An Integrated Game-Theoretic and Discrete Choice Framework to Evaluate the Benefits of Collaboration between Rideshare and Transit Service Providers

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Abstract

The idea of mobility provided as a service (MaaS) by rideshare service providers has gained significant traction in many cities due to attractive features such as on-demand mobility and door-to-door connectivity. However, there has been a concern that the increasing shares of MaaS modes can potentially increase traffic congestion. On the other hand, inadequate connectivity between travelers' residential or activity locations and transit stops discourages public transit usage. To address these issues, cities are considering collaborative mobility services. In such a collaborative service, the rideshare service providers facilitate transport for the first- and/or last-mile part of a journey and the transit agency facilitates transport for the long-haul part. This paper describes modeling methods that integrate game-theory and discrete choice theory to formulate and evaluate such collaborations. Application of the proposed models for a stylized network shows promising results in the form of increased market shares, higher profits, decreased travel times for both transit agency and rideshare service providers in addition to benefits for travelers.

Keywords: Transportation, First-mile, Last-mile, Game-theory, Discrete choice

1 Introduction

The idea of mobility provided as a service (MaaS) by rideshare service providers (RSPs) has disrupted the status quo with a significant impact on transportation systems in several cities worldwide. Various RSPs have gained popularity due to features like on-demand service, door-to-door connectivity and personalized rides. The increasing popularity and market share of MaaS, however, can potentially add significantly to traffic congestion (Erhardt et al., 2019; Roy et al., 2020). Also, these services are not necessarily affordable to the low-income population. On the other hand, the public transit agencies continue to lose market shares due to increased travelers' affluence and the emergence of different travel modes, including MaaS. Although available at generally affordable prices, inadequate first- and last-mile connectivity may cripple public transit systems (Krygsman et al., 2004).

Because of these issues, there is an interest in exploring collaboration between RSPs and transit agencies, where the RSPs provide first- and last-mile connectivity and the transit agencies provide the long-haul part of the travel (Shaheen & Chan, 2016; Stiglic et al., 2018). Such a collaborative service can potentially bring together the best of both operators' services by providing seamless door-to-door travel while keeping the overall travel prices low and reducing traffic congestion.

However, in many cities barring a few exceptions, neither the RSPs nor the transit agencies appear eager to co-operate with each other. This may be because of the lack of evidence on the potential benefits of such a collaboration. Further, since such collaboration will involve sharing of operations and demand data, there might be concerns that revealing such data to other transport operators might lead to loss of market shares.

There are also positive views that such collaboration can potentially benefit the RSPs and transit agencies in the form of increased market shares and revenues, and can benefit the travelers in the form of better connectivity, reduced travel time, and cheaper rides. However, the scientific literature lacks evidence and modeling methods to formulate, evaluate, and facilitate discussions for such collaboration. Therefore, the objectives of this study are to:

- Propose a tri-level model based on game-theory and discrete choice theory to evaluate collaboration mechanisms between RSPs and transit agency while considering traveler preferences and response to pricing.
- Use the proposed modeling methods to assist RSPs and transit agency in pricing the collaborative mobility service for enhancing market shares and profits to both operators.
- Apply the models for evaluating the potential benefits of such a collaborative mobility service between the RSPs and a transit agency for a major travel corridor in Bengaluru (a major metropolitan city in India).

2 Literature survey

This research paper discusses a tri-level model for collaboration between RSPs and a transit agency. However, the application of multi-level models is not new to transportation systems and several such models have been developed in the past to achieve some desired objectives. Y. Liu et al. (2019) discussed a framework to integrate mode choice models with mobility-on-demand (MoD) systems and public transit services using a Bayesian optimization approach. A stated preference (SP) survey was also conducted to determine the sensitivity of travelers to major determinants of mode choice. Both MoD's and transit's operational model were integrated with the mode choice model to maximize the MoD system's profit.

Another bi-level framework was proposed by Pinto et al. (2020) for the integration of transit and shared autonomous vehicle mobility services. They suggested that transit agencies can replace their usual fleet with shared autonomous vehicles for inefficient routes during certain times of the day and in some regions. This may be advantageous to the transit agencies in terms of operational cost. In the model, the upper level consisted of the transit network frequency setting problem to include autonomous vehicle fleet size as a decision variable. The lower level consisted of the dynamic combined mode choice traveler assignment problem.

A tri-level formulation was proposed by Dandl et al. (2021) for regulating mobility-on-demand services using a Bayesian optimization approach. The proposed framework had a policy maker (or regulator) at the top, a mobility service provider (MSP, which provides automated mobility-on-demand (AMOD) type of service) in the middle and travelers at the bottom. The policy maker tries to maximize the social welfare (sum of its revenue, tolls, parking costs, traveler's utility etc.), the mobility service provider tries to maximize its profit given the policies of the regulator and the traveler tries to maximize its utility given the services offered by the regulator and the MSP.

Optimal pricing models have also been widely studied in the transportation literature, e.g., the following research papers focused on congestion and/or dynamic pricing: Braid (1996), C.-C. Lu et al. (2008), Y. Liu et al. (2009), Lou et al. (2010), Guo and Yang (2010), Wu et al. (2011), Lawphongpanich and Yin (2012) and Do Chung et al. (2012). Supply-demand balance can also be used as a criterion for pricing: Banerjee et al. (2015), Y. Liu and Li (2017), Zha et al. (2018) and Fang et al. (2019). In the context of cooperative game theory, Rosenthal (2017) and Pantelidis et al. (2020) discussed cost allocation mechanisms for applications in the transportation field.

We have used a similar approach described in the above mentioned research papers to formulate a tri-level model for first- and last-mile services. This model is later applied to a travel corridor of a major metropolitan city in India to analyze the benefits of collaboration in the form of change in profits, market shares, and travel times of RSPs and bus agency. At the upper level we have employed a Stackelberg game between RSPs and bus agency to obtain optimal first- and last-mile service prices. The lower level consists of a mode choice model to determine the response of travelers to the prices set by RSPs and bus agency. Stackelberg game has been used in the past to model transportation markets. See Adler et al. (2020) for a review of the same.

There have been several efforts in the past to increase the use of public transport systems among the travelers, e.g., Melis (2022) discussed a demand responsive public transport system. The framework proposed by us also helps to increase the ridership of transit agencies by using an optimal pricing scheme for collaborative service where the RSPs provide first- and last-mile connectivity to the transit stops. The literature has some work on the pricing schemes for RSPs and transit agencies but only a few studies focus on the adaptation of these schemes for first- and last-mile services. The following is a brief description of the work that exists in the literature and attempts to solve a similar problem undertaken by us: Z. Liu et al. (2012) analyzed the causes for the failure of the first generation of public bicycle system as a last-mile transport in Beijing. Cheng et al. (2014) solved the last-mile problem by developing a dynamic and demand responsive mechanism to arrange rideshares for taxis/passenger vans. Wang and Odoni (2016) studied the performance of the last mile transportation system as a function of design parameters which is useful for the planning of such a transportation system. A pricing and collaboration model was discussed by Ko et al. (2018) for the last-mile delivery of goods to the consumers. Agussurja et al. (2019) proposed a two-level Markov decision process for the dispatch of last-mile service vehicles. Chen and Wang (2018) formulated a constrained non-linear optimization problem concerning social welfare to determine prices charged, vehicle capacity, and number of vehicles at each metro station for last-mile service. Wang (2019) formulated a mixed-integer programming model to minimize the passengers' waiting time and riding time in the context of last-mile travel. Heuristic approaches were also proposed to solve the same. Ma et al. (2019) customized the idle vehicle relocation and queueing-theory based vehicle dispatch algorithms to devise a ridesharing strategy where a private MoD service agency provides either or both first- and last-mile connectivity to transit. Bian and Liu (2019) designed a mechanism to determine customized prices, vehicle routing plan, and optimal vehicle-passenger matching for first-mile service. The objective function minimized the sum of passengers' inconvenience cost and the agency's transportation cost. As a follow-up work to Bian and Liu (2019), Bian et al. (2020) adapted the traditional Vickrey–Clarke–Groves (VCG) mechanism to develop a corresponding pricing scheme. A routing algorithm and an integrated scheduling procedure was proposed by Molenbruch et al. (2021) for the synchronization of dial-a-ride services and public transport. Ke et al. (2021) analyzed the cases where ride-sourcing service either substitutes or complements the transit service.

