; j_k \in \mathbb{J}_{\mathbb{k}} \,, k \in \{2,3,\dots,K\} \\ V_{qk,j_k} + \varepsilon_{qk,j_k} &< V_{q1} + \varepsilon_{q1} \; if \; t^*_{qk,j_k} = 0 \; \forall \; j_k \in \mathbb{J}_{\mathbb{k}} \,, k \in \{2,3,\dots,K\} \end{aligned} \tag{7}$$

where, $V_{qk,j_k} = \beta' \, z_{qk,j_k} - \ln\left(\frac{t^*_{qk,j_k}}{\gamma_{qk}}+1\right) \forall \; j_k \in \mathbb{J}_{\mathbb{k}} \,, k \in \{2,3,\dots,K\}$; and $V_{q1} = -\ln\left(t^*_{q1}\right)$.

These stochastic KKT conditions may be used to formulate the likelihood of observed time allocations. The specific form of the likelihood function depends on the distribution of the stochastic terms (ε_{qk,j_k}). For example, assuming the stochastic terms as independent and identically (*iid*) Gumbel distributed across choice alternatives and individuals gives rise to the standard MDCEV model of Bhat (2008), with choice alternatives as activity-episodes.

2.3 Activity Episode-level Model Formulation with Logical ordering of Activity-Episodes (The OMDCEV Model)

Defining the time-use choice alternatives as activity-episodes is a means to model time-use at an episode level and, thereby, implicitly model the frequency of occurrence of each activity, along with the episode-level durations of each activity. It is important to note, however, that the model formulation in the earlier section does not ensure a logical ordering of episodes for consistency in prediction of different episodes of an activity. That is, the model does not ensure that a higher-numbered episode of an activity should not occur without the occurrence of lower-numbered episodes of that activity. For example, the formulation does not preclude the possibility of predicting a 3rd episode of an activity without predicting the occurrence of 1st and 2nd episodes. To address this issue, the formulation ought to incorporate a labeling system for different episodes of an activity as well as ensure consistency in prediction. Both these aspects are discussed next, along with the formulation of a model and the derivation of its likelihood function. We label such a model that respects ordering of episodes as the ordered-MDCEV (OMDCEV) model.

2.3.1 Episode labeling/numbering

It is important that the episodes are labeled according to a logical criterion and the same labeling system is used for all activities and all individuals. If the episodes of an activity are not labelled in a consistent manner across all individuals (*i.e.*, if arbitrary labelling that is inconsistent across different individuals is employed), it will be difficult to estimate the model parameters.

Different approaches may be used to label the episodes of an activity. One approach is to label them based on attributes that differentiate one episode from another. For example, one can use time-of-day, location, etc. or a combination of these attributes to label and differentiate the episodes of an activity. As discussed earlier, doing so does not help in recognizing if multiple episodes of an activity occur for a single time-of-day period, location, or a combination of these attributes. The second approach is to use chronology of occurrence to differentiate and label the episodes. Note, however, that the objective of most MDC modelling applications to time-use analysis is activity generation (*i.e.*, analysis of what activities are undertaken and for how long), not activity scheduling (*i.e.*, when and in what sequence are activities undertaken).[5]

In view of the above discussion, the activity-episodes in our model ought to be agnostic to chronology, time-of-day of occurrence, or other attributes of the episodes. Instead, we label the episodes of an activity in accordance with the non-increasing order of durations of the episodes. That is, the longest duration episode of an activity is denoted the first episode, the second longest as second, and so on. This means that, for any activity k, the episode numbers ($j_k \in \{1,2, \ldots J_k\}$) are defined based on a non-ascending order of episode time-durations, such that:

$$t_{qk,1} \geq t_{qk,2} \geq \cdots \geq t_{qk,J_k} \; \forall \; k \in \{2,3, \ldots, K\} \tag{8}$$

[5] Indeed, most time-use and activity-based approaches to modeling travel focus on activity generation first followed by activity and travel scheduling; albeit there are a multitude of variants in which activity scheduling models are structured to influence activity generation. This is not to say that the chronology of occurrence of activities is not important. Should the analyst prefer to consider chronology alongside activity-episode generation, a different modeling framework must be adopted.

Defining episodes in this particular manner does not lose generality from a standpoint of activity-episode generation and duration modeling. This is because ordering activity-episodes by their duration allows the analyst to label the episodes without having to rely on other characteristics such as time-of-day or location. Besides, doing so, in combination with the fact that the γ_k parameter is same across different episodes of an activity k helps in maintaining tractability of the model and its estimation.

2.3.2 Consistency in prediction of episodes

Recall that the model should not allow a higher-numbered episode of an activity to occur without the occurrence of lower-numbered episodes of that activity. There are two ways to do so. One way is to explicitly incorporate the following set of constraints on time allocations in the model (these are in addition to the budget constraint and non-negativity constraints on time allocations):

$$t_{qk,(j_k+1)} = 0 \; if \; t_{qk,j_k} = 0 \; \forall \; k = \{2,3 \ldots, K\}, j_k \in \{1,2,3, \ldots J_k - 1\} \tag{9}$$

Accommodating the above constraints, however, complicates the utility maximization problem, as it requires mixed-integer programming techniques, and makes it difficult to derive the model's likelihood function. An alternative to such explicit constraints in the model is to express the utility function such that the individuals' preferences automatically ensure that no individual chooses a higher-numbered episode of an activity without choosing a lower-numbered episode. That is, to ensure that $(j_k + 1)^{th}$ episode is not chosen without the $j_k{}^{th}$ episode being chosen, we impose the following ordering condition on the baseline marginal utility parameters:

$$\psi_{qk,j_k} \geq \psi_{qk,(j_k+1)} \; \forall \; k \in \{2,3, \ldots, \mathrm{K}\}, j_k \in \{1,2,3, \ldots J_k - 1\} \tag{10}$$

To understand why imposing the above ordering on the baseline marginal utility parameters prevents the model from predicting the occurrence of a higher-numbered episode without the occurrence of a lower-numbered episode, it is useful to recall the work of Pinjari and Bhat (2011), who highlight an important property of MDC choice models with additively separable utility functions. Specifically, using the KKT conditions of optimality (as those in in Equations (4), (5), and (6)), they show that the baseline marginal parameter of any chosen alternative (*i.e.*, an episode with non-zero time allocation, other than the outside good) is greater than the Lagrange multiplier (λ_q), while that of the non-chosen activity-episodes is less than the Lagrange multiplier. This property implies that, if for a given activity k, an i^{th} episode is chosen, then all episodes j :

$\psi_{qk,j} \geq \psi_{qk,i}$ will also be chosen. Conversely, if we impose an ordering condition that $\psi_{qk,j_k} \geq \psi_{qk,(j_k+1)}$ $\forall$ $k \in \{2,3, \ldots \mathrm{K}\}, j_k \in \{1,2,3, \ldots J_k - 1\}$), then there would be no occurrence of a higher-numbered episode without that of a lower-numbered episode.

Although our proposed approach of imposing ordering conditions on the baseline utility parameters (as in Equation (10)) obviates the need for explicit constraints on time allocations (as in Equation (9)), a difficulty with our approach arises from the fact that the baseline marginal utility parameters are typically specified as unbounded stochastic distributions (such as log-extreme value or log-normal distributions). And it is difficult to impose ordering on parameters with unbounded distributions. To address this problem, we propose a conditioning method, where the stochastic KKT conditions obtained from the individuals' utility maximization problem (Equations (4), (5), and (6)) are conditioned on the ordering in Equation (10). That is, instead of deriving a likelihood function for the stochastic KKT conditions without regard to ordering of baseline marginal utility parameters, we derive a conditional likelihood function of the KKT conditions – conditioned on the ordering inequalities in Equation (10). An interpretation of such conditioning is that the distribution of baseline marginal utility parameters influencing the occurrence of each episode of an activity k is right-truncated by that of the previous episode (because we are conditioning on the inequality that $\psi_{qk,j_k} \geq \psi_{qk,(j_k+1)}$). In the next sub-section, we derive the conditional likelihood functions for use in parameter estimation.

Before proceeding further, note that the proposed utility maximization formulation with ordering conditions is not mathematically equivalent to the utility maximization model with explicit constraints on time allocations. However, the proposed alternative is much easier than the formulation with explicit constraints on time allocations as the latter entails mixed integer programs that are not easy to solve, thereby complicating the model formulation, estimation as well as application. Besides, with the baseline marginal utility parameters of the different episodes of an activity in a non-increasing order (after the condition that $\psi_{qk,j_k} \geq \psi_{qk,(j_k+1)}$) combined with the fact that the γ_{qk} parameters are same across all episodes of an activity, our model ensures ordering in time allocations across different episodes of an activity (as in Equation (9)).

2.3.3 Derivation of the likelihood expression for the OMDCEV model

Suppressing the index for induvial q for ease in notation, rewrite the KKT conditions in Equations (4), (5), and (6) as:

(a) For chosen episodes of chosen activities (m, i_m) other than outside good,

$$\frac{\psi_{m,i_m}}{\left(\frac{t^*_{m,i_m}}{\gamma_m}+1\right)} - \lambda = 0 \;\forall\, m \in \mathbb{M}\backslash\{1\}, i_m \in \mathbb{I}_{\mathbb{m}} \tag{11}$$

(b) For non-chosen episodes of the chosen activities (m, i'_m), which have zero time allocation,

$$\frac{\psi_{m,i'_m}}{\left(\frac{t^*_{m,i'_m}}{\gamma_m}+1\right)} - \lambda < 0 \;\Rightarrow\; \psi_{m,i'_m} < \lambda \;\forall\, m \in \mathbb{M}\backslash\{1\}, i'_m \in \mathbb{I}'_{\mathbb{m}} \tag{12}$$

(c) For episodes of non-chosen activities $(m', i'_{m'})$, all of which have zero time allocation,

$$\frac{\psi_{m',i'_{m'}}}{\left(\frac{t^*_{m'i'_{m'}}}{\gamma_{m'}}+1\right)} - \lambda < 0 \Rightarrow\; \psi_{m',i'_{m'}} < \lambda \;\forall\, m' \in \mathbb{M}', i'_{m'} \in \mathbb{I}'_{\mathbb{m}'} \tag{13}$$

(d) For the outside good,

$$\frac{\psi_1}{t^*_1} = \lambda \tag{14}$$

Next, the ordering conditions in Equation (10) are expressed for different activity episode pairs, as below:

(e) For chosen episodes of chosen activities other than outside good,

$$\psi_{m,i_m} > \psi_{m,i_m+1} \;\forall\, m \in \mathbb{M}\backslash\{1\}, i_m \in \mathbb{I}_{\mathbb{m}}\backslash\{I_m\} \tag{15}$$

(f) For I^{th}_m chosen episode of every chosen activity (except outside good) and the next, non-chosen (I^{th}_{m+1}) episode of that activity,

$$\psi_{m,I_m} > \psi_{m,I_m+1} \forall\, m \in \mathbb{M}\backslash\{1\} \tag{16}$$

(g) For non-chosen episodes of chosen activities other than outside good,

$$\psi_{m,i'_m} > \psi_{m,i'_m+1} \;\forall\, m \in \mathbb{M}\backslash\{1\}, i'_m \in \mathbb{I}'_{\mathbb{m}}\backslash\{J_m\} \tag{17}$$

(h) For episodes of non-chosen activities (m', i'_m), all of which have zero time-allocation,

$$\psi_{m',i'_{m'}} > \psi_{m',i'_{m'}+1} \; \forall \, m' \in \mathbb{M}', i'_{m'} \in \mathbb{I}'_{\mathbb{m}'} \setminus \{J_{m'}\} \tag{18}$$

Without loss of generality, let the first M of the K activities be the chosen activities, including the outside good. For each chosen activity other than the outside good (*i.e.*, $m \in \mathbb{M} \setminus \{1\}$), let the first I_m episodes be the chosen episodes. The likelihood that t_1^* time is allotted to the outside good, t_{m,i_m}^* amount of time is allotted to each of the chosen episodes of chosen activities, and zero time allocated to all other episodes is given by:

$$\begin{aligned} &\mathcal{L}\{t_1^*, \ldots, (t_{m,1}^*, t_{m,2}^*, \ldots, t_{m,I_m}^*), \ldots, (t_{M,1}^*, \ldots, t_{M,I_M}^*), 0, \ldots, 0\} \\ &\qquad\qquad = \mathcal{L}(KKT\ conditions | Ordering\ conditions) \end{aligned} \tag{19}$$

In the above equation, $\mathcal{L}(KKT\ conditions | Ordering\ conditions)$ is the conditional likelihood of the KKT conditions in Equations (11) through (14) conditioned on the ordering conditions in Equations (15) through (18).