The following are the key gaps in literature:

- Much of the literature focuses on first- and/or last-mile pricing schemes for RSPs alone. In our literature review we did not find joint pricing schemes (i.e., setting prices by both RSPs and transit agency) for the collaborative service, where RSPs provide the first- and last-mile services and transit agency provides the long-haul service. Joint pricing schemes offer an advantage in this context. If only some operators of the collaboration framework are allowed to set prices for the offered service, it may lead to disagreement among other operators who were not allowed to set prices. The joint pricing scheme will provide an opportunity to all the operators for pricing and structuring their services.

- To the best of our knowledge, no prior study evaluates the benefits of collaboration between the transit agency and the RSPs (for first- and last-mile services) by comparing their profits between the collaboration and the no-collaboration scenarios while taking into account the "usual service". The "usual service" is defined as the service provided by the RSPs to the travelers when there is no collaboration between them and transit agency.

Our work attempts to fill these gaps using an integrated game-theoretic and discrete choice modeling framework to evaluate the benefits of collaboration between RSPs and bus agency to both these operators and to travelers. In our model, the RSPs are allowed to provide usual service between travelers' origin and destination, while also providing first- and last-mile connectivity to/from transit stops. In addition, while the modeling framework allows the presence of multiple RSPs operating their services, we apply the framework for the following cases prevalent in most cities: (a) a single RSP and (b) two RSPs. Using these models, one can not only analyze optimal pricing of collaborative RSP+bus services, but also assess the pros and cons of any or both of the RSPs not collaborating with the bus agency.

3 Model formulation

Notation

Choice sets of travelers in different cases:

C_1^{nco} Choice set of travelers in the no-collaboration case when only one RSP is present, $C_1^{nco} = \{r, b, pv\}$, where r denotes RSP, b denotes bus and pv denotes personal vehicle.

C_1^{co} Choice set of travelers in the collaboration case when only one RSP is present, $C_1^{co} = \{r, b, pv, r+b\}$, where $r+b$ denotes RSP+bus collaborative service.

C_2^{nco} Choice set of travelers in the no-collaboration case when two RSPs are present, $C_2^{nco} = \{r_1, r_2, b, pv\}$, where r_1 denotes RSP_1 and r_2 denotes RSP_2.

C_2^{co} Choice set of travelers in the collaboration case when two RSPs are present, $C_2^{co} = \{r_1, r_2, b, pv, r_1+b, r_2+b\}$, where r_1+b denotes collaborative service involving RSP_1 and bus. Similarly, r_2+b denotes collaborative service involving RSP_2 and bus.

Mode choice models' inputs, parameters, and other related notations:

p_i Travel price associated with i^{th} mode alternative, $p_i \in \mathbb{R}_+$, $i \in \{C_1^{nco}, C_1^{co}, C_2^{nco}, C_2^{co}\}$, depending on the scenario under study, e.g., when only one RSP is present, p_r, p_b and p_{pv} are the travel costs in the no-collaboration case and p_{r+b} is the travel cost of RSP+bus collaborative service. Note that p_{r+b} is the sum of travel prices charged by the RSP and the bus agency for their part of the collaborative service. It is not known a priori and is a *decision variable*. Please refer to the formulated models for the details on its determination.

t_i Travel time associated with i^{th} mode alternative, $t_i \in \mathbb{R}_+$, $i \in \{C_1^{nco}, C_1^{co}, C_2^{nco}, C_2^{co}\}$, e.g., when only one RSP is present, t_r, t_b and t_{pv} are the travel times in the no-collaboration case and t_{r+b} is the travel time of RSP+bus collaborative service.

β_i^c Alternative specific constant of i^{th} mode alternative, $\beta_i^c \in \mathbb{R}$, $i \in \{C_1^{nco}, C_1^{co}, C_2^{nco}, C_2^{co}\}$.

β_p Magnitude of price sensitivity of travelers, $\beta_p \in \mathbb{R}_+$.

β_t Magnitude of time sensitivity of travelers, $\beta_t \in \mathbb{R}_+$.

V_i Deterministic part of traveler's utility for i^{th} mode alternative, $V_i = \beta_i^c - \beta_p p_i - \beta_t t_i$, $V_i \in \mathbb{R}$, $i \in \{C_1^{nco}, C_1^{co}, C_2^{nco}, C_2^{co}\}$.

θ_k Nesting parameter for nest k present in the mode choice model, $\theta_k \in [0, 1]$.

λ_{ik} Allocation parameter for mode alternative i, which represents how much mode alternative i is a member of nest k, $\lambda_{ik} \in [0, 1]$, $i \in \{C_1^{co}, C_2^{nco}, C_2^{co}\}$.

B_k k^{th} nest (subset of mode alternatives) present in the mode choice model.

α_i Probability of choosing i^{th} mode alternative (or share of that mode alternative), $\alpha_i \in [0, 1]$, $i \in \{C_1^{nco}, C_1^{co}, C_2^{nco}, C_2^{co}\}$, e.g., when only one RSP is present, α_r, α_b and α_{pv} are the shares of mode alternatives in the no-collaboration case and α_{r+b} is the share of RSP+bus collaborative service.

$\bar{\alpha}$ A tuple containing shares of the mode alternatives, $\bar{\alpha} = (\alpha_i | i \in \{C_1^{nco}, C_1^{co}, C_2^{nco}, C_2^{co}\})$.

Network variables:

D Total demand (number of trips) between origin o and destination d, $D \in \mathbb{Z}_+$.

FFT_{ik} Free flow travel time for i^{th} alternative on link-k in the network. See Figure (1) for the details. For link-1 and link-3, $i \in \{r, pv\}$. For link-2, $i \in \{r, b, pv\}$. In case of two RSPs, both are assumed to have same values of free flow travel times on all the links in the network.

Δ_{walk}^k Time taken by the traveler to walk on link-k, $k \in \{1, 3\}$.

Profit optimization variables:

m_i Operator's cost for i^{th} mode alternative for a trip, $m_i \in \mathbb{R}_+$. When only one RSP is present, $i \in \{r, rco, b\}$. Note that for the collaborative service, m_{rco} represents the RSP's operating cost for first- and last-mile services added together and m_b represents the bus operating cost. When two RSPs are present, $i \in \{r_1, r_2, rco_1, rco_2, b\}$. Here also, for the RSP$_1$+bus collaborative service, m_{rco_1} represents the RSP$_1$'s operating cost for first- and last-mile services added together and m_b represents the bus operating cost. The same is true for RSP$_2$+bus collaborative service.

$m_{i(bal)}$	Operator's cost for rebalancing the vehicle of i^{th} mode alternative back to the trip origin, $m_{i(bal)} \in \mathbb{R}_+$. When only one RSP is present, $i \in \{r, rco, b\}$. For the collaborative service, $m_{rco(bal)}$ represents the RSP's rebalancing cost for first- and last-mile services added together and $m_{b(bal)}$ represents the bus rebalancing cost. When two RSPs are present, $i \in \{r_1, r_2, rco_1, rco_2, b\}$. Here also, for the RSP$_1$+bus collaborative service, $m_{rco_1(bal)}$ represents the RSP$_1$'s rebalancing cost for first- and last-mile services added together and $m_{b(bal)}$ represents the bus rebalancing cost. The same is true for RSP$_2$+bus collaborative service.
Occ_r	Occupancy of RSP's vehicle, $Occ_r \in \mathbb{Z}_+$. In case of two RSPs, both are assumed to have Occ_r as the vehicle occupancy.
Occ_b	Occupancy of bus agency's vehicle, $Occ_b \in \mathbb{Z}_+$.
Occ_{pv}	Occupancy of personal vehicle, $Occ_{pv} \in \mathbb{Z}_+$.
n_0, n_1, n_2	Constants, $n_0, n_1, n_2 \in (1, \infty)$. These are the multipliers used to set upper bounds on collaborative service prices.

Let us consider a four-node network as shown in Figure (1). All the models developed in this research paper are for this stylized network to gain insights on the benefits of collaboration between the RSPs and the bus agency. They also help to address the question of revenue-sharing mechanisms for such services without facing the computational complexities of solving the problem for a real-sized framework. Of course, scaling up the proposed framework and its solutions methods for large-sized networks is an important avenue for future work. It can be noted that although the proposed models may not be directly applicable to a general network, they are still useful to make a case to do the pilot test of the collaborative service for a corridor in a city before deploying this service in the whole city.

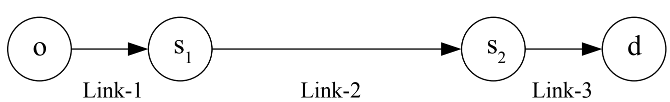

Figure 1: A stylized four-node network considered in this study.

The mode alternatives available to a traveler at node o are:

1. Rideshare service (RSP) from node o to node d. The RSP uses cars to serve the demand.

2. Bus service (transit agency) from node s_1 (a bus stop) to node s_2 (another bus stop); the travelers need to walk from o to s_1 and s_2 to d.

3. Personal vehicle from node o to node d. The travelers use cars as their personal vehicles.

4. Collaborative service (RSP+bus) in which RSP provides first-mile service from node o to node s_1 and last-mile service from node s_2 to node d. The bus provides long-haul service from node s_1 to node s_2. This mode alternative is available only when RSP and bus agency collaborate.

We have assumed the following while formulating the models:

- Total demand (D number of trips between o and d) is inelastic to travel prices and travel times.

- For each trip, only travel price and travel time of the available travel modes between o and d influence the mode choice. An implication of this assumption is that travelers making different trips between o and d are homogeneous in their mode choice preferences. This assumption can be easily relaxed by enhancing the mode choice utility functions to include additional covariates representing traveler characteristics and mode attributes.

- After the RSP or bus takes the travelers from a start node to an end node, the same vehicle needs to return to the start node (without passengers) and the cost incurred to them in this process is called the rebalancing cost. This assumption arises because the demand is not spatially dispersed given the stylized network in Figure (1). In real networks, the demand for travel has spatio-temporal distribution, offering opportunities for the transport operators to reduce rebalancing costs. Although it is a strong assumption that 50% of the total miles traveled are empty (i.e., miles without passengers that add to operator costs but do not generate revenue), some studies estimate that the empty miles covered by ride-sharing vehicles may reach 35% to 50% of the total miles traveled by them (Cramer & Krueger, 2016; Henao & Marshall, 2019; Komanduri et al., 2018). Nevertheless, to assess the robustness of our findings, we conduct additional investigations for smaller percentages of empty miles.

To evaluate the benefits of collaboration between RSPs and bus agency, this paper formulates a sequence of three models discussed below. Each model in the sequence is obtained by relaxing assumptions in the preceding model.

- *Model with congestion-independent travel times, fixed bus price, and mode shares based on the multinomial logit model for mode choice (Model-1)*: Here, to fix ideas, we have made the simplistic assumption that travel times do not depend on traffic conditions. For the collaborative mode (RSP+bus) of travel, we have assumed that the bus agency will charge the same price it would charge to other travelers boarding at s_1 and alighting at s_2. Further, mode shares of the trips between o and d are given by the multinomial logit expressions due to assumption of independent and identically distributed (IID) Gumbel random utility terms governing travelers' mode choice. Under such assumptions, an optimization problem (RSP's profit maximization) can be solved to obtain the RSP's price for the first- and last-mile services while keeping the bus price fixed. This leads to closed form expressions that are useful for comparing the RSP's and bus agency's profits in no-collaboration and with-collaboration cases.

- *Model with congestion-dependent travel times, fixed bus price, and mode shares based on the cross-nested logit model for mode choice (Model-2)*: Here, we have allowed travel times to vary with the mode shares to bring in the effect of congestion while keeping the bus price fixed and utilized the cross-nested logit expressions to determine the shares of mode alternatives. In this setting, due to congestion-dependent travel times, the RSP's profit maximization problem does not yield closed form expressions for optimal profits and therefore we have to resort to numerical methods. The cross-nested logit formulation has been explored for mode choice to recognize that the random utility component of the collaborative mode would likely be correlated with that of the bus mode and the RSP mode (with different levels of correlation). Ignoring these correlations could have potentially led to distorted estimates of optimal prices, mode shares, and profits (and therefore, the benefits of collaboration).

- *Stackelberg game with congestion-dependent travel times and mode shares based on the cross-nested logit model for mode choice (Model-3 and 4)*: Here, we have formulated a Stackelberg game where, for the collaborative mode, the bus agency optimizes its price for the long-haul service (s_1 to s_2) and the RSP optimizes its price for the first- and last-mile service (o to s_1 and s_2 to d). The bus agency has been designated as the leader in the Stackelberg game. The RSP observes the bus agency's strategy (price set by the bus agency) and then proceeds to decide its own strategy (price set by the RSP). The cross-nested logit expression gives the shares of mode alternatives.

We have used a non-cooperative game approach to model the problem instead of a cooperative one, although a cooperative game may give better payoffs to the players. The non-cooperative game mirrors reality because the RSPs and bus agency are competitors in the transportation market who want to attract as many travelers as possible. Therefore, a non-cooperative game serves as a good starting point to model such competition in the market. Note that although the RSPs and the bus agency are playing a non-cooperative game, it does not mean that they are not collaborating. The RSPs and the bus agency are concerned with the maximization of their profit only while setting the price for their part of the collaborative service (non-cooperative game). However, once the prices are set they need to co-ordinate with each other (e.g., synchronization of schedules etc.) for providing the collaborative service to the travelers. The competition is structured through a Stackelberg game framework.

While the frameworks described so far are for a single RSP, it is straightforward to extend the framework to include multiple RSPs and other modes of travel. Since many major cities across the world have at least two major RSPs, we have formulated a Stackelberg game with two RSPs, each of them collaborating with the bus agency to offer first- and last-mile services. To differentiate between the two cases, let the Stackelberg game that includes only one RSP be called Model-3 and that with two RSPs Model-4. Using this formulation, one can not only assess the benefits of both the RSPs collaborating with the bus agency, but also examine a scenario where only one of the two RSPs collaborates with the bus agency (and the other does not).

3.1 Model with congestion-independent travel times, fixed bus price, and mode shares based on the multinomial logit model for mode choice (Model-1)

3.1.1 No-collaboration case

Based on the theory of random utility maximization (Ben-Akiva & Lerman, 1985) and assuming that the random utility terms are IID across alternatives and travelers, the market shares of various travel mode alternatives are given by the multinomial logit (MNL) expression, where the share of mode i ($i \in C_1^{nco}$) is:

$$\alpha_i = \frac{e^{\beta_i^c - \beta_p p_i - \beta_t t_i}}{\sum_{i \in C_1^{nco}} e^{\beta_i^c - \beta_p p_i - \beta_t t_i}} \tag{1}$$

The resulting profits of RSP and bus agency when they do not collaborate are provided by the following expressions (Wischik (2018) discusses integration of mode choice expressions with profit expressions for RSPs):

$$\text{Profit}_r = D\alpha_r \left(p_r - \frac{m_r + m_{r(bal)}}{Occ_r} \right) \tag{2}$$

$$\text{Profit}_b = D\alpha_b \left(p_b - \frac{m_b + m_{b(bal)}}{Occ_b} \right) \tag{3}$$

These profit expressions are straightforward to derive and will be used in comparisons later.