Now, we expand the conditional likelihood $\mathcal{L}(KKT\ conditions | Ordering\ conditions)$. To do so, we first rewrite it as:

$$\begin{aligned} &\mathcal{L}(KKT\ conditions | Ordering\ conditions) \\ &\qquad\qquad = \frac{\mathcal{L}(KKT\ conditions\ \boldsymbol{AND}\ Ordering\ conditions)}{\mathcal{L}(Ordering\ conditions)} \end{aligned} \tag{20}$$

The numerator in the above expression requires one to eliminate any redundancies among the KKT conditions in Equations (11) through (14) and the ordering conditions in Equations (15) through (18). First, the ordering conditions in Equation (16) become redundant because the KKT conditions in Equations (11) and (12) imply that $\psi_{m,I_m} > \psi_{m,I_m+1} \forall \, m \in \{2,3, \ldots, \mathrm{K}\}$. This is because the KKT conditions in Equation (11) imply $\psi_{m,i_m} > \lambda$ for all chosen activity-episodes and those in Equation (12) imply $\psi_{m,i'_m} < \lambda$ for all non-chosen episodes of chosen activities. As discussed in Pinjari and Bhat (2011), combining these two sets of inequalities results in $\psi_{m,i_m} > \lambda > \psi_{m,i'_m}$ for each chosen activity m. Next, the KKT conditions in Equation (12) and the ordering conditions in Equation (17) – both for non-chosen episodes of chosen activities – may be combined to result in the following ordering condition:

$$\lambda > \psi_{m,I_m+1} > \psi_{m,I_m+2} \ldots > \psi_{m,J_m} \forall \, m \in \mathbb{M} \setminus \{1\} \tag{21}$$

Similarly, the KKT conditions in Equation (13) and the ordering conditions in Equation (18) – both for episodes of non-chosen activities – may be combined to establish the following ordering:

$$\lambda > \psi_{m',1} > \psi_{m',2} > \cdots > \psi_{m',J_{m'}}, \forall\, m' \in \mathbb{M}' \tag{22}$$

Eliminating the above redundancies, one may rewrite the numerator in the likelihood expression of Equation (20) as the joint likelihood of conditions in Equations (11), (14), (15), (21), and (22). Accordingly, the likelihood expression in Equation (20) may be rewritten as:

$$\mathcal{L}(KKT\ conditions | Ordering\ conditions)$$
$$= \frac{\mathcal{L}\left(\begin{array}{c} \dfrac{\psi_{m,i_m}}{\left(\dfrac{t^*_{m,i_m}}{\gamma_m}+1\right)} = \dfrac{\psi_1}{t^*_1}\ \forall\, m \in \mathbb{M}\backslash\{1\}, i_m \in \mathbb{I}_{\mathbb{m}}\ , \\ \left(\psi_{m,1} > \psi_{m,2} \ldots > \psi_{m,I_m}\right) \forall\, m \in \mathbb{M}\backslash\{1\}, \\ \left(\dfrac{\psi_1}{t^*_1} > \psi_{m,I_m+1} > \psi_{m,I_m+2} \ldots > \psi_{m,J_m}\right)\ \forall\, m \in \mathbb{M}\backslash\{1\}, \\ \left(\dfrac{\psi_1}{t^*_1} > \psi_{m',1} > \psi_{m',2} > \cdots > \psi_{m',J_{m'}}\right) \forall\, m' \in \mathbb{M}' \end{array}\right)}{\mathcal{L}\left(\psi_{k,1} > \psi_{k,2} > \cdots > \psi_{k,j_k} \ldots > \psi_{k,J_k}\right) \forall\, k \in \mathbb{K}\backslash\{1\}} \tag{23}$$

Next, consider the consider the second condition, *i.e.*, $\left(\psi_{m,1} > \psi_{m,2} \ldots > \psi_{m,I_m}\right) \forall\, m \in \mathbb{M}\backslash\{1\}$ in the numerator of the above expression. For any two chosen episodes i_m and j_m such that $i_m < j_m$ of an activity m, the KKT conditions in Equation (11) imply that the marginal utilities of optimal time allocations are equal. Mathematically, this implies the following:

$$\frac{\psi_{m,i_m}}{\psi_{m,j_m}} = \frac{\left(\dfrac{t^*_{m,i_m}}{\gamma_m}+1\right)}{\left(\dfrac{t^*_{m,j_m}}{\gamma_m}+1\right)} = \frac{t^*_{m,i_m}+\gamma_m}{t^*_{m,j_m}+\gamma_m}\ \forall\, m \in \mathbb{M}\backslash\{1\}\ \&\ \forall\, (i_m\, , j_m \in \mathbb{I}_{\mathbb{m}} : i_m < j_m) \tag{24}$$

For forming the likelihood of observed data, where the observed durations of episodes are in a non-increasing order, the above equation implies that the baseline utility values must be in the same order. Specifically, since in the observed data $t^*_{m,i_m} > t^*_{m,j_m}$ and the satiation parameter γ_m in the model is same across different episodes of an activity, it automatically implies that $\psi_{m,i_m} > \psi_{m,j_m}$. Therefore, the second condition, *i.e.*, $\left(\psi_{m,1} > \psi_{m,2} \ldots > \psi_{m,I_m}\right) \forall\, m \in \mathbb{M}\backslash\{1\}$ in the numerator of Equation (23) becomes redundant. As a result, the proposed model's likelihood for an individual's observed time allocations may be written as:

$$\mathcal{L}\{t_1^*,..,(t_{m,1}^*,t_{m,2}^*,..,t_{m,I_m}^*),..,(t_{M,1}^*,..,t_{M,I_M}^*),0,..,0\}$$

$$= \frac{\mathcal{L}\left(\begin{array}{c} \dfrac{\psi_{m,i_m}}{\left(\dfrac{t_{m,i_m}^*}{\gamma_m}+1\right)} = \dfrac{\psi_1}{t_1^*}\ \forall\ m \in \mathbb{M}\backslash\{1\}, i_m \in \mathbb{I}_{\mathbb{m}}\ , \\ \left(\dfrac{\psi_1}{t_1^*} > \psi_{m,I_m+1} > \psi_{m,I_m+2} \dots > \psi_{m,J_m}\right)\ \forall\ m \in \mathbb{M}\backslash\{1\}, \\ \left(\dfrac{\psi_1}{t_1^*} > \psi_{m',1} > \psi_{m',2} > \cdots > \psi_{m',J_{m'}}\right) \forall\ m' \in \mathbb{M}' \end{array}\right)}{\mathcal{L}\left(\psi_{k,1} > \psi_{k,2} > \cdots > \psi_{k,j_k} \dots > \psi_{k,J_k}\right)\ \forall\ k \in \mathbb{K}\backslash\{1\}} \tag{25}$$

With the baseline marginal utilities parameterized as a function of observed covariates and unobserved factors (*i.e.*, $\psi_{k,j_k} = \exp\left(\beta'\, z_{k,j_k} + \varepsilon_{k,j_k}\right)$), the above likelihood function may be written as:

$$\mathcal{L}\{t_1^*,..,(t_{m,1}^*,t_{m,2}^*,..,t_{m,I_m}^*),..,(t_{M,1}^*,..,t_{M,I_M}^*),0,..,0\}$$

$$= \frac{\mathcal{L}\left(\begin{array}{c} \left(V_{m,i_m} + \varepsilon_{m,i_m}\ = V_1 + \varepsilon_1\right) \forall\ m \in \mathbb{M}\backslash\{1\}, i_m \in \mathbb{I}_{\mathbb{m}}\ , \\ \left(U_1 > \bar{U}_{m,I_m+1} > \bar{U}_{m,I_m+2} \dots > \bar{U}_{m,J_m}\right) \forall\ m \in \mathbb{M}\backslash\{1\}, \\ \left(U_1 > \bar{U}_{m',1} > \bar{U}_{m',2} > \cdots > \bar{U}_{m',J_{m'}}\right) \forall\ m' \in \mathbb{M}' \end{array}\right)}{\mathcal{L}\left(\bar{U}_{k,1} > \bar{U}_{k,2} > \cdots > \bar{U}_{k,j_k} \dots > \bar{U}_{k,J_{k'}}\right) \forall\ k \in \mathbb{K}\backslash\{1\}} \tag{26}$$

where, $V_{k,j_k} = \beta'_{\ k,j_k}\, z_{k,j_k} - \ln\left(\frac{t_{k,j_k}^*}{\gamma_k} + 1\right)$, $V_1 = -\ln(t_1^*)$, $U_1 = V_1 + \varepsilon_1$, $\bar{V}_{k,j_k} = \beta'_{\ k,j_k}\, z_{k,j_k}$ and $\bar{U}_{k,j_k} = \bar{V}_{k,j_k} + \varepsilon_{k,j_k}$. Assuming the stochastic terms ε_{k,j_k} to be *iid*-Gumbel distributed ($\varepsilon_{k,j_k} \sim iid\ Gumbel(0,\mu)$), the numerator in the above expression may be written as:

$$\begin{aligned} &\int_{\varepsilon_1=-\infty}^{\infty} \prod_{m=2}^{M} \prod_{i_m=1}^{I_m} \{g_\varepsilon\left(V_1 - V_{m,i_m} + \varepsilon_1\right)\} * |J| \\ &* \prod_{m=2}^{M} P\left(U_1 > \max_{i'_m \in \mathbb{I}'_{\mathbb{m}}} (\bar{U}_{m,i'_m})\right) * \prod_{m=2}^{M} \prod_{i'_m=I_m+1}^{J_m} \left(\frac{e^{\mu \bar{V}_{m,i_m}}}{\sum_{r=i'_m}^{J_m} e^{\mu \bar{V}_{m,r}}}\right) \\ &* \prod_{m'=M+1}^{K} P\left(U_1 > \max_{i'_{m'} \in \mathbb{I}'_{\mathbb{m}'}} (\bar{U}_{m',i'_{m'}})\right) * \prod_{m'=M+1}^{K} \prod_{i'_{m'}=1}^{J_{m'}} \left(\frac{e^{\mu \bar{V}_{m',i'_{m'}}}}{\sum_{r'=i'_m}^{J_m} e^{\mu \bar{V}_{m',r'}}}\right) g_{\varepsilon_1} d\varepsilon_1 \end{aligned} \tag{27}$$

In this expression, g_ε is the type-I extreme value density and $|J|$ is the determinant of the Jacobian, which has a similar structure (see Appendix A) as derived in Bhat (2005), given by:[6]

$$|J| = \left[\prod_{m=1}^{M}\prod_{i_m=1}^{I_m}\left(\frac{1}{t^*_{m,i_m}+\gamma_m}\right)\right]\left[\sum_{m=1}^{M}\sum_{i_m=1}^{I_m}\left(t^*_{m,i_m}+\gamma_m\right)\right] \quad (28)$$

The denominator in Equation (26) is a product of rank-ordered logits for each activity $k: k \in \mathbb{K}\backslash\{1\}$ as:

$$\prod_{k=2}^{K}\prod_{j_k=1}^{J_K}\left(\frac{e^{\mu\bar{V}_{k,j_k}}}{\sum_{s=j_k}^{J_k} e^{\mu\bar{V}_{k,s}}}\right) \quad (29)$$

The ratio of the expressions in Equations (27) and (29) is further simplified (see Appendix B) to arrive at a closed-form likelihood expression for the observed time allocations, given by:

$$\mathcal{L}\{t^*_1,..,(t^*_{m,1},t^*_{m,2},..,t^*_{m,I_m}),..,(t^*_{M,1},..,t^*_{M,I_M}),0,..,0\}$$
$$= |J|\,\mu^{\delta-1}(\delta-1)! * \frac{\prod_{m=1}^{M}\prod_{i_m=1}^{I_m}\exp\left(\mu\, V_{m,i_m}\right)}{\prod_{k=1}^{M}\prod_{j_k=1}^{I_k}\left(\frac{e^{\mu\bar{V}_{k,j_k}}}{\sum_{s=j_k}^{J_k} e^{\mu\bar{V}_{k,s}}}\right) * \left(\sum_{k=1}^{K}\sum_{j_k=1}^{J_k}\exp\left(\mu\, V_{k,j_k}\right)\right)^{\delta}} \quad (30)$$

where, δ is the total number of chosen episodes across all activities.

It is noteworthy that the above likelihood expression for the OMDCEV model is same as the likelihood expression of the traditional MDCEV model (with activity-episodes as choice alternatives) normalized by the rank-ordered logit terms for the probability of the baseline marginal utilities (ψ_{k,j_k}) of the chosen activity-episodes being in a non-increasing order for all chosen activities. This is intuitive, since the ordering of non-chosen activity-episodes should not affect the likelihood of observed data. Further, the similarity of the likelihood expression with that of the traditional MDCEV model makes the estimation of the above formulation not much more difficult than that of the MDCEV model (coding of the likelihood function is also a simple extension of the MDCEV model code). Also, in cases where the observed time allocations across all activities occur in a single episode for each activity, the above formulation collapses to the traditional MDCEV

[6] The Jacobian here has the same structure as that in the traditional MDCEV model (Bhat, 2005), except that the choice alternatives here are activity-episodes. Also, in notation, since the outside good does not have any episodes, $I_1 = 1$.

model. This follows as δ in Equation (30) becomes M, the Jacobian becomes that of the MDCEV model, and the rank-ordered logit terms in the denominator reduce to 1.

2.3.4 Model with different satiation parameters across different episodes of an activity

Recall that the model formulation so far imposes the satiation parameter (γ_k) to be same across all episodes of the same activity. Let us, for a simple thought experiment, consider that different episodes of an activity had different satiation parameters (γ_{k,j_k}). Applying the KKT conditions of optimality for two chosen episodes i_m and j_m of an activity m, one can write the following expression similar to Equation (24):

$$\frac{\psi_{m,i_m}}{\psi_{m,j_m}} = \frac{\left(\frac{t^*_{m,i_m}}{\gamma_{m,i_m}}+1\right)}{\left(\frac{t^*_{m,j_m}}{\gamma_{m,j_m}}+1\right)} = \frac{t^*_{m,i_m}+\gamma_{m,i_m}}{t^*_{m,j_m}+\gamma_{m,j_m}} * \frac{\gamma_{mj_m}}{\gamma_{mi_m}} \forall\, m \in \mathbb{M}\backslash\{1\}\ \&\ \forall\, i_m, j_m \in \mathbb{I}_{\mathbb{m}}:\ i_m < j_m \qquad (31)$$

From the setup in the above expression, $i_m < j_m$ or $t^*_{m,i_m} > t^*_{m,j_m}$ implies that $\psi_{m,i_m} > \psi_{m,j_m}$. To ensure this ordering in baseline marginal utilities, the ratio of $\left(\frac{t^*_{m,i_m}}{\gamma_{m,i_m}}+1\right)$ to $\left(\frac{t^*_{m,j_m}}{\gamma_{m,j_m}}+1\right)$ in the above equation must be greater than 1. This implies that $\frac{\gamma_{mi_m}}{\gamma_{mj_m}}$ must be less than $\frac{t^*_{m,i_m}}{t^*_{m,j_m}}$. However, the inequality $\frac{\gamma_{mi_m}}{\gamma_{mj_m}} < \frac{t^*_{m,i_m}}{t^*_{m,j_m}}$ might not hold for all observations if we allow γ_{mi_m} to be different from γ_{mj_m} (particularly, when $\gamma_{mi_m} > \gamma_{mj_m}$), which might cause estimation problems. An easy way to ensure this is to set $\gamma_{mi_m} = \gamma_{mj_m}$, which is innocuous (see discussion in Section 2.2) as well as circumvents difficulties in parameter estimation.