3.1.2 With-collaboration case

Let us consider the case when both RSP and bus agency collaborate to provide a service. The RSP provides the first-mile service from node o to node s_1 and the last-mile service from node s_2 to node d at a price p_{rco} for both first- and last-mile services. The bus agency provides the long-haul service from node s_1 to node s_2 at the same price p_b that it charges to other bus travelers from s_1 to s_2. Therefore, the traveler choosing the collaborative mode incurs a total travel cost: $p_{r+b} = p_{rco} + p_b$. Since, p_b and all other parameters are fixed, this problem involves determining the optimal price p_{rco} of RSP which can be subsequently used to obtain the market shares and profits of RSP and bus agency. The share of RSP's usual service as a function of p_{rco} is:

$$\alpha_r(p_{rco}) = \frac{e^{\beta_r^c - \beta_p p_r - \beta_t t_r}}{\sum_{i \in C_1^{co}} e^{\beta_i^c - \beta_p p_i - \beta_t t_i}} = \frac{e^{\beta_r^c - \beta_p p_r - \beta_t t_r}}{\sum_{i \in C_1^{nco}} e^{\beta_i^c - \beta_p p_i - \beta_t t_i} + e^{\beta_{r+b}^c - \beta_p (p_{rco} + p_b) - \beta_t t_{r+b}}} \tag{4}$$

The other mode shares can be similarly written. Consequently, the profit of RSP can be expressed as:

$$\text{Profit}_r(p_{rco}) = D\alpha_r(p_{rco}) \left(p_r - \frac{m_r + m_{r(bal)}}{Occ_r} \right) + D\alpha_{r+b}(p_{rco}) \left(p_{rco} - \frac{m_{rco} + m_{rco(bal)}}{Occ_r} \right) \tag{5}$$

It can be shown that the value of above profit at optimal first- and last-mile service price is greater than the profit of RSP in no-collaboration case. Appendix-A in the supplementary material provides a derivation of this result.

The profit of bus as a function of p_{rco} is:

$$\begin{aligned}\text{Profit}_b(p_{rco}) &= D\alpha_b(p_{rco})\left(p_b - \frac{m_b + m_{b(bal)}}{Occ_b}\right) + D\alpha_{r+b}(p_{rco})\left(p_b - \frac{m_b + m_{b(bal)}}{Occ_b}\right) \\ &= D(\alpha_b(p_{rco}) + \alpha_{r+b}(p_{rco}))\left(p_b - \frac{m_b + m_{b(bal)}}{Occ_b}\right)\end{aligned} \tag{6}$$

Here also, it can be shown that at the optimum value of first- and last-mile service price for which the profit of RSP is maximum, the profit of bus agency is also greater than the no-collaboration case. Appendix-B in the supplementary material provides a derivation of this result.

In summary, we see that both the RSP and the bus agency get additional profit by collaborating and therefore have an incentive to collaborate. It can be noted that the assumptions in this model may hold true during the off-peak period of travel demand. During such period, the number of vehicles present on the network links will be relatively less. Therefore, the travel times on the links will be equal to their free flow travel times, which are constant. In this case, the RSP and the bus agency are guaranteed to see an increase in profit. This shows the robustness of the model to changes in input variables and parameters.

3.2 Model with congestion-dependent travel times, fixed bus price, and mode shares based on the cross-nested logit model for mode choice (Model-2)

3.2.1 No-collaboration case

In this setting, the market share of an alternative $i (i \in C_1^{nco})$ is given by the multinomial logit expression which is as follows:

$$\alpha_i = \frac{e^{V_i}}{\sum_{i \in C_1^{nco}} e^{V_i}} \tag{7}$$

Here, $V_i = \beta_i^c - \beta_p p_i - \beta_t t_i(\bar{\alpha})$. The travel times are no longer constant and depend on market shares. Therefore, the market share of any mode alternative depends on the market share of all the other mode alternatives. It can be noted that the equations corresponding to market shares constitute a set of fixed-point equations. These can be solved using numerical methods to determine the market shares. Let (7) be represented as: $\alpha_i = f_i(\bar{\alpha})$, $i \in C_1^{nco}$. In this case, Brouwer's fixed-point theorem ensures the existence of fixed-points, since $f_i : [0,1]^{|C_1^{nco}|} \to [0,1] \ \forall \ i \in C_1^{nco}$. The market shares can be estimated as a solution to the following optimization problem by recognizing that at the fixed-point, the value of the function to be optimized will be least (zero):

$$\begin{aligned}
\min_{\bar{\alpha}} \quad & \sum_{i \in C_1^{nco}} (\alpha_i - f_i(\bar{\alpha}))^2 \\
\text{s.t.} \quad & \sum_{i \in C_1^{nco}} \alpha_i = 1 \\
& 0 \le \alpha_i \le 1, i \in C_1^{nco}
\end{aligned} \tag{8}$$

The obtained market shares can be substituted in profit and travel time expressions to get these values.

3.2.2 With-collaboration case

The market share of an alternative $i (i \in C_1^{co})$ is given by the cross-nested logit (CNL) expression which not only depends on $\bar{\alpha}$ but also on the first- and last-mile price p_{rco}:

$$\alpha_i(p_{rco}) = \frac{\sum_k (\lambda_{ik} e^{V_i})^{\frac{1}{\theta_k}} (\sum_{j \in B_k} (\lambda_{jk} e^{V_j})^{\frac{1}{\theta_k}})^{\theta_k - 1}}{\sum_{l=1}^{K} (\sum_{j \in B_l} (\lambda_{jl} e^{V_j})^{\frac{1}{\theta_l}})^{\theta_l}} \tag{9}$$

Here also, $V_i = \beta_i^c - \beta_p p_i - \beta_t t_i(\bar{\alpha})$. Figure (2) illustrates the nesting structure of the mode choice model. The RSP+bus collaborative mode is assumed to belong to two nests – one with RSP and another with bus. This is so because RSP+bus mode combines both RSP and bus modes. Therefore, it may have some modal attributes common to RSP and bus modes which have not been included in the utility expressions. This will create correlation among the unobserved components of utilities. Personal vehicle alternative is not included in any of the nests for ease of analysis. However, the model can be easily adapted to the case where personal vehicle also belongs to the existing nests or a new nest (while keeping in view parameter identifiability).

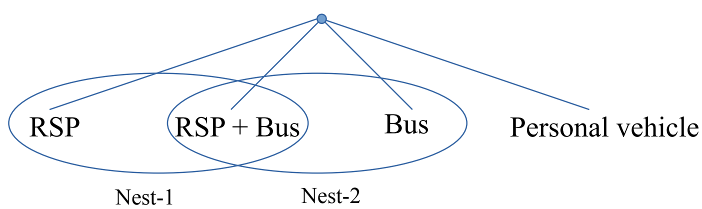

Figure 2: Cross-nested logit model for travel modes.

The profit optimization problem of the RSP can be represented as follows:

$$\begin{aligned}
\max_{p_{rco}} \quad & D\alpha_r(p_{rco})\left(p_r - \frac{m_r + m_{r(bal)}}{Occ_r}\right) + D\alpha_{r+b}(p_{rco})\left(p_{rco} - \frac{m_{rco} + m_{rco(bal)}}{Occ_r}\right) \\
\text{s.t.} \quad & \frac{m_{rco} + m_{rco(bal)}}{Occ_r} \le p_{rco} \le n_0 \frac{m_{rco} + m_{rco(bal)}}{Occ_r}
\end{aligned} \tag{10}$$

The inequality constraint in (10) can be explained as follows: The price charged to a traveler should be at least the RSP's total operating cost per traveler and it should be upper bounded by a value which is some multiplier ($n_0 > 1$) times the RSP's total operating cost per traveler. The provided optimization problem can at best be solved using a numerical method due to its complex nature. At each iteration of the algorithm, fixed-point equations have to be solved (for a given value of p_{rco}) to determine the market shares as described earlier.

3.3 Stackelberg game with congestion-dependent travel times and mode shares based on the cross-nested logit model for mode choice (for one RSP): Model-3

In the earlier version of the model, the bus agency's price for the RSP+bus collaborative mode remained constant and fixed at a price charged to regular bus users. Only the RSP had the flexibility to adjust its price. The bus agency did not get an opportunity to adjust its price for that mode in the previous model. This may act as a discouraging factor for collaboration. The Stackelberg game-theoretic framework can be used to enable both players to set their prices in the collaborative service. The bus sets the long-haul service price as p_{bco} and the RSP sets the first- and last-mile services price as p_{rco}. The traveler choosing the collaborative mode sees the total price of collaborative service from o to d: $p_{r+b} = p_{rco} + p_{bco}$.

3.3.1 No-collaboration case

This case is the same as presented in Section-3.2.1. The market shares of mode alternatives are given by the multinomial logit expressions and the fixed point equations corresponding to market shares have to be solved using the optimization problem provided in (8).

3.3.2 With-collaboration case

The market share of an alternative i ($i \in C_1^{co}$) is given by the cross-nested logit expression as given in (11), which is now a function of both p_{rco} and p_{bco}:

$$\alpha_i(p_{rco}, p_{bco}) = \frac{\sum_k (\lambda_{ik} e^{V_i})^{\frac{1}{\theta_k}} (\sum_{j \in B_k} (\lambda_{jk} e^{V_j})^{\frac{1}{\theta_k}})^{\theta_k - 1}}{\sum_{l=1}^{K} (\sum_{j \in B_l} (\lambda_{jl} e^{V_j})^{\frac{1}{\theta_l}})^{\theta_l}} \tag{11}$$

In this game, we have designated the bus agency as a leader and the RSP as a follower. This recognizes that the bus agency may not have sufficient dynamic information, unlike the RSP, to change p_{bco} dynamically. Having the bus agency as a lead player can also enable the extension of the game to any number of RSPs. An illustration of the Stackelberg game is given in Figure (3).

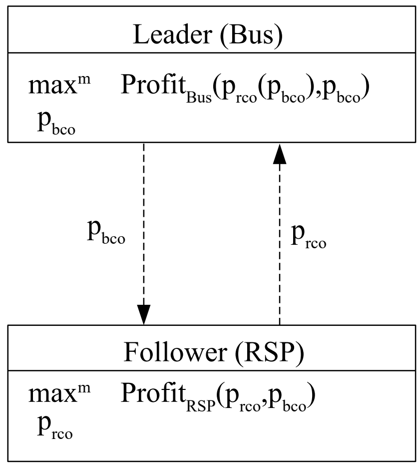

Figure 3: Stackelberg game for one RSP.

The resulting profit optimization problem of the bus agency is:

$$\max_{p_{bco}} \quad D\alpha_b(p_{rco}(p_{bco}), p_{bco})\left(p_b - \frac{m_b + m_{b(bal)}}{Occ_b}\right) + D\alpha_{r+b}(p_{rco}(p_{bco}), p_{bco})\left(p_{bco} - \frac{m_b + m_{b(bal)}}{Occ_b}\right) \tag{12}$$

$$\text{s.t.} \quad \frac{m_b + m_{b(bal)}}{Occ_b} \leq p_{bco} \leq n_1 \frac{m_b + m_{b(bal)}}{Occ_b}$$

where p_{rco} as a function of p_{bco} is given by the solution of the profit optimization problem of the RSP, which is:

$$\max_{p_{rco}} \quad D\alpha_r(p_{rco}, p_{bco})\left(p_r - \frac{m_r + m_{r(bal)}}{Occ_r}\right) + D\alpha_{r+b}(p_{rco}, p_{bco})\left(p_{rco} - \frac{m_{rco} + m_{rco(bal)}}{Occ_r}\right) \tag{13}$$

$$\text{s.t.} \quad \frac{m_{rco} + m_{rco(bal)}}{Occ_r} \leq p_{rco} \leq n_2 \frac{m_{rco} + m_{rco(bal)}}{Occ_r}$$

The bus agency optimizes anticipating the RSP's actions. The RSP as the follower optimizes knowing the bus agency's action. We have used the differential evolution heuristic algorithm to solve the above two-level optimization problem. The fixed-point equations have to be solved (for a given value of p_{rco} and p_{bco}) in this case as well to determine the market shares while performing iterations corresponding to differential evolution.

3.4 Stackelberg game with congestion-dependent travel times and mode shares based on the cross-nested logit model for mode choice (for two RSPs): Model-4

The formulated Stackelberg game for one RSP can be easily extended to the case when two RSPs are present. It will also require the calculation of a Nash equilibrium between the two RSPs in terms of prices for first- and last-mile services.

3.4.1 No-collaboration case

The nesting structure in the no-collaboration case is shown in Figure (4). Both the RSPs are assumed to be present in a nest. The market share of an alternative $i (i \in C_2^{nco})$ is given by the cross-nested logit expression:

$$\alpha_i = \frac{\sum_k (\lambda_{ik} e^{V_i})^{\frac{1}{\theta_k}} (\sum_{j \in B_k} (\lambda_{jk} e^{V_j})^{\frac{1}{\theta_k}})^{\theta_k - 1}}{\sum_{l=1}^{K} (\sum_{j \in B_l} (\lambda_{jl} e^{V_j})^{\frac{1}{\theta_l}})^{\theta_l}} \tag{14}$$

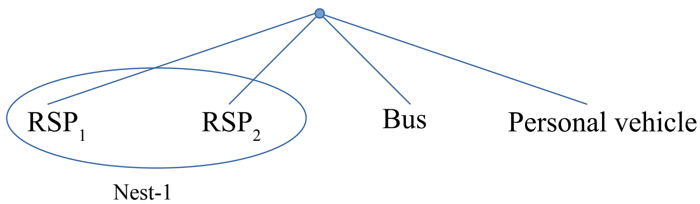

Figure 4: Cross-nested logit model for travel modes in the presence of two RSPs (no-collaboration case).

This case requires calculation of fixed-points corresponding to the market shares. Again, for numerical evaluation we have to solve (15) to identify feasible solution, which is similar to the discussion presented in section 3.2.1.

$$\begin{aligned} \min_{\bar{\alpha}} \quad & \sum_{i \in C_2^{nco}} (\alpha_i - f_i(\bar{\alpha}))^2 \\ \text{s.t.} \quad & \sum_{i \in C_2^{nco}} \alpha_i = 1 \\ & 0 \leq \alpha_i \leq 1, i \in C_2^{nco} \end{aligned} \tag{15}$$

3.4.2 With-collaboration case

The market share of an alternative i $(i \in C_2^{co})$ is given by the cross-nested logit expression provided in (16), which is now a function of p_{rco_1}, p_{rco_2} and p_{bco}:

$$\alpha_i(p_{rco_1}, p_{rco_2}, p_{bco}) = \frac{\sum_k (\lambda_{ik} e^{V_i})^{\frac{1}{\theta_k}} (\sum_{j \in B_k} (\lambda_{jk} e^{V_j})^{\frac{1}{\theta_k}})^{\theta_k - 1}}{\sum_{l=1}^{K} (\sum_{j \in B_l} (\lambda_{jl} e^{V_j})^{\frac{1}{\theta_l}})^{\theta_l}} \tag{16}$$

The nesting structure of the model is given in Figure (5). A total of eight nests are present in this case. We have assumed RSP$_1$ to be present in three nests with the following mode alternatives: RSP$_2$, RSP$_1$+bus and RSP$_2$+bus. Similarly, RSP$_2$ is assumed to be present in the nests with following mode alternatives: RSP$_1$+bus and RSP$_2$+bus. RSP$_1$+bus and RSP$_2$+bus are also present in a nest and each of these mode alternatives is present in a separate nest with the bus mode alternative.