2.3.5 Forecasting procedure for the proposed OMDCEV model

To apply the proposed model for prediction, we proposed a minor modification to the forecasting procedure proposed by Pinjari and Bhat (2011). Specifically, the entire procedure of Pinjari and Bhat (2011) is employed, except that an accept-reject method is employed to use only the baseline marginal utilities that follow the ordering conditions in Equation (10). This is because, the proposed model can be interpreted as a variant of the traditional MDCEV model applied at the activity-episode level, albeit with the baseline marginal utility parameters influencing the occurrence of each episode of an activity k is right-truncated by that of the previous episode (because we are conditioning on the inequality that $\psi_{k,j_k} \geq \psi_{k,(j_k+1)}$). The procedure is outlined below for the sake of completion.

Step 1: Let the number of chosen alternatives be 1, *i.e.* δ =1.

Step 2: For each activity k, draw the stochastic terms (ε_{k,j_k}) for all activity-episode alternatives available to the individual. Using these stochastic terms, model parameters and data on observed covariates, compute baseline marginal utilities (ψ_{k,j_k}) for all activity-episode alternatives.

Step 3: For each activity k, until the simulated baseline marginal utilities of the episodes are in a non-increasing order given in Equation (10), reject the draws of ε_{k,j_k} for all episodes of that activity. Go back to *Step 2* for those activities which the ordering is not satisfied.

Step 4: Arrange the baseline marginal utilities of all activity-episodes in their decreasing order, starting with the outside good in the first place.

Step 5: Compute Lagrange multiplier (λ), using KKT conditions as in Pinjari and Bhat (2011).

Step 6: If $\lambda > \psi_{\delta+1}$, compute the optimal consumption of the first δ alternatives (*i.e.*, activity-episodes). Set all other consumptions to 0 and stop.

Else, go to step 7.

Step 7: $\delta = \delta + 1$.

If $\delta = \sum_{k=1}^{K} \sum_{j_k=1}^{J_k} 1$, compute optimal consumptions for all alternatives and stop.

Else, go to step 5.

In steps 2 and 3 of the above procedure, note that only the error term draws of those activities for which the baseline marginals are not in a non-increasing order from the lowest-numbered episode to the highest-numbered episode are rejected and redrawn (while retaining the draws of other activities for which the ordering condition is satisfied). This helps in reducing the number of draws and time needed to simulate the baseline marginal distributions that follow the ordering condition. A second approach is to reject all error draws (of all activities) even if the baseline marginal utilities of a single activity do not satisfy the ordering condition. While this approach works in theory, it can become very time- and simulation-intensive in situations with large number of activities and/or large number of episodes per activity. In any case, either approaches are correct in that the above-outlined forecasting procedure is consistent with the proposed model and ensures no illogical predictions where a higher-numbered episode is predicted without the occurrence of a lower-numbered episode. Therefore, the proposed algorithm can be applied with large number of error draws (that follow the ordering condition on baseline marginal utilities) to forecast the distributions of time allocations to different activity-episodes.

3. SIMULATION EXPERIMENTS WITH THE OMDCEV MODEL

This section presents simulation experiments conducted to: (a) verify if the proposed formulation precludes illogical predictions while allowing for time allocations at an activity-episode level, (b) verify if the proposed forecasting procedure and the model formulation are consistent with each other, and (c) the retrievability of model parameters.

3.1. Simulation Experiment Design

We performed simulation experiments on synthetic data with 3 activities. The first alternative was assumed to be the essential Hicksian outside good. The other two activities were assumed to have a maximum of 3 episodes each. This leads to a choice set of seven alternatives, including an essential Hicksian outside good and six activity-episodes (three episodes for each of two activities) The utility function in this set up is written as:

$$\begin{aligned} U = \psi_1 \ln(t_1) &+ \psi_{2,1}\gamma_2 \ln\left(\frac{t_{2,1}}{\gamma_2}+1\right) + \psi_{2,2}\gamma_2 \ln\left(\frac{t_{2,2}}{\gamma_2}+1\right) + \psi_{2,3}\gamma_2 \ln\left(\frac{t_{2,3}}{\gamma_2}+1\right) \\ &+ \psi_{3,1}\gamma_3 \ln\left(\frac{t_{3,1}}{\gamma_3}+1\right) + \psi_{3,2}\gamma_3 \ln\left(\frac{t_{3,2}}{\gamma_3}+1\right) + \psi_{3,3}\gamma_3 \ln\left(\frac{t_{3,3}}{\gamma_3}+1\right) \end{aligned} \quad (32)$$

where, $\psi_1 = \exp(\varepsilon_1)$, and

$$\begin{aligned} \psi_{2,1} &= \exp\left(ASC_{2,1} + \beta_{2,1}^{X_a} X_a + \beta_{2,1}^{X_b} X_b + \varepsilon_{2,1}\right), \\ \psi_{2,2} &= \exp\left(ASC_{2,2} + \beta_{2,2}^{X_a} X_a + \beta_{2,2}^{X_b} X_b + \varepsilon_{2,2}\right), \\ \psi_{2,3} &= \exp\left(ASC_{2,3} + \beta_{2,3}^{X_a} X_a + \beta_{2,3}^{X_b} X_b + \varepsilon_{2,3}\right), \\ \psi_{3,1} &= \exp\left(ASC_{3,1} + \beta_{3,1}^{X_a} X_a + \beta_{3,1}^{X_b} X_b + \varepsilon_{3,1}\right), \\ \psi_{3,2} &= \exp\left(ASC_{3,2} + \beta_{3,2}^{X_a} X_a + \beta_{3,2}^{X_b} X_b + \varepsilon_{3,2}\right), \\ \psi_{3,3} &= \exp\left(ASC_{3,3} + \beta_{3,3}^{X_a} X_a + \beta_{3,3}^{X_b} X_b + \varepsilon_{3,3}\right). \end{aligned}$$

Note that the superscripts to the beta parameters in the above equations are not exponents to the parameters; they are used to represent the variable which the corresponding parameter is associated with. For example, the parameter $\beta_{3,1}^{X_a}$ is associated with the effect of variable X_a on participation in activity-episode (3,1); *i.e.*, 3rd episode of 1st activity. And the model covariates influencing the baseline marginal utilities are simulated as $X_a \sim Normal(4,3)$ and $X_b \sim Bernoulli(0.5)$. The satiation parameters are expressed as $\gamma_2 = \exp(\theta_2)$ and $\gamma_3 = \exp(\theta_3)$.

The forecasting procedure proposed in the previous section was used to simulate time allocations for a synthetic sample of 5000 individuals and was repeated for 100 simulated draws of error terms (*i.e.*, 100 datasets of sample size 5000 were created). Accept-reject method was used to simulate error terms so that the baseline marginal utilities of every activity followed the ordering condition. We verified that in no single instance was a higher-numbered episode of an activity predicted without predicting lower-numbered episodes. Table 2 presents the simulated participation rates and average durations at both activity and activity-episode levels.

Table 2. Participation rates and episode durations across the 100 simulated datasets.

Sample size = 5000; Number of simulated datasets = 100				
Activity purpose	**% of individuals participating in the row activity purpose**	**% participation at episode level**		
		1 or more	**2 or more**	**3**
Outside good (Activity purpose 1)	100.0	NA		
Activity purpose 2	46.1	46.1	24.2	8.7
Activity purpose 3	55.6	55.6	41.7	29.9

Next, for each of the 100 datasets, model parameters were estimated by maximizing likelihood function in Equation (30). To measure parameter retrievability, we computed the following metrics:

- For each parameter, we calculated the mean of its corresponding estimate across the 100 datasets in the experiment and computed the absolute percentage bias (APB) as:

$$APB = \left|\frac{mean\ estimate - true\ value}{true\ value}\right| \times 100$$

- For each parameter, we calculated the standard deviation (across the 100 datasets) of the corresponding estimates and reported them as finite sample standard error (FSSE).
- For each parameter, the average value of the standard error computed across the 100 datasets is reported as asymptotic standard error (ASE).

- For each parameter, root mean standard error (RMSE) is reported, which is computed as:

$$RMSE = \sqrt{(Mean\ estimate - True\ Parameter)^2 + (FSSE)^2}$$

- For each parameter, the coverage probability (CP) is computed as:

$$CP = \frac{1}{N}\sum_{r=1}^{N} I\left[\hat{\beta}_X^r - t_\alpha * se\left(\hat{\beta}_X^r\right) \le \beta_X \le \hat{\beta}_X^r + t_\alpha * se\left(\hat{\beta}_X^r\right)\right]$$

  where, $\hat{\beta}_X^r$ is the estimated parameter in dataset r, $se\left(\hat{\beta}_X^r\right)$ is the corresponding asymptotic standard error of the parameter, t_α is the t-statistic value for the given confidence interval. We computed coverage probability at 90% and 95% confidence intervals. Ideally, the coverage probability should be equal to the corresponding confidence level.

3.2. Performance Evaluation Results

Table 3 presents an overall summary of the accuracy and precision of parameter recovery from the 100 simulated datasets. It is evident from the simulation results that it is possible to accurately retrieve the parameters as indicated by the low APB values and low RMSE values. Further, the FSSE and ASE values for each parameter are small and reasonably close suggesting good precision in parameter recovery. Lastly, the coverage probabilities are close to the corresponding confidence intervals, suggesting that the coverage of the parameter estimates around the true parameter is neither conservative nor permissive. Overall, the results help verify that the proposed model is consistent with its prediction procedure and demonstrate the ease with which its parameters can be recovered. As importantly, the proposed formulation allows the modeling and prediction of episode-level activity participation and duration while not violating a logical ordering of episodes.

Along with the parameter retrievability, the simulation experiment sheds light on an important property of the model. Recall that the baseline marginal utilities of episodes of an activity are in a decreasing order. This ordering condition asserts a non-increasing order across the average rate of occurrence of episodes of an activity. That is, the overall participation in the first episode of an activity will be greater than the overall participation in the second episode, which again will be greater than the overall participation in the third episode and so on. This follows intuitively from the fact that the model does not allow the occurrence of a higher-numbered episode of an activity without all the lower-numbered episodes. That is, the third episode of an activity cannot occur without the first and second episodes. However, this property might be wrongfully interpreted that the model does not allow higher participation in a higher-numbered episode as compared to participation only in a lower-numbered episode. There are various practical situations where individuals are more likely to participate in two episodes as compared to just one. An example of such situation is escorting, where individuals may participate in both pick-up and drop-off (*i.e.* two episodes) than just doing either one. As can be observed from Table 2, the simulated participation rates suggest a higher participation in three episodes of the third activity than participation in only the first or only first and second episodes of the same activity (nearly 30% participation in the all the three episodes as compared to 14% in only the first and 12% in first and second). The fact that the model allows such allocation bundles and that the parameters are well recovered suggests that the model is flexible enough to accommodate such situations.

Table 3. Summary of parameter retrievability from simulated datasets.