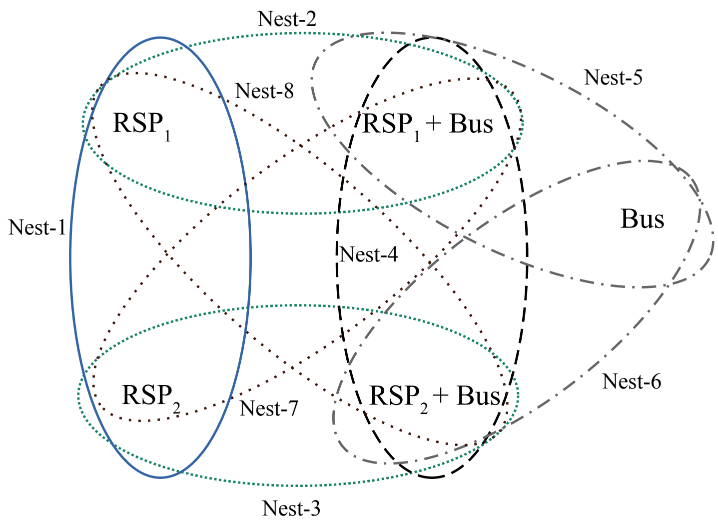

Figure 5: Cross-nested logit model for travel modes in the presence of two RSPs (with-collaboration case).

An illustration of Stackelberg game with two RSPs is given in Figure (6). In this setting, the optimization problem of the bus agency is:

$$\begin{aligned} \max_{p_{bco}} \quad & D\alpha_b(p_{rco_1}(p_{bco}), p_{rco_2}(p_{bco}), p_{bco})\left(p_b - \frac{m_b + m_{b(bal)}}{Occ_b}\right) + \\ & D\alpha_{r_1+b}(p_{rco_1}(p_{bco}), p_{rco_2}(p_{bco}), p_{bco})\left(p_{bco} - \frac{m_b + m_{b(bal)}}{Occ_b}\right) + \\ & D\alpha_{r_2+b}(p_{rco_1}(p_{bco}), p_{rco_2}(p_{bco}), p_{bco})\left(p_{bco} - \frac{m_b + m_{b(bal)}}{Occ_b}\right) \\ \text{s.t.} \quad & \frac{m_b + m_{b(bal)}}{Occ_b} \leq p_{bco} \leq n_1 \frac{m_b + m_{b(bal)}}{Occ_b} \end{aligned} \tag{17}$$

where p_{rco_1} and p_{rco_2} are obtained by the Nash equilibrium solution (for a given value of p_{bco}) of the optimization problems of RSP_1 and RSP_2 provided in (18) and (19) respectively:

$$\begin{aligned} \max_{p_{rco_1}} \quad & D\alpha_{r_1}(p_{rco_1}, p_{rco_2}, p_{bco})\left(p_{r_1} - \frac{m_{r_1} + m_{r_1(bal)}}{Occ_r}\right) + \\ & D\alpha_{r_1+b}(p_{rco_1}, p_{rco_2}, p_{bco})\left(p_{rco_1} - \frac{m_{rco_1} + m_{rco_1(bal)}}{Occ_r}\right) \\ \text{s.t.} \quad & \frac{m_{rco_1} + m_{rco_1(bal)}}{Occ_r} \leq p_{rco_1} \leq n_2 \frac{m_{rco_1} + m_{rco_1(bal)}}{Occ_r} \end{aligned} \tag{18}$$

$$\begin{aligned}\max_{p_{rco_2}} \quad & D\alpha_{r_2}(p_{rco_1}, p_{rco_2}, p_{bco})\left(p_{r_2} - \frac{m_{r_2} + m_{r_2(bal)}}{Occ_r}\right) + \\ & D\alpha_{r_2+b}(p_{rco_1}, p_{rco_2}, p_{bco})\left(p_{rco_2} - \frac{m_{rco_2} + m_{rco_2(bal)}}{Occ_r}\right) \\ \text{s.t.} \quad & \frac{m_{rco_2} + m_{rco_2(bal)}}{Occ_r} \leq p_{rco_2} \leq n_2 \frac{m_{rco_2} + m_{rco_2(bal)}}{Occ_r}\end{aligned} \tag{19}$$

Note that the provided Stackelberg game has three optimization problems. Each of these optimization problems depends on the outcome of other optimization problems and can at best be solved using a numerical method. We have used the differential evolution heuristic algorithm to solve these optimization problems. The fixed-point equations have to be solved (for a given value of p_{rco_1}, p_{rco_2} and p_{bco}) in this case as well to determine the market shares.

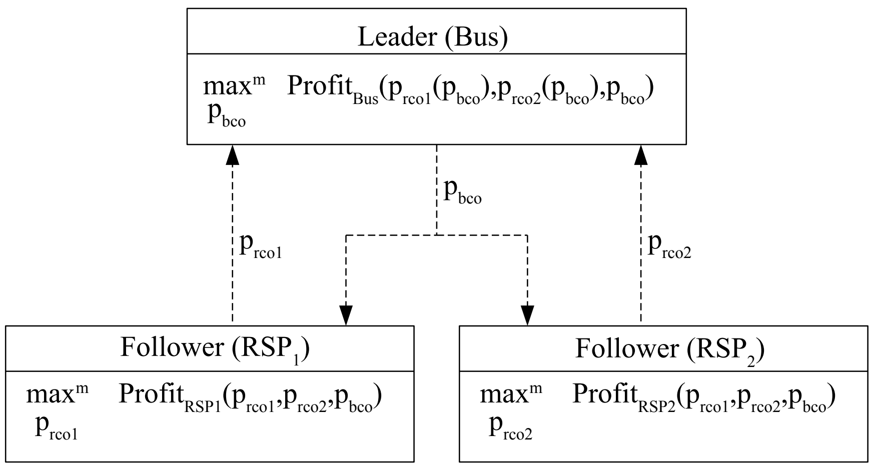

Figure 6: Stackelberg game for two RSPs.

4 Case studies for travel corridors in Bengaluru

For applying the formulated models to a real-world situation, we have considered the following travel corridors:

- *Corridor-1*: Indian Institute of Science (IISc) campus (node o) to the Airport (node d) in Bengaluru, India (total length of the corridor is around 30 km). The intermediate bus stop is the *Mekhri* circle bus stop (node s_1), which is around 2.2 km from the IISc campus.
- *Corridor-2*: IISc campus (node o) to the Banashankari locality (node d) in Bengaluru, India (total length of the corridor is around 15.3 km). The *Yeswanthpur* circle (node s_1) is considered as the intermediate bus stop, which is around 1.3 km from the IISc campus.

Since the last-mile section of the journey is not present in these cases, for the sake of demonstration, we have replicated the first-mile section's characteristics for the last mile. Note that Corridor-1 is our major travel corridor and the simulation results have been discussed in detail for this corridor. We have used the other travel corridor to test the reliability and applicability of the models.

The Bureau of Public Roads (BPR) functions (Z. Lu et al., 2016) can be used to express travel times as a function of traffic volumes and roadway capacity. A general form of BPR functions is given as follows:

$$t(x) = FFT\left[1 + u\left(\frac{x}{C}\right)^{v}\right] \tag{20}$$

Here, $t(x)$ is the actual flow-dependent travel time and FFT is the free flow travel time on a link. x is the link flow and C is the capacity of the link. u and v are empirically determined constants ($u = 0.15$ and $v = 4.0$). To render the travel times of various mode alternatives to depend on congestion, the BPR function can be expressed as follows:

$$t(\bar{\alpha}) = FFT\left[1 + u\left(\frac{x(\bar{\alpha})}{C}\right)^{v}\right] \tag{21}$$