Parameter	True Value	Mean Estimated Value	Absolute Percentage Bias (APB)	Finite Sample Standard Error (FSSE)	Asymptotic Standard Error (ASE)	RMSE	$CP_{90\%}$	$CP_{95\%}$
$\boldsymbol{ASC_{21}}$	-1.00	-0.991	0.89	0.12	0.11	0.12	0.90	0.94
$\boldsymbol{ASC_{21}}$	-1.50	-1.505	0.30	0.10	0.10	0.10	0.90	0.94
$\boldsymbol{ASC_{21}}$	-2.00	-1.991	0.43	0.08	0.08	0.08	0.91	0.95
$\boldsymbol{ASC_{21}}$	-0.50	-0.511	2.26	0.12	0.14	0.12	0.94	0.98
$\boldsymbol{ASC_{21}}$	-0.80	-0.784	1.99	0.10	0.10	0.10	0.86	0.97
$\boldsymbol{ASC_{21}}$	-1.00	-1.008	0.76	0.05	0.06	0.05	0.93	0.98
$\boldsymbol{\beta_{21}^{X_a}}$	-1.10	-1.109	0.79	0.04	0.04	0.04	0.91	0.96
$\boldsymbol{\beta_{22}^{X_a}}$	-0.90	-0.899	0.13	0.04	0.04	0.04	0.89	0.94
$\boldsymbol{\beta_{23}^{X_a}}$	-0.80	-0.804	0.47	0.03	0.03	0.03	0.96	0.97
$\boldsymbol{\beta_{31}^{X_a}}$	-1.50	-1.506	0.40	0.04	0.05	0.05	0.90	0.94
$\boldsymbol{\beta_{32}^{X_a}}$	-1.20	-1.198	0.18	0.03	0.03	0.03	0.95	0.97
$\boldsymbol{\beta_{33}^{X_a}}$	-1.00	-1.001	0.07	0.01	0.02	0.01	0.92	0.96
$\boldsymbol{\beta_{21}^{X_b}}$	-1.00	-0.989	1.14	0.16	0.17	0.16	0.93	0.98
$\boldsymbol{\beta_{22}^{X_b}}$	-0.80	-0.812	1.50	0.17	0.17	0.17	0.89	0.95
$\boldsymbol{\beta_{23}^{X_b}}$	-0.50	-0.492	1.50	0.12	0.12	0.12	0.91	0.96
$\boldsymbol{\beta_{31}^{X_b}}$	0.60	0.600	0.74	0.20	0.19	0.20	0.91	0.91
$\boldsymbol{\beta_{32}^{X_b}}$	0.90	0.890	1.00	0.12	0.13	0.12	0.91	0.94
$\boldsymbol{\beta_{33}^{X_b}}$	1.10	1.108	0.77	0.06	0.07	0.06	0.90	0.94
$\boldsymbol{\theta_2}$	0.80	0.796	0.48	0.03	0.03	0.03	0.90	0.95
$\boldsymbol{\theta_3}$	0.50	0.503	0.64	0.03	0.03	0.03	0.89	0.95

4. EMPIRICAL APPLICATION

4.1. Empirical Data

An application of the proposed model is demonstrated through an empirical analysis of activity participation and time use decisions of individuals at a disaggregate, episode level. The intent of this empirical analysis is to demonstrate the applicability of the proposed OMDCEV model and it's benefits over the traditional MDCEV model for analyzing activity participation and time use. For this analysis, we focus our attention on disaggregate, episode-level activity time-use of non-working adults using empirical data drawn from the 2013 regional Household Travel Survey conducted by Southern California Association of Governments (SCAG) in the six county Los Angeles region of California. The data was extensively cleaned for incorrect/missing information and the final sample had time-use information for 2936 non-workers.

The travel survey data was used to generate data on a) in-home (IH) activities, and b) out-of-home (OH) activities. The OH activities were further classified into one of the following eight types: 1) Escorting (pick-up and drop-off activities), 2) Shopping (buying household goods), 3) Maintenance (include family obligations such as visiting a bank, post office, etc.), 4) Social (going to a social/civic event, religious activity, etc.), 5) Entertainment (going for a cinema, sports event, etc.), 6) Active recreation (yoga, sports or any physical activity), 7) Visiting family and friends, and 8) Eat out. In all, there were nine activity purposes, with the in-home activity serving as the essential "outside good". The data was used to extract time use information for each of these OH activities at an episode level, *i.e.*, time spent on each of these activities at each instance of occurrence on a typical weekday. Because the model is formulated such that the episodes are defined in the decreasing order of their allocations, the episodes for each of the eight OH activities were defined in the decreasing order of their allocation (and not in chronology of their occurrence). The maximum number of episodes observed for each of these activities varied from 2 episodes for entertainment and eat-out to as high as 6 episodes for maintenance and shopping activities. The details of activity-episode participation rates and their time investments are presented in Table 3. The total budget considered in the analysis is 1080 minutes for every individual, which is obtained after subtracting 6 hours of time for minimum needed sleep and subsistence activities at home.[7]

The first numeric column in Table 4 gives the percentage of non-workers in the data participating in the row activity (regardless of the number of episodes of that activity they

[7] The data did not have detailed information on in-home activities.

participated in). The next set of columns gives the percentage of individuals who participated in that episode or more episodes of the row activity. It can be observed from the data that for every row activity, any higher-numbered episode has not occurred without all the lower-numbered episodes (because left side columns have greater percentages of individuals than right side columns). One can also simply subtract the percentage participation in one column from that in the immediately following column to get the percentage of individuals participating in only the number of episodes until that column. For example, for escorting activity, the percentage of individuals participating in only first and second episodes is 11.3 (18.4 – 7.1). Note that this percentage is greater than those participating in only one escorting episode, which is 9.8 (28.2 – 18.4). As illustrated in the simulation experiments, the proposed model can accommodate these types of differences in activity participation rates across different episodes of an activity; at least in theory. The last two columns of the table report average time allocations in each of the row-activity at both activity level and episode level for individuals who participated in the row activity. As expected, average time spent in escorting episodes is the lowest amongst all the activities. The highest time allocation is to entertainment (157 minutes) at an episode level and to visiting friends and family at the activity level (169 minutes). Further, as expected, out-of-home recreational activities, such as entertainment and socialization have higher time allocations than obligatory activities such as maintenance and shopping.

Table 4. Participation rates and time-investment in the sample data (N = 2936 non-workers).

	Activity-level participation	Episode-level participation (% of individuals participating in at least these no. of episodes)						Average time allocation, if non-zero (minutes)	
Activity purpose	**% of individuals participating in the activity**	**1 or more**	**2 or more**	**3 or more**	**4 or more**	**5 or more**	**6**	**Episode-level**	**Activity-level**
Escorting	28.2	28.2	18.4	7.1	4.2	--	--	11.6	23.8
Shopping	46.9	46.9	13.6	4.3	1.3	0.4	0.1	43.8	62.2
Maintenance	40.9	40.9	12.9	4.1	1.1	0.3	0.1	51.4	74.5
Social	7.6	7.6	0.8	0.2	--	--	--	123.2	139.2
Entertainment	6.3	6.3	0.3	--	--	--	--	157.4	164.3
Visit family/friends	18.5	18.5	3.3	0.7	0.1	--	--	138.8	169.5
Active Recreation	17.6	17.6	1.9	0.2	--	--	--	99.5	107.8
Eat Out	19.3	19.3	1.5	--	--	--	--	69.1	74.4

4.2. Estimation results

The model estimation results are reported in Tables 5a, 5b, and 5c for the proposed OMDCEV model and the traditional MDCEV model estimated at an episode level (referred to as MDCEV model for brevity). The final empirical specifications reported in these tables were carefully built by systematically including explanatory variables one after the other and dropping statistically insignificant parameters while retaining those that offered an intuitive interpretation as long as the t-statistic was greater than 1. Note that the scale of the error term was fixed to one in both the models. However, as pointed out in Bhat (2018), the scale parameter need not be fixed to 1 and is potentially estimable for the gamma-profile utility function used in this paper. This, along with other extensions such as separating the marginal utilities for discrete and continuous decisions (Bhat, 2018) can be explored in the future.

4.2.1. Baseline preference constants

The baseline preference constants are estimated for each activity-episode except the outside good (which serves as the base category) for both the models. The constants are specified such that the constant corresponding to the first episode of each activity is interpreted as the "activity-level constant" (since it enters the baseline utility of all episodes of that activity). This constant for a given activity may be interpreted as the aggregate influence of unobserved factors on the likelihood of taking part in one or more episodes of this activity. The subsequent constants (which will be referred to as "episode-specific constants" in the subsequent discussion) for each episode of an activity capture the differential influence for that episode with respect to the "activity-level" constant. For example, from Table 5a, the baseline preference constant for the third episode of escorting activity is -8.55 for the OMDCEV model, which is the sum of the corresponding activity-level constant (-9.59) and the episode-specific constant for the third episode (1.04). Similarly, for the MDCEV model, the baseline preference constant for the third episode of escorting activity is -10.30 (*i.e.*, -8.35 + (-1.95) = -10.30).

Recall that an important difference between the two models is that the baseline utility parameter across different episodes of an activity are conditioned to be in decreasing order for the OMDCEV model, which is not explicitly done in the MDCEV model. Because of this property, the overall baseline preferences are always in a decreasing order across the different episodes of an activity, even if the episode-specific constants are zero or positive. This obviates the need for estimating each episode-specific constant in the OMDCEV model. This is also a reason why not many episode-specific constants were estimated in the OMDCEV model (they turned out to be statistically insignificant), keeping the model parsimonious in terms of the number of parameters.

Since the above property is not explicitly accommodated in the MDCEV model, it's episode-specific constants for any activity are all negative and in a decreasing order from the second episode to the last possible episode, such that the baseline preference constants across episodes are in a decreasing order. This is simply reflective of the fact that the participation rates across episodes are in a decreasing order (*i.e.*, observed data would not have situations where individuals take part in the third episode without doing in the third episode). This is also a reflection of the large number of empirical parameters needed to capture such trend in the data without an explicit ordering in the baseline utilities across different episodes of an activity. Yet, when the MDCEV model is applied to situations outside the estimation sample, there is no guarantee of an order in predicted episodes.

Note that the baseline preference constants do not have a substantive interpretation because of the influence of differences in scales of the continuous explanatory variables in the model and the measurement errors therein. However, all the activity-level constants are negative in the OMDCEV model (which makes sense since the base case is outside activity which has 100% participation rate). Interestingly, some of the episode-level constants in the OMDCEV model are positive; for example, the third episode-specific constant is positive for the escorting activity. Not including these parameters in the model resulted in a greater value for the estimate of the activity-level parameter. This would lead to a uniform increase in the overall baseline utility for all episodes, because the activity-level constant would enter the baseline utility for all episodes, and substantially deteriorated the model fit (because of uniform increase in baseline utility of all episodes). A positive value for the third episode-specific constant implies a higher value of the baseline utility constants for the first, second, and third episodes (by the value of the third episode-specific constant) than those for the fourth episode.

In the MDCEV model, the activity-level constant is highest for shopping activities, which reflects its highest participation rate across all activities, followed by maintenance and escorting. Also, entertainment has the lowest participation of all the activities, as reflected in the lowest activity-level constant. Similar trend was observed in the OMDCEV model with activity-level constants only, but not after including episode-specific constants. This is evident in the estimates of OMDCEV, where visiting friends and family has the lowest activity-level constant.

Table 5a. Estimation results: Baseline preference constants

Baseline preference constants						
	Episodes					
Activity purpose (Base: In home discretionary)	Activity-level (*i.e.*, 1 or more episodes)	2	3	4	5	6
OMDCEV model						
Escorting	-9.59 *(-52.07)*	-	1.04 (2.94)	-	NA	NA
Shopping	-9.27 *(-44.11)*	-	-	1.46 (3.53)	-	-
Maintenance	-9.35 *(-39.27)*	-3.82 *(-0.11)*	-	1.76 (4.29)	-	-
Social	-10.23 *(-109.72)*	-	-	NA	NA	NA
Entertainment	-10.49 *(-79.08)*	-	NA	NA	NA	NA
Visiting family and friends	-11.01 *(-18.87)*	-	2.73 (3.93)	-	NA	NA
Active recreation	-9.74 *(-107.70)*	-	-	NA	NA	NA
Eat out	-9.06 *(-35.91)*	-	NA	NA	NA	NA
MDCEV model						
Escorting	-8.35 *(-112.14)*	-0.55 *(-5.07)*	-1.95 *(-9.97)*	-2.71 *(10.77)*	NA	NA
Shopping	-7.00 *(-128.53)*	-1.52 *(-18.57)*	-2.70 *(20.93)*	-4.01 *(17.16)*	-5.14 *(11.87)*	-6.23 *(9.88)*
Maintenance	-7.10 *(-120.64)*	-1.31 *(-13.82)*	-2.60 *(19.93)*	-3.95 *(15.69)*	-5.23 *(-9.48)*	-6.04 *(8.28)*
Social	-9.22 *(-91.48)*	-2.35 *(-7.77)*	-3.50 *(-7.32)*	NA	NA	NA
Entertainment	-9.84 *(-72.55)*	-2.88 *(-6.45)*	NA	NA	NA	NA
Visiting family and friends	-8.20 *(-115.95)*	-1.80 *(-12.00)*	-3.44 *(11.03)*	-4.99 *(-8.04)*	NA	NA
Active recreation	-8.67 *(-94.59)*	-2.54 *(-12.79)*	-2.23 *(-4.24)*	NA	NA	NA
Eat out	-8.35 *(-100.44)*	-11.30 *(-32.56)*	NA	NA	NA	NA

-: Insignificant. NA: Not applicable

4.2.2. Individual and household specific effects

The effects of individual and household specific variables on time-use decisions are reported in Table 5b. In the context of gender effects, the parameter estimates in the OMDCEV model indicate that women are more likely to participate in escorting and shopping activities whereas men were more likely to participate in active recreation. This is reflective of the traditional gender roles where women take greater responsibility in household activities (Calastri *et al.*, 2017; Pinjari *et al.*, 2016; Astroza *et al.*, 2018). Interestingly, females are more likely to participate in social activities and visiting friends and family as well, which maybe because women are more family oriented and sociable than men (Kapur & Bhat, 2007; Siegling *et al.*, 2012). However, we did not find gender differences for participation in maintenance, entertainment, and eat-out activities. Interestingly, other individual-specific variables such as age did not show statistically significant influence on time-use decisions in either the OMDCEV or the MDCEV model.

The next set of covariates correspond to household-specific variables. Among these, the number of pre-school children in the household had a negative influence on the participation of all out of home activities, except escorting and active recreation. This result is reflective of the additional childcare responsibilities in households with young children (Bernardo *et al.*, 2015). In the context of escorting activities, as expected, the OMDCEV model suggests that more pre-school children in the household leads to greater number of escorting episodes; perhaps for child daycare pickup and/or drop-off purposes. Interestingly, the activity-specific parameter was insignificant for escorting activity, while the episode-specific parameters were significant and positive for the OMDCEV model. However, for the MDCEV model, the activity specific parameter was positive and significant, while the episode-specific coefficients were insignificant. An important point to note here is that while the activity-specific parameter estimate for number of pre-school children is insignificant for escorting in the OMDCEV model, one should not infer that participation in escorting is not influenced by the number of children in the household. Since the episode-specific parameters for the second, third and fourth episodes are significant (and positive) for escorting, it implies that an increase in number of children leads to higher chances of more number of escorting episodes, which indirectly increases the participation in the first episode (or activity participation) as well. This is because of the condition that any higher-numbered episode cannot occur without all the lower-numbered episodes. However, the MDCEV model does not show any differential effect of number of pre-school children across episodes of escorting. While most time-use studies based on the application of MDCEV models at an activity level (e.g., Pinjari *et al.* 2016; Calastri *et al.* 2017) report that increase in pre-school children leads to increase in escorting activities, the literature does not shed light on the influence of the number of escorting episodes. In this context, the OMDCEV model not only provides insight but also appears to offer interpretations that are more plausible than those from the MDCEV model.