The travel times associated with different mode alternatives can be expressed as the sum of travel times on the three links shown in Figure (1) and are given by (22)-(25):

$$t_r(\bar{\alpha}) = FFT_{r1}\left[1 + u\left(\frac{x'(\bar{\alpha})}{C}\right)^{v}\right] + FFT_{r2}\left[1 + u\left(\frac{x(\bar{\alpha})}{C}\right)^{v}\right] + FFT_{r3}\left[1 + u\left(\frac{x'(\bar{\alpha})}{C}\right)^{v}\right] \tag{22}$$

$$t_b(\bar{\alpha}) = \Delta^1_{walk} + FFT_{b2}\left[1 + u\left(\frac{x(\bar{\alpha})}{C}\right)^{v}\right] + \Delta^3_{walk} \tag{23}$$

$$t_{pv}(\bar{\alpha}) = FFT_{pv1}\left[1 + u\left(\frac{x'(\bar{\alpha})}{C}\right)^{v}\right] + FFT_{pv2}\left[1 + u\left(\frac{x(\bar{\alpha})}{C}\right)^{v}\right] + FFT_{pv3}\left[1 + u\left(\frac{x'(\bar{\alpha})}{C}\right)^{v}\right] \tag{24}$$

$$t_{r+b}(\bar{\alpha}) = FFT_{r1}\left[1 + u\left(\frac{x'(\bar{\alpha})}{C}\right)^{v}\right] + FFT_{b2}\left[1 + u\left(\frac{x(\bar{\alpha})}{C}\right)^{v}\right] + FFT_{r3}\left[1 + u\left(\frac{x'(\bar{\alpha})}{C}\right)^{v}\right] \tag{25}$$

Here, FFT_{ik} is the free flow travel time of i^{th} mode alternative on link-k and Δ^k_{walk} is the time taken by the traveler to walk on link-k. See the notations for more details. The term $x'(\bar{\alpha})$ represents the traffic flow on the first- and last-mile links whereas $x(\bar{\alpha})$ is the traffic flow on the long-haul link. Both these terms are expressed as:

$$x(\bar{\alpha}) = PCU_{car} \times \frac{D\alpha_1}{Occ_r} + PCU_{bus} \times \frac{D\alpha_2}{Occ_b} + PCU_{car} \times \frac{D\alpha_3}{Occ_{pv}} + PCU_{bus} \times \frac{D\alpha_4}{Occ_b} \tag{26}$$

$$x'(\bar{\alpha}) = PCU_{car} \times \frac{D\alpha_1}{Occ_r} + PCU_{car} \times \frac{D\alpha_3}{Occ_{pv}} + PCU_{car} \times \frac{D\alpha_4}{Occ_r} \tag{27}$$

Here, PCU refers to the passenger car units of a vehicle. $PCU_{car} = 1.0$ and $PCU_{bus} = 2.2$ have been adopted from IRC (1990). The expression for $x(\bar{\alpha})$ can be explained as follows: Consider the bus mode alternative. The division of the number of travelers choosing this mode alternative ($D\alpha_2$) by the occupancy of bus (Occ_b) provides the required number of vehicles to serve this demand $\left(\frac{D\alpha_2}{Occ_b}\right)$. The multiplication of the corresponding number of vehicles by PCU_{bus} provides the equivalent passenger car units of bus $\left(PCU_{bus} \times \frac{D\alpha_1}{Occ_b}\right)$. Similar equivalent passenger car units can be calculated for all the mode alternatives present on the link and summed up to obtain the link flow. The expression for $x'(\bar{\alpha})$ can be explained in a similar manner.

We have used the following metrics to evaluate the benefits of collaboration:

• Change in the profits of the RSP(s) and the bus agency from no-collaboration to with-collaboration scenario. The profits are reported as profit per unit demand (or profit per traveler).

• Change in the effective mode shares (effective mode shares are defined as the sum of mode shares of usual service and collaborative service) for both the RSP(s) and the bus agency.

• Change in the travel times for the RSP(s) and the bus agency. It is used as a way to measure change in traffic congestion.

• Change in the expected maximum utility (EMU) of travelers associated with travel mode choice. McFadden et al. (1978) provides an expression for the expected maximum utility of generalized extreme value models given as:

$$\bar{U} = \ln\left(\sum_{l=1}^{K}\left(\sum_{j\in B_l}(\lambda_{jl}e^{V_j})^{\frac{1}{\theta_l}}\right)^{\theta_l}\right) \tag{28}$$

It can be divided by the coefficient of income in the utility expression to get the monetary equivalent which is also known as consumer surplus. Since income is not considered as an influencing variable in the current case study we have reported only the change in the value of expected maximum utility from no-collaboration to with-collaboration scenario.

4.1 Simulation results for Stackelberg game with congestion-dependent travel times and mode shares based on the cross-nested logit model for mode choice (for one RSP)

The overall empirical setup has considered travel prices faced by the travelers, free flow travel times of the links, vehicle occupancies, operators' (RSP and bus agency) costs used in profit calculations, assumed demand and walking times whose values are given in Table (1) for both corridors. Since the estimated price and time sensitivity values are not available (due to non-availability of empirical data), we have used a suitable range of values for both price and time sensitivity (Ben-Akiva and Lerman (1985) and de Dios Ortúzar and Willumsen (2011) discuss the estimation of these parameters). The Stackelberg game is solved by choosing some pairs of sensitivity values from these ranges. The costs of moving a vehicle and rebalancing a vehicle are taken to be equal for any mode alternative under consideration. The total operating cost is calculated as follows for the i^{th} alternative:

$$\begin{aligned} m_i + m_{i(bal)} = {} & 2 \times \text{fuel cost (Rs./km) incurred per vehicle to serve the demand} \times \\ & \text{distance from the start node to the end node} \end{aligned} \tag{29}$$

We have assumed following values of the parameters related to the nesting structure: θ_1=0.7, θ_2=0.5, λ_{r+b1}=0.2 and λ_{r+b2}=0.8, which are typical values based on the type of mode alternatives considered and because of the cross-nesting structure the results become less sensitive to change in these values. The constants n_1 and n_2 are used to set upper bounds on the collaborative service prices for the bus and the RSP respectively. In the simulation setting, n_1 and n_2 are assumed equal to 10 and 15 respectively. The values of nesting structure parameters, n_1 and n_2 are same for both corridors.

Table 1: Parameters for numerical optimization

Parameter	Values	Corridor-1	Corridor-2
Prices (Rupees)	p_r	708.15	549.82
	p_b	235.00	30.00
	p_{pv}	776.15	583.82
Free Flow Travel time (hours)	FFT_{r1}	0.083	0.05
	FFT_{pv1}	0.083	0.05
	FFT_{r2}	0.50	0.55
	FFT_{b2}	1.00	1.08
	FFT_{pv2}	0.50	0.55
	FFT_{r3}	0.083	0.05
	FFT_{pv3}	0.083	0.05
Vehicle occupancy	Occ_r	2	2
	Occ_b	30	30
	Occ_{pv}	2	2
Operator's cost (Rupees)	$m_r + m_{r(bal)}$	247.57	130.46
	$m_b + m_{b(bal)}$	937.74	485.04
	$m_{rco} + m_{rco(bal)}$	32.61	19.27
Others	D	5000	5000
	Δ^1_{walk}	0.44 hr	0.26 hr
	Δ^3_{walk}	0.44 hr	0.26 hr
	Range of β_p	[-0.01,-0.001]	[-0.01,-0.001]
	Range of β_t	[-1.5,-0.1]	[-1.5,-0.1]

4.1.1 Simulation results for Corridor-1

Table (2) provides the comparison of profits, effective shares and travel times of RSP and bus agency in both no-collaboration (denoted as N.C.) and with-collaboration (denoted as W.C.) cases.

The profit of RSP increases due to collaboration with bus agency. Although the introduction of a new service by RSP leads to a decline in the share of the usual service (and therefore the corresponding profit), the optimization procedure helps to set prices in such a way that the RSP sees a net profit. This is because now there is a new collaborative travel mode which provides less expensive (than RSP's usual service) and faster (than bus only service) rides. This increases effective shares for the RSP and the bus agency by drawing away travelers from the personal vehicle mode as well as those who were walking (from o to s_1 and s_2 to d) to the RSP mode (from o to s_1 and s_2 to d). At the pair $(-0.001, -1.5)$ of sensitivity values, a minor decline is seen in RSP's profit because RSP's usual service has a higher share and thus higher profit at these values. The profit generated from collaborative service is not able to compensate for the decline in the profit of usual service due to reduction of mode share of the usual service. However, the value of time

(VoT) at this pair is Rs. 1500/hour, which is very high and will be found in only a smaller segment of the population.