As expected, number of children in the household who go to school but cannot drive reduce the likelihood of participation in shopping, maintenance, active recreation, visiting family and friends and eat out. Interestingly, social and entertainment activities had no influence of number of school-going children who cannot drive. However, higher number of school-going children makes participation in escorting activities more likely for both the models. Also, the episode-specific parameter for the fourth episode was significant and positive for the OMDCEV model, implying that more number of school-going children leads to higher activity participation as well as more number of escorting episodes. Similar trend was also observed for the MDCEV model, where along with the activity- specific parameter, episode-specific parameter for the third and fourth episodes were also positive. However, it should be noted here that unlike OMDCEV model, the episode-specific parameter for fourth episode in the MDCEV model does not influence the participation in third or second episode.

Table 5b. Estimation results: Effect of individual and household specific variables

	Gender (Base case: Female)						**Number of pre-school children**					
	Episodes						**Episodes**					
Activity purpose (Base: In home discretionary)	**Activity-level**	**2**	**3**	**4**	**5**	**6**	**Activity-level**	**2**	**3**	**4**	**5**	**6**
OMDCEV model												
Escorting	-0.43 *(-7.78)*	-	-	-	NA	NA	-	0.57 *(2.03)*	0.44 *(2.58)*	0.63 *(6.03)*	NA	NA
Shopping	-0.16 *(-2.80)*	-	-	-	-	-	-0.28 *(-4.52)*	-	-	-	-	-
Maintenance	-	-	-	-	-	-	-0.27 *(-4.72)*	-	-	-	-	-
Social	-0.27 *(-2.12)*	-	-	NA	NA	NA	-0.26 *(-2.18)*	-	-	NA	NA	NA
Entertainment	-	-	NA	NA	NA	NA	-0.20 *(-1.33)*	-	NA	NA	NA	NA
Visiting family/friends	-0.14 *(-1.76)*	-	-	-	NA	NA	-0.16 *(-2.20)*	-	-	-	NA	NA
Active recreation	0.14 *(1.54)*	-	-	NA	NA	NA	-	-	-	NA	NA	NA
Eat out	-	-	NA	NA	NA	NA	-0.30 *(-3.41)*	-	NA	NA	NA	NA
MDCEV model												
Escorting	-0.48 *(-9.33)*	-	-	-	NA	NA	0.56 *(13.62)*	-	-	-	NA	NA
Shopping	-0.23 *(-4.70)*	-	-	-	-	-	-0.34 *(-5.82)*	-	-0.38 *(-1.23)*	-	-	-
Maintenance	-	-	-	-	-	-	-0.33 *(-6.39)*	-	-	-	-	-
Social	-0.27 *(-2.14)*	-	-	NA	NA	NA	-0.27 *(-2.25)*	-	-	NA	NA	NA
Entertainment	-	-	NA	NA	NA	NA	-0.21 *(-1.37)*	-	NA	NA	NA	NA
Visiting family/friends	-0.14 *(-1.76)*	-	-	-	NA	NA	-0.18 *(-2.54)*	-	-	-	NA	NA
Active recreation	0.16 *(1.80)*	-	-	NA	NA	NA	-	-	-	NA	NA	NA
Eat out	-	-	NA	NA	NA	NA	-0.33 *(-3.76)*					

-: Insignificant. NA: Not applicable

Table 5b (Contd.). Estimation results: Effect of individual and household specific variables

	Number of children who go to school but cannot drive						Income *(Base case: Low income (less than $50k))*											
							Medium income ($50k-$100k)						High income (more than $100k)					
	Episodes						Episodes						Episodes					
Activity purpose (Base: In home discretionary)	**Activity-level**	**2**	**3**	**4**	**5**	**6**	**Activity-level**	**2**	**3**	**4**	**5**	**6**	**Activity-level**	**2**	**3**	**4**	**5**	**6**
Ordered MDCEV (OMDCEV)																		
Escorting	0.45 (6.36)	-	-	0.55 (5.25)	NA	NA	0.10 (1.85)	-	-	-	NA	NA	0.24 (3.47)	-	-	-	NA	NA
Shopping	-0.09 (2.02)	-	-	-	-	-	-	-	-	-	-	-	0.19 (2.53)	-	-	-	-	-
Maintenance	-0.20 (-4.63)	-	-	-	-	-	-0.16 (-2.66)	-	-	-	-	-	-0.11 (-1.41)	-	-	-	-	-
Social	-	-	-	NA	NA	NA	-	-	-	NA	NA	NA	0.21 (1.43)	-	-	NA	NA	NA
Entertainment	-	-	NA	NA	NA	NA	0.46 (2.64)	-	NA	NA	NA	NA	0.71 (3.75)	-	NA	NA	NA	NA
Visiting family and friends	-0.19 (-3.06)	-	-	-	NA	NA	-	-	-	-	NA	NA	-	-	-	-	NA	NA
Active recreation	-0.09 (-1.40)	-	-	NA	NA	NA	0.43 (4.31)	-	-	NA	NA	NA	-	0.60 (4.43)	-	NA	NA	NA
Eat out	-0.21 (-3.67)	-	NA	NA	NA	NA	-	0.35 (1.86)	NA	NA	NA	NA	-	0.70 (2.00)	NA	NA	NA	NA
Traditional disaggregate MDCEV model																		
Escorting	0.69 (3.72)	-	0.21 (1.67)	0.31 (2.10)	NA	NA	0.13 (2.14)	-	0.17 (1.37)	0.27 (1.80)	NA	NA	0.27 (4.23)	-	-	-	NA	NA
Shopping	-0.10 (-2.77)	-	-	-	-	-	-	-	-	-	-	-	0.21 (3.12)	-	-	-	-	-
Maintenance	-0.19 (-4.25)	-0.19 (1.55)	-	-	-	-	-0.22 (-3.00)	-	-	-	-	-	-0.15 (-2.13)	-	-	-	-	-
Social	-	-	-	NA	NA	NA	-	-	-	NA	NA	NA	0.21 (1.39)	-	-	NA	NA	NA
Entertainment	-	-	NA	NA	NA	NA	0.47 (2.67)	-	NA	NA	NA	NA	0.76 (4.03)	-1.67 (-1.39)	NA	NA	NA	NA
Visiting family and friends	-0.19 (-3.60)	-	-	-	NA	NA	-	-	-	-	NA	NA	-	-	-	-	NA	NA
Active recreation	-0.09 (-1.55)	-	-	NA	NA	NA	0.44 (4.60)	-	-	NA	NA	NA	0.35 (2.70)	0.66 (2.12)	-	NA	NA	NA
Eat out	-0.19 (-3.00)	-0.20 (1.38)	NA	NA	NA	NA	0.19 (1.73)	0.56 (1.90)	NA	NA	NA	NA	0.55 (4.54)	1.12 (2.88)	NA	NA	NA	NA

The last set of variables in the baseline utility functions relate to household income, with income less than $50,000 per annum (low-income) as the base category. The effects of income variables are mainly manifested in the form of financial affordability and availability of resources, translated into activity participation preferences. Individuals form medium (with income between $50,000 and $100,000) and high-income (with income more than $100,000) households are more likely to participate in entertainment, active recreation and eat out activities. This result can be attributed to higher spending capabilities of medium- and high-income households as compared to low-income households. The same effect was observed for escorting activities as well where medium and high-income households show higher propensity to participate in escorting when compared to those from low-income households (He, 2013; Vovsha *et al.*, 2004). On the other hand, individuals from low-income households are more likely to participate in maintenance activities than those from medium and high-income households.

In the OMDCEV model, specifically, for eat-out activity, the activity specific parameter is insignificant, but the parameter specific to second episode is significant and positive. As discussed before, this makes medium- and high-income individuals more likely to participate in eat out activities as well as do more episodes of it when compared to low-income individuals. The same is true for active recreation activities; specifically, for high-income individuals. Also, individuals from high-income households are more likely to participate in shopping related activities. Interestingly, in the MDCEV model, the episode-specific parameter for entertainment corresponding to the second episode is negative, implying that high-income individuals are less likely to participate in the second episode of entertainment. However, in the OMDCEV model, the second episode-specific parameter is statistically insignificant; maybe because the ordering condition imposed on baseline utilities of different episodes in the model already accounts for this.

4.2.3. Satiation parameters

Table 5c reports the satiation parameter estimates from both the models. As discussed earlier, for the OMDCEV model, the satiation parameters are same across all episodes of an activity, *i.e.*, $\gamma_{kj_k} = \gamma_k \, \forall \, k = 2,3, \dots, K$. In the MDCEV model, however, a separate satiation parameter is estimated for each activity-episode. *Ceteris paribus*, choice alternatives with higher satiation parameters (γ_{kj_k}) are associated with lower satiation and, therefore, higher time allocation. Recall from the descriptive statistics of the estimation data that entertainment, visiting friends and family, and active recreation have relatively higher time investment while escorting has the lowest. This trend is reflected in the satiation parameter estimates of both the models. Interestingly, at an aggregate (activity) level, average time allocation is highest for visiting family and friends. However, at an episode level, entertainment has the highest allocation. Since our analysis is at an episode level, the parameter estimate for entertainment is highest in both the models.

Table 5c. Estimation results: Translation parameters

Activity purpose (Base: In home discretionary)	Translation parameter (Parameterized as exp(theta))					
		Episodes				
	Activity-level	2	3	4	5	6
OMDCEV model						
Escorting	5.01 (20.74)	NA				
Shopping	29.69 (19.12)	NA				
Maintenance	21.21 (23.64)	NA				
Social	97.72 (6.94)	NA				
Entertainment	139.79 (5.54)	NA				
Visiting family and friends	105.83 (11.76)	NA				
Active recreation	72.84 (9.75)	NA				
Eat out	52.85 (8.87)	NA				
MDCEV model						
Escorting	5.97 *(11.04)*	3.32 *(7.59)*	2.67 *(4.23)*	2.02 *(3.27)*	NA	NA
Shopping	24.95 *(12.93)*	18.87 *(6.45)*	14.38 *(3.43)*	10.19 *(1.82)*	-	-
Maintenance	22.38 *(13.29)*	9.96 *(7.44)*	6.82 *(3.65)*	5.29 *(1.70)*	-	-
Social	105.66 *(5.90)*	58.20 *(1.54)*	-	NA	NA	NA
Entertainment	145.64 *(5.03)*	28.52 *(1.37)*	NA	NA	NA	NA
Visiting family and friends	114.10 *(9.34)*	46.46 *(3.78)*	33.20 *(1.38)*	-	NA	NA
Active recreation	72.87 *(8.72)*	35.79 *(3.10)*	-	NA	NA	NA
Eat out	50.08 *(8.03)*	28.28 *(2.70)*	NA	NA	NA	NA

-: Insignificant. NA: Not applicable

Overall, the effects of exogenous variables on time use captured by both MDCEV and OMDCEV models are intuitive and behaviorally consistent. In the context of a few covariates, the OMDCEV model offers more plausible interpretations than the MDCEV model. Also, the number of parameters needed in the OMDCEV model is much smaller than those for the MDCEV model, making the former more parsimonious than the latter. This is attributable to the structure of the

OMDCEV model in that it accommodates inherent ordering in the baseline utility parameters across different episodes of an activity. On the other hand, despite the larger number of parameters in the MDCEV model, as demonstrated later, it does not guarantee a logical occurrence of different episodes of an activity.

4.3. Comparison of Model Performance

4.3.1. Goodness of fit measures

The goodness of fit measures for both the OMDCEV and MDCEV models are presented in Table 6. As can be observed from the table, the OMDCEV model has a higher log-likelihood than the MDCEV model at convergence, even with less number of parameters (55 parameters for OMDCEV and 94 for MDCEV). Of course, one cannot compare the log-likelihood values directly as the two models are non-nested. Therefore, Akaike information criterion (AIC) and Bayesian information criterion (BIC) are used to compare the goodness of fit of the two models. Both these measures suggest a superior fit of the OMDCEV model when compared to the MDCEV model.

Table 6. Goodness of fit measures across the estimation and prediction samples

Goodness of fit measures		
	Traditional disaggregate MDCEV model	**Ordered MDCEV**
Data fit measures for the estimation sample		
Number of cases	2936	2936
Number of parameters	94	55
Null log-likelihood	-99995.29	-89559.77
Log-likelihood for constants only model	-56397.33	-52275.77
Log-likelihood at convergence	-55881.82	-51517.11
Akaike Information Criterion (AIC)	111951.64	103172.40
Bayesian Information Criterion (BIC)	112089.61	103253.13
Non-nested statistical test for model selection		
Vuong test statistic	36.04 (Since φ (36.04)>>0.999, we reject the null hypothesis and OMDCEV is the preferred model	
Data fit measures for the prediction sample*		
Number of cases	200	200
Predictive log-likelihood value	-3834.75	-3554.63
Akike Information Criterion (AIC)	7857.51	7219.27
Bayesian Information Criterion (BIC)	8167.55	7400.68
Non-nested statistical test for model selection		
Vuong test statistic	33.57 (Since φ (33.57)>>0.999, we reject the null hypothesis and OMDCEV is the preferred model	

*10 outside samples were considered for prediction analysis. The reported values are average across the 10 samples.

As a supplement to the AIC and BIC measures, we employ Vuong test (Vuong, 1989), a non-parametric test, to compare the two models. The Vuong test statistic is given by:

$$n^{-1/2} LR_n(\hat{\theta}, \hat{\gamma}) \Big/ \hat{\omega} \backsim N(0,1) \tag{33}$$

where, $LR_n(\hat{\theta}, \hat{\gamma})$ is the difference in the log-likelihood values of the two competing models $F(\hat{\theta})$ and $G(\hat{\gamma})$, with the former model having a higher log-likelihood value; $\hat{\omega}$ is the variance of the difference in the individual-level log-likelihood values and n is the sample size. The test states that under the null hypothesis, the two non-nested models $F(\hat{\theta})$ and $G(\hat{\gamma})$ are equally close to the true model (or fit equally well) and the test statistic is asymptotically normal. Thus, for a critical value c (for a given significance level), if the test statistic is greater than c, one rejects the null hypothesis in favor of $F(\hat{\theta})$ being closer to the true model than $G(\hat{\gamma})$. The test statistic along with the result of the test is reported in Table 6. For the estimation sample, the test statistic is 36.06, which implies that there is enough evidence to reject the null hypothesis in favor of the OMDCEV model as the preferred model even at very low significance levels.

To compare model performance on outside samples, we randomly selected 10 samples, each of sample size of 2736 for model estimation and used the remaining 200 data points to validate the model. This 10-fold validation strategy is used to analyze the proposed model's performance on outside samples and compare it with that of the traditionally used MDCEV model.

The average[8] predictive log-likelihood across the 10 outside samples along with AIC and BIC are reported in the second set of rows in Table 6. These measures suggest that the proposed OMDCEV model is preferred over the disaggregate MDCEV model. Vuong test performed on these average predictive log-likelihood values again points out the superiority of the OMDCEV model over the disaggregate MDCEV model. Next, we compare the predictive performance of the two models on the 10 outside samples.

4.3.2. Prediction performance evaluation using k-fold validation

To evaluate the efficacy of the proposed OMDCEV model on outside samples, we apply the forecasting algorithm proposed in this paper to 10 outside samples of 200 individuals. That is, model parameters were estimated from a random sample of 2736 individuals were applied on the remaining sample of 200 individuals. This exercise was repeated 10 times for a 10-fold validation. We do the same using the forecasting algorithm of Pinjari and Bhat (2011) to the same 10 outside

[8] Ideally, the k-fold validation results should be tabulated for each of the k-outside samples. However, for brevity, we report the average values across these 10 outside samples. Similar trends were observed for each of the 10 datasets independently as well.

samples to predict time allocations from the MDCEV model. For both the models, 20 sets of error draws were simulated for each of the 200 individuals to cover the distribution of error terms.

As discussed in Section 2.3.5, an issue with the implementation of the forecasting procedure for the proposed OMDCEV model is a large rate of rejection of error term draws to ensure appropriate ordering (Equation (10)) of the baseline marginal utilities across episodes, especially when the number of choice alternatives – either number of activities or number of episodes for each activity – is large. In the current empirical context with a total of 31 choice alternatives (activity-episode alternatives), redrawing error terms of all 31 choice alternatives every time the ordering conditioning was violated for any activity turned out to be extremely time consuming. Therefore, we rejected and re-drew only the error terms of those activities for which the ordering condition among the baseline marginal utilities of different episodes was violated (while retaining the error draws for the activities for which the ordering condition was satisfied).

Recall that the traditional MDCEV model with activity-episodes as choice alternatives does not ensure logical ordering or consistency in prediction of activity-episodes. For example, using the MDCEV one can predict the occurrence of a second or a third episode of an activity, without allocating time to the first episode. The extent of this problem is demonstrated in the prediction results of this model in Table 7. As can be observed form this table, for escorting, shopping, and maintenance activities, the percentage of the individuals for which such illogical predictions happened is 27%, 20%, and 19% respectively. Overall, such inconsistent predictions were observed for more than 32% of individuals in the data. On the other hand, the proposed OMDCEV model does not result in a single instance of such illogical predictions.

Table 7. MDCEV predictions with illogical occurrence of episodes.

% of individuals with of illogical occurrence of episodes for each activity (Avg. across 10 outside samples of N = 200)								**% of individuals with illogical occurrence of episodes**
Escorting	**Shopping**	**Maintenance**	**Social**	**Entertainment**	**Visiting friends & family**	**Active recreation**	**Eat out**	
27	20	19	1	0	5	3	2	32.60%

The predicted episode-level activity participation rates for both the OMDCEV and MDCEV models as well as the corresponding observed values averaged across the 10 samples are presented in Table 8a. Similarly, averages of predicted and observed episode durations (when the specific episode was predicted) are presented Table 8b. The first numeric column in Table 8a reports predicted and observed activity participation rates for different types of activities (activity participation refers to a situation with non-zero time allocated to any episode of an activity). The next set of columns report the episode-level participation rates (with episode participation, we refer to situations where one participated at least in that episode. For example, for second episode of an activity, we count all instances one participated in the second episode, irrespective of whether they participate in the third episode or not). As one can see form the table, the activity-level participation rates are same as the first episode participation rates for the OMDCEV model, which is expected. However, this is not true for the MDCEV model because, the model can predict an individual to participate in the second episode without ensuring participation in the first episode.

An aspect worth discussing here is the participation rate in a specific episode. That is, it is possible that the number of individuals who participate in exactly one episode is less than the number of individuals who participate in exactly two episodes. In the data sample, escorting has higher participation in exactly two episodes than just one episode. However, the same trend is not predicted by the OMDCEV model (nor the MDCEV model). This is possibly because the episode-specific constant for second episode could not be estimated (standard error on that parameter was high). Recall, however, that the same issue was not faced in the simulated datasets in Section 3, where participation in exactly three episodes of the second simulated activity was higher than participation in exactly first or second. The fact that such trend could be simulated and corresponding parameters could be retrieved demonstrates that the proposed model can indeed accommodate such conditions. The inability of the current empirical model to replicate such trend observed in the data is possibly specific to the empirical dataset we worked with.

The last column in Table 8a reports the total number of episodes observed and predicted for all 200 individuals in the validation samples (averaged across the 10 samples). Both the OMDCEV and MDCEV models underpredict the number of episodes compared to the observed numbers. This maybe because, as can be observed in Table 8b, both the models overpredict the episode-level activity durations than those observed in the data. And overpredicting episode-level activity durations exhausts the time budget available to individuals sooner than necessary for accurate prediction of the number of episodes. This is also why activity-level time allocations predicted by both the models are higher than those observed in the data. Therefore, it is important to extend our model formulation to accommodate upper bounds on episode-level activity durations. Doing so might help improve the predictions of episode-level durations and the number of chosen episodes.

Table 8a. Prediction Results: Observed and predicted shares of activity participation at an episode level.

Activity		Observed and predicted % participation in row activity (N = 200)							Number of Episodes
		Average across 10 outside samples with 20 repetitions							
		Activity-level participation	Episode 1 or more	Episode 2 or more	Episode 3 or more	Episode 4 or more	Episode 5 or more	Episode 6	
Escorting	Observed	30.0	30.0	19.9	7.1	4.1	--	--	121.9
	OMDCEV	30.5	30.5	10.9	4.4	1.8	--	--	99.1
	MDCEV	34.2	24.0	16.4	6.7	4.3	--	--	98.6
Shopping	Observed	46.0	46.0	14.1	4.4	1.1	0.3	0.1	131.8
	OMDCEV	41.5	41.5	12.7	3.3	0.7	0.0	0.0	109.7
	MDCEV	45.2	38.3	11.5	3.4	1.3	0.3	0.1	111.8
Maintenance	Observed	39.9	39.9	12.4	4.4	1.2	0.3	0.2	116.6
	OMDCEV	37.0	37.0	10.2	2.7	0.6	0.0	0.0	99.6
	MDCEV	40.7	32.9	10.8	3.3	1.2	0.3	0.2	99.1
Social	Observed	8.1	8.1	0.7	0.4	--	--	--	18.2
	OMDCEV	6.8	6.8	0.2	0.0	--	--	--	14.1
	MDCEV	6.0	5.3	0.5	0.2	--	--	--	13.9
Entertainment	Observed	7.0	7.0	0.0	--	--	--	--	13.9
	OMDCEV	5.4	5.4	0.1	--	--	--	--	11.1
	MDCEV	4.5	4.4	0.2	--	--	--	--	10.5
Visiting family and friends	Observed	19.6	19.6	3.9	0.8	0.2	--	--	48.8
	OMDCEV	16.5	16.5	1.8	0.1	0.0	--	--	36.7
	MDCEV	16.7	14.7	2.4	0.4	0.1	--	--	37.4
Active recreation	Observed	18.2	18.2	2.0	0.2	--	--	--	40.6
	OMDCEV	15.1	15.1	1.1	0.0	--	--	--	32.6
	MDCEV	16.4	15.3	1.5	0.1	--	--	--	32.7
Eat out	Observed	19.6	19.6	1.7	--	--	--	--	42.4
	OMDCEV	16.9	16.9	1.2	--	--	--	--	34.1
	MDCEV	16.2	15.3	1.2	--	--	--	--	34.9

--: Not applicable

Table 8b. Prediction Results: Observed and predicted time investment at an episode level.

Activity		Observed and predicted average time allocation (minutes) to each row activity per episode per individual who participated in the row activity (N = 200)						
		Average across 10 outside samples with 20 repetitions						
		Aggregate activity-level time allocation	Duration of Episode 1	Duration of Episode 2	Duration of Episode 3	Duration of Episode 4	Duration of Episode 5	Duration of Episode 6
Outside good	Observed	927.91						
	OMDCEV	883.04						
	MDCEV	873.29						
Escorting	Observed	22.9	17.3	6.3	4.7	2.6	--	--
	OMDCEV	40.9	34.8	9.6	6.7	5.1	--	--
	MDCEV	41.4	39.3	21.4	16.4	11.3	--	--
Shopping	Observed	61.9	51.3	27.7	18.5	13.9	4.7	0.3
	OMDCEV	111.2	109.5	28.3	15.1	7.0	0.5	0.1
	MDCEV	106.9	101.2	69.7	45.5	30.7	2.4	0.8
Maintenance	Observed	74.0	65.8	21.2	10.0	6.8	4.4	2.0
	OMDCEV	92.1	84.9	20.9	11.3	5.2	0.3	0.0
	MDCEV	89.6	91.6	44.8	30.1	27.9	9.5	1.3
Social	Observed	136.3	131.7	32.1	3.0	--	--	--
	OMDCEV	178.3	176.3	12.9	0.1	--	--	--
	MDCEV	182.8	189.4	55.8	21.2	--	--	--
Entertainment	Observed	142.7	142.7	0.0	--	--	--	--
	OMDCEV	220.0	219.4	2.7	--	--	--	--
	MDCEV	213.7	216.4	20.6	--	--	--	--
Visiting family and friends	Observed	157.2	141.4	68.7	44.7	3.0	--	--
	OMDCEV	203.0	199.9	55.1	7.4	0.0	--	--
	MDCEV	198.1	207.2	111.9	50.3	1.1	--	--
Active recreation	Observed	113.1	107.9	46.0	7.0	--	--	--
	OMDCEV	161.2	158.2	38.8	0.6	--	--	--
	MDCEV	158.9	162.6	103.2	27.7	--	--	--
Eat out	Observed	72.8	69.3	42.3	--	--	--	--
	OMDCEV	137.6	131.6	31.2	--	--	--	--
	MDCEV	126.7	129.7	65.6	--	--	--	--

--: Not applicable

To compare the predictive accuracy of the two models, we computed the Root Mean Squared Error (RMSE) of the predictions vis-à-vis the observed data, as below:

$$RMSE = \frac{\sum_k \sum_{j_k} p_{k,j_k} * r_{k,j_k}^2}{\sum_k \sum_{j_k} p_{k,j_k}} \tag{34}$$

where, p_{k,j_k} and o_{k,j_k} are the predicted and observed average participation rates (or time-allocation) for activity episode (k, j_k), and r_{k,j_k} is the relative error in prediction and is given by $r_{k,j_k} = \frac{\left(p_{k,j_k} - o_{k,j_k}\right)}{o_{k,j_k}}$. The RMSEs calculated for participation rates and time allocation both at activity-level and episode-level are reported in Table 9. As can be observed from the table, the RMSE values for episode-level activity participation and time allocation are lower for the OMDCEV model (0.198 for participation rate and 0.474 for time allocation) than those for the MDCEV model (0.201 for participation and 1.022 for time-investment). At an aggregate, activity level, activity participation was predicted better by the proposed OMDCEV model than the MDCEV model, whereas the activity-level time allocation is predicted better by MDCEV.

Table 9. RMSE of predictions from OMDCEV and MDCEV models

	Participation rate		Time allocation	
Model	**Activity-level participation**	**Episode-level participation**	**Episode-level time allocation**	**Activity-level time allocation**
OMDCEV	0.113	0.198	0.474	0.543
MDCEV	0.121	0.201	1.022	0.491

The comparison of the RMSE values suggest that the proposed OMDCEV model performed better than the traditional MDCEV when the comparison is done at a disaggregate, activity-episode level. Besides, irrespective of the predictive performance of the two models, OMDCEV should be the preferred model for disaggregate episode-level analysis because it accommodates logical consistency in the prediction of episodes.

5. SUMMARY AND CONCLUSIONS

This paper formulates a novel modeling framework to analyze multiple discrete continuous (MDC) choices at a disaggregate level, including the number of instances different alternatives are chosen and the amount of consumption at each instance of choice. The model is formulated for disaggregate, episode-level analysis of individuals' time-use, where all the episodes of each activity-type an individual undertakes in a day are analyzed along with the duration of each episode. In this model, choice alternatives are defined as episodes of different activity-types (*i.e.*, activity-type and episode number forms a choice alternative), so that time-allocation can be modeled at an episode level. In doing so, the formulation also ensures that a higher-numbered episode of an activity-type does not occur without the occurrence of all lower-numbered episodes of that activity-type (*i.e., j^{th}* episode of an activity does not occur without the *$(j\text{-}1)^{th}$* episode occurring). Accommodating such conditions via explicit constraints in the utility maximization framework is difficult, for it leads to mixed integer constraints. To circumvent this problem, we exploit the properties of MDC choice models with additive utility functions to condition on the baseline utility parameters. Specifically, instead of explicit constraints on time allocations of different episodes of an activity, we impose non-increasing ordering conditions on the baseline marginal utility functions of all episodes of that activity; with that of the first episode being first in the order, followed by the second episode, and so on. Since it is difficult to ensure such ordering among baseline utility parameters that include unbounded stochastic distributions, we develop a conditional likelihood function where the likelihood arising from the optimality conditions of the utility maximization problem is conditioned on the ordering of baseline marginal utilities.

A combination of the above-mentioned strategies, including ordering of baseline marginal utilities and conditioning on such ordering, and carefully specified utility functional form and IID type-1 extreme value distributions for the baseline marginal utility parameters results in the proposed OMDCEV model with a closed-form likelihood expression for the observed data. Interestingly, the likelihood expression derived for our proposed model is same as the likelihood expression of the traditional MDCEV model (with activity-episodes as choice alternatives) normalized by the rank-ordered logit terms for the probability of the baseline marginal utilities of the chosen activity-episodes of each activity being in a non-increasing order. The similarity of the likelihood expression with that of the traditional MDCEV model makes the estimation of the above formulation not much more difficult than that of the MDCEV model. Further, the forecasting procedure for the proposed model involves a simple modification of that for the MDCEV models.

In addition to the model formulation, likelihood function derivation, and a forecasting procedure, a simulation experiment (using synthetic datasets) is presented to verify the formulation as well as the retrievability of model parameters using the maximum likelihood method. The simulation results suggest the ability to retrieve model parameters with little bias using the familiar maximum likelihood method.

As a demonstration of the applicability of the proposed OMDCEV model and its benefits over the traditional MDCEV model, we apply both the models for analyzing episode-level activity participation and time allocation behavior of non-working adults in the Los Angeles region of California. The empirical data used for this analysis is drawn from the 2013 regional Household Travel Survey conducted by Southern California Association of Governments (SCAG) in the six-county region of Los Angeles. Both the models provide intuitive and behaviorally consistent explanations of the effect of various socio-demographic variables on individual's time use choices. In the context of a few covariates, the OMDCEV model offers more plausible interpretations than the MDCEV model. Also, the number of parameters needed in the OMDCEV model is much smaller than those for the MDCEV model, making the former more parsimonious than the latter. This is attributable to the structure of the OMDCEV model in that it accommodates inherent ordering in the baseline utility parameters across different episodes of an activity. Despite the larger number of parameters in the MDCEV model, the proposed OMDCEV model provides better fit to estimation data (as measured by AIC, BIC, and Vuong's test). In addition, the proposed model exhibits better performance in all the 10 validation samples in a 10-fold validation exercise. Furthermore, a comparison of prediction accuracies between the two models (in the 10-fold validation exercise) suggests a better prediction performance of the proposed OMDCEV model; especially when evaluated at the disaggregate, episode level. Besides, the traditional MDCEV model, despite the larger number of parameters, does not guarantee a logical occurrence of different episodes of an activity. It resulted in illogical predictions (where a higher-numbered episode was predicted without predicting a lower-numbered episode) for more than 32% of the times it was applied for prediction on the validation samples.

Overall, the proposed framework provides a simple yet conceptually appealing framework to analyze episode-level time allocation, while recognizing a logical ordering in the occurrence of episodes. The framework is applicable for not only disaggregate (*i.e.*, episode level) activity participation and time-use analysis but also a variety of other MDC choice contexts where it is important to analyze choice and resource allocation at a disaggregate level. Examples include the analysis of household vehicle ownership and utilization, while considering the number of vehicles of each type and the mileage on each type of the vehicle owned. Also, the framework can be easily extended to accommodate flexible stochastic specifications such as correlations across different episodes of an activity, unobserved heterogeneity in parameters etc. through mixing distributions.

In the context of future work, recall that both the OMDCEV and MDCEV models exhibited overprediction of the episode-level activity durations, due to which both the models underpredicted the number of activity-episodes an individual participates in a day. It would be useful to extend the proposed framework to accommodate maximum and minimum bounds on episode-level time allocation. The novelty of the proposed model framework lies in the use of conditioning of stochastic parameters in lieu of explicit constraints on consumptions; because, the latter might lead to complications in deriving model likelihood, parameter estimation, and prediction. Such a conditioning approach opens possibilities for other research, where inequalities or ordering conditions among stochastic parameters of the model can be explored to accommodate different behaviors. One such application for MDC models is to model situations where a certain alternative 'B' should be chosen only when another alternative 'A' is also chosen. In the context of transportation expenditures, for example, expenditure in vehicle maintenance should happen only with a non-zero expenditure to fuel, since, it is unlikely for vehicles to not be used but incur maintenance costs. A possible solution to accommodate such situations in MDC models is to impose appropriate ordering conditions among the baseline marginal utilities while also allowing different satiation parameters (current formulation imposes same satiation parameters across alternatives that involve an ordering of baseline marginal utilities). It is likely that such situations can be handled better by combining the proposed strategy with Bhat's (2018) flexible utility form, where separate baseline utility parameters are employed for discrete and continuous components of consumption. Although such extension might lead to complicated likelihood expressions, it might be a viable and simpler alternative to having explicit constraints on consumptions. This, along with other extensions such as imposing bounds on consumptions are being currently explored by the authors.

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APPENDIX A: DERIVATION OF THE DETERMINANT OF JACOBIAN

From Equation (27), the Jacobian is written for the following transformation:

$$\varepsilon_{m,i_m} = V_1 - V_{m,i_m} + \varepsilon_1 = -\ln(t_1^*) - \beta'_{m,i_m} z_{m,i_m} + \ln\left(\frac{t_{m,i_m}^*}{\gamma_m} + 1\right) + \varepsilon_1$$

Writing $t_1^* = T - \sum_{m=2}^{M} \sum_{i_m=1}^{I_m} t_{m,i_m}^*$, $V_1 - V_{m,i_m} + \varepsilon_1$ becomes:

$$V_1 - V_{m,i_m} + \varepsilon_1 =$$

$$-\ln\left(T - \sum_{m=2}^{M} \sum_{i_m=1}^{I_m} t_{m,i_m}^*\right) - \beta'_{m,i_m} z_{m,i_m} + \ln\left(\frac{t_{m,i_m}^*}{\gamma_m} + 1\right) + \varepsilon_1 \tag{A1}$$

The the Jacobian matrix of dimension $\left((\delta - 1) \times (\delta - 1)\right)$, with all of its off-diagonal elements as $\frac{1}{t_1^*}$ and the diagonal elements as:

$$\frac{1}{t_1^*} + \frac{1}{t_{m,i_m}^* + \gamma_m} \tag{A2}$$

Specifically, the matrix takes the following structure:

$$\begin{bmatrix} \frac{1}{t_1^*} + \frac{1}{t_{21}^* + \gamma_2} & \frac{1}{t_1^*} & \cdots & \frac{1}{t_1^*} \\ \frac{1}{t_1^*} & \frac{1}{t_1^*} + \frac{1}{t_{22}^* + \gamma_2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \\ \frac{1}{t_1^*} & \frac{1}{t_1^*} & \cdots & \frac{1}{t_1^*} + \frac{1}{t_{M,I_m}^* + \gamma_M} \end{bmatrix}_{(\delta-1)\times(\delta-1)}$$

This is similar to the Jacobian derived in traditional MDC models (Bhat, 2008), except with episode-level (as opposed to activity-level) alternatives. The determinant of the Jacobian is:

$$\left[\prod_{m=1}^{M} \prod_{i_m=1}^{I_m} \left(\frac{1}{t_{m,i_m}^* + \gamma_m}\right)\right] \left[\sum_{m=1}^{M} \sum_{i_m=1}^{I_m} \left(t_{m,i_m}^* + \gamma_m\right)\right] \tag{A3}$$

APPENDIX B: DERIVATION OF THE OMDCEV MODEL LIKELIHOOD FUNCTION

The integrand (conditioned on ε_1) in Equation (27) is:

$$\prod_{m=2}^{M}\prod_{i_m=1}^{\mathrm{I}_m}\{g_{\varepsilon}(V_1 - V_{m,i_m} + \varepsilon_1)\} * |\mathrm{J}| * \prod_{m=2}^{M}\{P(U_1 > \max_{i'_m \in \mathbb{I}'_{\mathbb{m}}}(\bar{U}_{m,i'_m}))\}$$
$$* \{\prod_{m=2}^{M}\prod_{i'_m=I_m+1}^{J_m}\left(\frac{e^{\mu\bar{V}_{m,i'_m}}}{\sum_{r=i'_m}^{J_m} e^{\mu\bar{V}_{m,r}}}\right)\} * \prod_{m'=M+1}^{K}\{P\left(U_1 > \max_{i'_{m'} \in \mathbb{I}'_{\mathbb{m}'}}(\bar{U}_{m',i'_{m'}})\right)\} * \prod_{m'=M+1}^{K}\prod_{i'_{m'}=1}^{J_{m'}}\left(\frac{e^{\mu\bar{V}_{m',i'_{m'}}}}{\sum_{r'=i'_{m'}}^{J_{m'}} e^{\mu\bar{V}_{m',r'}}}\right) \quad \text{(B1)}$$

In the above expression, the terms $\prod_{m=2}^{M}\prod_{i'_m=I_m+1}^{J_m}\left(\frac{e^{\mu\bar{V}_{m,i'_m}}}{\sum_{r=i'_m}^{J_m} e^{\mu\bar{V}_{m,r}}}\right)$ and $\prod_{m'=M+1}^{K}\prod_{i'_{m'}=1}^{J_{m'}}\left(\frac{e^{\mu\bar{V}_{m',i'_{m'}}}}{\sum_{r'=i'_{m'}}^{J_{m'}} e^{\mu\bar{V}_{m',r'}}}\right)$ are constants for the integral as they don't depend on ε_1. Therefore, let $\prod_{m=2}^{M}\prod_{i'_m=I_m+1}^{J_m}\left(\frac{e^{\mu\bar{V}_{m,i'_m}}}{\sum_{r=i'_m}^{J_m} e^{\mu\bar{V}_{m,r}}}\right) * \prod_{m'=M+1}^{K}\prod_{i'_{m'}=1}^{J_{m'}}\left(\frac{e^{\mu\bar{V}_{m',i'_{m'}}}}{\sum_{r'=i'_{m'}}^{J_{m'}} e^{\mu\bar{V}_{m',r'}}}\right)$ be denoted $as\ A$.

Further, we know that $\max_{i'_m \in \mathbb{I}'_{\mathbb{m}}}(\bar{U}_{m,i'_m})$ follows a type I extreme value distribution with a location parameter given by: $\frac{1}{\mu}\ln\left(\sum_{i'_m=I_m+1}^{J_m} e^{\mu\bar{V}_{m,i'_m}}\right)$ and scale parameter by: $\frac{1}{\mu}$. So, let $\max_{i'_m}(\bar{U}_{m,i'_m}) = \bar{U}_m^*$, whose distribution is given by:

$F_{U_m^*}(\varepsilon) = \exp\left(-e^{-\mu\left(\varepsilon - \frac{1}{\mu}\ln\left(\sum_{i'_m=I_m+1}^{J_m} e^{\mu\bar{V}_{m,i'_m}}\right)\right)}\right)$. Therefore,

$$\prod_{m=2}^{M} P(U_1 > \max_{i'_m \in \mathbb{I}'_{\mathbb{m}}}(\bar{U}_{m,i'_m})) = \prod_{m=2}^{M} F_{U_m^*}(U_1) = \prod_{m=2}^{M}\exp\left(-\frac{\sum_{i'_m=I_m+1}^{J_m} e^{\mu\bar{V}_{m,i'_m}}}{e^{\mu U_1}}\right) = \exp\left(-\frac{\sum_{m=2}^{M}\sum_{i'_m=I_m+1}^{J_m}\mu\bar{V}_{m,i'_m}}{e^{\mu U_1}}\right) \quad \text{(B2)}$$

Similarly, let $\max_{i'_{m'} \in \mathbb{I}'_{\mathbb{m}'}} (\bar{U}_{m',i'_{m'}}) = \bar{U}^*_{m'}$, which also follows a type-1 extreme value distribution. Therefore,

$$\prod_{m'=M+1}^{K} P(U_1 > \max_{i'_{m'} \in \mathbb{I}'_{\mathbb{m}'}} (\bar{U}_{m',i'_{m'}})) = \prod_{m'=M+1}^{K} F_{U^*_{m'}}(U_1) = \prod_{m'=M+1}^{K} \exp\left(-\frac{\sum_{i'_{m'}=1}^{J_{m'}} e^{\mu \bar{V}_{m',i'_{m'}}}}{e^{\mu U_1}}\right)$$
$$= \exp\left(-\frac{\sum_{m'=M+1}^{K} \sum_{i'_{m'}=1}^{J_{m'}} e^{\mu \bar{V}_{m',i'_{m'}}}}{e^{\mu U_1}}\right) \tag{B3}$$

The product of the two expressions $\prod_{m=2}^{M} P(U_1 > \max_{i'_m \in \mathbb{I}'_{\mathbb{m}}} (\bar{U}_{m,i'_m}))$ and $\prod_{m'=M+1}^{K} P(U_1 > \max_{i'_{m'} \in \mathbb{I}'_{\mathbb{m}'}} (\bar{U}_{m',i'_{m'}}))$ is given by:

$$\exp\left(-\frac{\sum_{m=2}^{M} \sum_{i'_m=I_m+1}^{J_m} e^{\mu \bar{V}_{m,i'_m}} + \sum_{m'=M+1}^{K} \sum_{i'_{m'}=1}^{J_{m'}} e^{\mu \bar{V}_{m',i'_{m'}}}}{e^{\mu U_1}}\right) \tag{B4}$$

The integrand (conditioned on ε_1) in Equation (27) may now be written as:

$$\prod_{m=2}^{M} \prod_{i_m=1}^{\mathrm{I}_m} \{g_\varepsilon(V_1 - V_{m,i_m} + \varepsilon_1)\} * |\mathrm{J}| * \exp\left(-\frac{\sum_{m=2}^{M} \sum_{i'_m=I_m+1}^{J_m} e^{\mu \bar{V}_{m,i'_m}} + \sum_{m'=M+1}^{K} \sum_{i'_{m'}=1}^{J_{m'}} e^{\mu \bar{V}_{m',i'_{m'}}}}{e^{\mu U_1}}\right) * A \tag{B5}$$

Next, after expanding the first term of the above expression using the following density function for Gumbel distribution:

$$\{g_\varepsilon(V_1 - V_{m,i_m} + \varepsilon_1) = \mu e^{-\mu(V_1 - V_{m,i_m} + \varepsilon_1)} * \exp\left(-e^{-\mu(V_1 - V_{m,i_m} + \varepsilon_1)}\right)$$

The integrand (conditioned on ε_1) in Equation (27) may be written as:

$$|J| * A * \prod_{m=2}^{M} \prod_{i_m=1}^{\mathrm{I}_m} \mu e^{-\mu\left(V_1 - V_{m,i_m} + \varepsilon_1\right)} * \exp\left(-e^{-\mu\left(V_1 - V_{m,i_m} + \varepsilon_1\right)}\right) * \exp\left(-\frac{\sum_{m=2}^{M} \sum_{i'_m=I_m+1}^{J_m} e^{\mu \bar{V}_{m,i'_m}} + \sum_{m'=M+1}^{K} \sum_{i'_{m'}=1}^{J_{m'}} e^{\mu \bar{V}_{m',i'_{m'}}}}{e^{\mu U_1}}\right) \tag{B6}$$

The above expression further simplifies as (where, $\delta = \sum_{m=1}^{M} \sum_{i_m=1}^{\mathrm{I}_m} 1$):

$$|J| * A * \mu^{\delta-1} * \exp\left(\sum_{m=2}^{M} \sum_{i_m=1}^{\mathrm{I}_m} \left[-\mu\left(V_1 - V_{m,i_m}\right)\right]\right) * \exp(-\mu\varepsilon_1)^{\delta-1} * \exp\left(-\left(\frac{\sum_{m=2}^{M} \sum_{i_m=1}^{\mathrm{I}_m} e^{\mu\left(V_{m,i_m} - V_1\right)} + \sum_{m=2}^{M} \sum_{i'_m=I_m+1}^{J_m} e^{\mu\left(\bar{V}_{m,i'_m} - V_1\right)} + \sum_{m'=M+1}^{K} \sum_{i'_{m'}=1}^{J_{m'}} e^{\mu\left(\bar{V}_{m',i'_{m'}} - V_1\right)}}{e^{\mu\varepsilon_1}}\right)\right) \tag{B7}$$

Let $B = \sum_{m=2}^{M} \sum_{i_m=1}^{\mathrm{I}_m} e^{\mu\left(V_{m,i_m} - V_1\right)} + \sum_{m=2}^{M} \sum_{i'_m=I_m+1}^{J_m} e^{\mu\left(\bar{V}_{m,i'_m} - V_1\right)} + \sum_{m'=M+1}^{K} \sum_{i'_{m'}=1}^{J_{m'}} e^{\mu\left(\bar{V}_{m',i'_{m'}} - V_1\right)}$

The above expression, which is the integrand (conditioned on ε_1) in Equation (27) may now be written as:

$$|J| * A * \mu^{\delta-1} * \exp\left(\sum_{m=2}^{M} \sum_{i_m=1}^{\mathrm{I}_m} \left[-\mu\left(V_1 - V_{m,i_m}\right)\right]\right) * \exp(-\mu\varepsilon_1)^{\delta-1} * \exp\left(-\frac{B}{e^{\mu\varepsilon_1}}\right) \tag{B8}$$

Next, the above expression should be integrated over the distribution of ε_1 to obtain an expression for the numerator in the likelihood expression of Equation (26).

Start of integration to simplify the expression in Equation (27)

Considering the expression in Equation (B8), the expression in Equation (27) may be written as:

$$\int_{\varepsilon_1=-\infty}^{\infty} |\mathrm{J}| * A * \mu^{\delta-1} * \exp\left(\sum_{m=2}^{M}\sum_{i_m=1}^{\mathrm{I}_m}\left[-\mu\left(V_1 - V_{m,i_m}\right)\right]\right) * \exp(-\mu\varepsilon_1)^{\delta-1} * \exp\left(-\frac{B}{e^{\mu\varepsilon_1}}\right) * \mu e^{-\mu(\varepsilon_1)} * \exp\left(-e^{-\mu(\varepsilon_1)}\right) d\varepsilon_1$$

This can be re-written as:

$$|\mathrm{J}| * A * \mu^{\delta-1} * \exp\left(\sum_{m=2}^{M}\sum_{i_m=1}^{\mathrm{I}_m}\left[-\mu\left(V_1 - V_{m,i_m}\right)\right]\right) \int_{\varepsilon_1=-\infty}^{\infty} \exp(-\mu\varepsilon_1)^{\delta-1} * \exp\left(-\left(\frac{B}{e^{\mu\varepsilon_1}}\right)\right) * \mu e^{-\mu(\varepsilon_1)} * \exp\left(-e^{-\mu(\varepsilon_1)}\right) d\varepsilon_1$$

Considering just the integrand, let us focus on the following integral:

$$\int_{\varepsilon_1=-\infty}^{\infty} \exp(-\mu\varepsilon_1)^{\delta-1} * \exp\left(-(B+1)e^{-\mu(\varepsilon_1)}\right) * \mu e^{-\mu(\varepsilon_1)} d\varepsilon_1$$

In the above integral, since $B = \sum_{m=2}^{M}\sum_{i_m=1}^{\mathrm{I}_m} e^{\mu\left(V_{m,i_m}-V_1\right)} + \sum_{m=2}^{M}\sum_{i'_m=I_m+1}^{J_m} e^{\mu\left(\bar{V}_{m,i'_m}-V_1\right)} + \sum_{m'=M+1}^{K}\sum_{i'_{m'}=1}^{J_{m'}} e^{\mu\left(\bar{V}_{m',i'_{m'}}-V_1\right)}$,

$B + 1 = \sum_{m=1}^{M}\sum_{i_m=1}^{\mathrm{I}_m} e^{\mu\left(V_{m,i_m}-V_1\right)} + \sum_{m=2}^{M}\sum_{i'_m=I_m+1}^{J_m} e^{\mu\left(\bar{V}_{m,i'_m}-V_1\right)} + \sum_{m'=M+1}^{K}\sum_{i'_{m'}=1}^{J_{m'}} e^{\mu\left(\bar{V}_{m',i'_{m'}}-V_1\right)} = \sum_{k=1}^{K}\sum_{j_k=1}^{J_k} e^{\mu\left(V_{k,j_k}-V_1\right)}$.

Let $B+1$ be a. The above integral thus becomes,

$$\int_{\varepsilon_1=-\infty}^{\infty} \exp(-\mu\varepsilon_1)^{\delta-1} * \exp\left(-(a) * e^{-\mu(\varepsilon_1)}\right) * \mu e^{-\mu(\varepsilon_1)} d\varepsilon_1$$

Now, let $e^{-\mu(\varepsilon_1)}$ be t. Therefore $dt = -\mu \exp(-\mu\varepsilon_1)\ d\varepsilon_1$. The above integral then becomes: $-\int_{t=\infty}^{0} t^{\delta-1} * \exp(-(a) * t) * dt$. Further, let $u = -at$. Therefor $du = -adt$. The above integral may be rewritten as:

$$-\int_{u=-\infty}^{0} (-1)^{\delta-1} \left(\frac{u}{a}\right)^{\delta-1} * \exp(u) * \left(-\frac{du}{a}\right) = (-1)^{\delta+1} \frac{1}{a^{\delta}} \int_{u=-\infty}^{0} u^{\delta-1} * \mathrm{e}^{u} * du$$

This integral $\int_{u=-\infty}^{0} u^{\delta-1} * \mathrm{e}^{u} * du$ may be recursively calculated as: $\int_{u=-\infty}^{0} u^{\delta-1} * \mathrm{e}^{u} * du = (-1)^{\delta-1} * (\delta-1)!$.

Therefore,

$$\int_{\varepsilon_1=-\infty}^{\infty} \exp(-\mu\varepsilon_{1,1})^{\delta-1} * \exp\left(-(a) * e^{-\mu(\varepsilon_{1,1})}\right) * \mu e^{-\mu(\varepsilon_{1,1})} d\varepsilon_{1,1} = \frac{(-1)^{2\delta} * (\delta-1)!}{a^{\delta}}$$

Feeding the value of $a = \sum_{k=1}^{K} \sum_{j_k=1}^{J_k} e^{\mu\left(V_{k,j_k} - V_1\right)}$, the above integral may be written as:

$$\frac{(\delta-1)!}{\left(\sum_{k=1}^{K} \sum_{j_k=1}^{J_k} e^{\mu\left(V_{k,j_k} - V_1\right)}\right)^{\delta}}$$

End of integration

Given the above closed-from integral, the numerator in the likelihood expression of Equation (26) becomes:

$$\frac{|J| * \mu^{\delta-1} * (\delta-1)! * \prod_{m=2}^{M} \prod_{i'_m=I_m+1}^{J_m} \left(\frac{e^{\mu \bar{V}_{m,i'_m}}}{\sum_{r=i'_m}^{J_m} e^{\mu \bar{V}_{m,r}}} \right) * \prod_{m'=M+1}^{K} \prod_{i'_{m'}=1}^{J_{m'}} \left(\frac{e^{\mu \bar{V}_{m',i'_{m'}}}}{\sum_{r'=i'_{m'}}^{J_{m'}} e^{\mu \bar{V}_{m',r'}}} \right) * \exp\left(\sum_{m=1}^{M} \sum_{i_m=1}^{\mathrm{I}_m} \left[\mu\left(V_{m,i_m} - V_1 \right) \right] \right)}{\left(\sum_{k=1}^{K} \sum_{j_k=1}^{J_k} e^{\mu\left(V_{k,j_k} - V_1 \right)} \right)^{\delta}} \tag{B9}$$

Since, the denominator in Equation (26) is simply a product of rank-ordered logit expressions, the likelihood expression in Equation (26) becomes:

$$\frac{|J| * \mu^{\delta-1} * (\delta-1)! * \prod_{m=2}^{M} \prod_{i'_m=I_m+1}^{J_m} \left(\frac{e^{\mu \bar{V}_{m,i'_m}}}{\sum_{r=i'_m}^{J_m} e^{\mu \bar{V}_{m,r}}} \right) * \prod_{m'=M+1}^{K} \prod_{i'_{m'}=1}^{J_{m'}} \left(\frac{e^{\mu \bar{V}_{m',i'_{m'}}}}{\sum_{r'=i'_{m'}}^{J_{m'}} e^{\mu \bar{V}_{m',r'}}} \right) * \exp\left(\sum_{m=1}^{M} \sum_{i_m=1}^{\mathrm{I}_m} \left[\mu\left(V_{m,i_m} - V_1 \right) \right] \right)}{\prod_{k=1}^{K} \prod_{j_k=1}^{J_\mathrm{K}} \left(\frac{e^{\mu \bar{V}_{k,j_k}}}{\sum_{s=j_k}^{J_k} e^{\mu \bar{V}_{k,s}}} \right) \left(\sum_{k=1}^{K} \sum_{j_k=1}^{J_k} e^{\mu\left(V_{k,j_k} - V_1 \right)} \right)^{\delta}} \tag{B10}$$

In the above expression, the exploded logit terms corresponding to all non-chosen alternatives cancel out, and hence the final likelihood expression, after simplification is:

$$|J| * \mu^{\delta-1} * (\delta-1)! * \frac{\exp\left(\sum_{m=1}^{M} \sum_{i_m=1}^{\mathrm{I}_m} \left[\mu\left(V_{m,i_m} \right) \right] \right)}{\prod_{k=1}^{M} \prod_{j_k=1}^{\mathrm{I}_k} \left(\frac{e^{\mu \bar{V}_{k,j_k}}}{\sum_{s=j_k}^{J_k} e^{\mu \bar{V}_{k,s}}} \right) * \left(\sum_{k=1}^{K} \sum_{j_k=1}^{J_k} e^{\mu\left(V_{k,j_k} \right)} \right)^{\delta}} \tag{B11}$$

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