The bus agency also sees an increase in profit. This can be explained as the travelers who shift from the usual bus alone alternative to collaborative service are again using the bus service and therefore no decline is seen in bus agency's profit. In addition, some of the personal vehicle users have also shifted to the collaborative mode now. It can be noted that although the bus agency has first-mover advantage and RSP is the follower, the equilibrium has settled in favor of both agents.

Table 2: Comparison of profits (Rs./traveler), effective shares (%) and travel times (hr.) of RSP and bus agency for Corridor-1

	$-\beta_p$	$-\beta_t$	Profit (N.C.)	Profit (W.C.)	Share (N.C.)	Effective share (W.C.)	Travel time (N.C.)	Travel time (W.C.)
RSP	-0.010	-1.5	29.48	40.70	5.05	26.71	0.67	0.67
	-0.010	-0.8	13.08	20.42	2.24	16.16	0.67	0.67
	-0.010	-0.1	5.71	8.58	0.98	6.90	0.67	0.67
	-0.005	-1.5	155.67	167.48	26.64	37.69	0.72	0.71
	-0.005	-0.8	86.31	94.29	14.77	21.51	0.68	0.68
	-0.005	-0.1	42.89	48.35	7.34	12.93	0.67	0.67
	-0.001	-1.5	278.25	277.66	47.62	53.86	1.24	1.15
	-0.001	-0.8	237.69	242.46	40.68	51.28	1.00	0.94
	-0.001	-0.1	174.64	189.02	29.89	42.09	0.79	0.78
Bus agency	-0.010	-1.5	188.26	203.83	92.40	95.01	1.88	1.88
	-0.010	-0.8	196.88	205.60	96.63	97.84	1.88	1.88
	-0.010	-0.1	200.74	205.49	98.53	99.04	1.88	1.88
	-0.005	-1.5	112.12	128.47	55.03	60.58	1.97	1.94
	-0.005	-0.8	152.93	163.30	75.06	77.14	1.90	1.90
	-0.005	-0.1	178.51	185.40	87.61	88.96	1.88	1.88
	-0.001	-1.5	16.11	39.83	7.91	15.68	2.76	2.53
	-0.001	-0.8	43.45	74.36	21.32	30.81	2.40	2.22
	-0.001	-0.1	85.94	109.75	42.18	48.43	2.07	2.02

Table 3: RSP+bus collaborative service attributes (prices (Rs.), travel times (hr.) and shares (%)) and change in EMU of travelers for Corridor-1

$-\beta_p$	$-\beta_t$	Travel price	Travel time	Share	Change in EMU
-0.010	-1.5	395.72	1.17	23.89	0.16
-0.010	-0.8	384.20	1.17	15.01	0.11
-0.010	-0.1	398.36	1.17	6.41	0.05
-0.005	-1.5	511.88	1.25	14.74	0.12
-0.005	-0.8	530.89	1.18	8.30	0.05
-0.005	-0.1	492.48	1.17	6.73	0.04
-0.001	-1.5	555.08	1.97	10.40	0.23
-0.001	-0.8	546.92	1.61	15.80	0.17
-0.001	-0.1	546.50	1.34	15.85	0.10

Table 4: Comparison of profits (Rs./traveler) of RSP and bus agency for CNL and MNL models (Corridor-1)

$-\beta_p$	$-\beta_t$	Profit of RSP (N.C.)	Profit of RSP with CNL model (W.C.)	Profit of RSP with MNL model (W.C.)	Profit of bus (N.C.)	Profit of bus with CNL model (W.C.)	Profit of bus with MNL model (W.C.)
-0.010	-1.5	29.48	40.70	52.72	188.26	203.83	205.26
-0.010	-0.8	13.08	20.42	31.72	196.88	205.60	210.33
-0.010	-0.1	5.71	8.58	18.81	200.74	205.49	210.40
-0.005	-1.5	155.67	167.48	167.02	112.12	128.47	151.10
-0.005	-0.8	86.31	94.29	112.23	152.93	163.30	176.79
-0.005	-0.1	42.89	48.35	68.47	178.51	185.40	192.83
-0.001	-1.5	278.25	277.66	263.46	16.11	39.83	60.03
-0.001	-0.8	237.69	242.46	229.19	43.45	74.36	98.60
-0.001	-0.1	174.64	189.02	187.52	85.94	109.75	134.51

At all pairs of values of price and time sensitivities, an increase in the effective shares can also be seen for both RSP and bus agency. Due to the introduction of collaborative service, a decline is there in the shares of usual service of RSP and bus agency, but the share of collaborative service is able to compensate for this decline and increases the effective shares. An improvement in travel times is also seen except at -0.01 value of price sensitivity (at this value of price sensitivity, travel times are same in both no-collaboration and with-collaboration cases).

The advantages of collaborative service for the travelers can be seen from Table (3). The prices charged for this service are less as compared to RSP's usual service. Although collaborative service travel times are higher than personal vehicle travel times, they are still less as compared to bus travel times. We also see an increase in travelers' expected maximum utility because the new mode provides a cost and time-effective option to them.

Table (4) shows the comparison of RSP's and bus agency's profits for MNL and CNL choice models to present the advantages associated with using a CNL model. We have used a CNL model since the RSP+bus mode alternative may have some attributes common to those of RSP and bus mode alternatives which have not been included in the utility expression thereby resulting in correlation among the unobserved components of utilities. Another important reason is that if we use an MNL model it may lead to over-estimation of the profits of RSP and bus agency in the with-collaboration case at many possible VoT values (which is evident from Table (4)). This is because the RSP+bus mode alternative will draw in equal proportions from all the other modes in the case of an MNL model. On the other hand, the CNL model will allow the RSP+bus mode alternative to draw more from the RSP and bus mode alternatives than the personal vehicle. As a result, the profits of RSP and bus agency implied by the CNL model are more conservative (and more realistic) than those from the MNL model.

4.1.2 Simulation results for Corridor-2

Table 5: Comparison of profits (Rs./traveler), effective shares (%) and travel times (hr.) of RSP and bus agency for Corridor-2

	$-\beta_p$	$-\beta_t$	Profit (N.C.)	Profit (W.C.)	Share (N.C.)	Effective share (W.C.)	Travel time (N.C.)	Travel time (W.C.)
RSP	-0.010	-1.5	10.82	17.34	2.23	13.29	0.65	0.65
	-0.010	-0.8	5.62	11.52	1.16	11.44	0.65	0.65
	-0.010	-0.1	3.00	6.43	0.62	8.52	0.65	0.65
	-0.005	-1.5	82.14	87.62	16.95	27.38	0.67	0.67
	-0.005	-0.8	48.88	54.00	10.09	18.15	0.66	0.65
	-0.005	-0.1	27.39	31.00	5.65	10.99	0.65	0.65
	-0.001	-1.5	219.66	214.68	45.33	51.09	1.15	1.06
	-0.001	-0.8	183.77	181.48	37.92	48.19	0.93	0.86
	-0.001	-0.1	139.15	143.28	28.72	41.64	0.76	0.74
Bus agency	-0.010	-1.5	13.30	21.39	96.18	97.34	1.60	1.60
	-0.010	-0.8	13.56	20.07	98.01	98.70	1.60	1.60
	-0.010	-0.1	13.69	19.32	98.94	99.38	1.60	1.60
	-0.005	-1.5	9.54	23.05	68.99	72.58	1.64	1.63
	-0.005	-0.8	11.28	22.00	81.55	83.66	1.61	1.61
	-0.005	-0.1	12.40	20.32	89.66	90.86	1.61	1.61
	-0.001	-1.5	1.50	14.80	10.87	17.73	2.45	2.25
	-0.001	-0.8	3.52	24.02	25.42	33.78	2.07	1.93
	-0.001	-0.1	6.02	28.16	43.53	50.02	1.80	1.74

Table 6: RSP+bus collaborative service attributes (prices (Rs.), travel times (hr.) and shares (%)) and change in EMU of travelers for Corridor-2

$-\beta_p$	$-\beta_t$	Travel price	Travel time	Share	Change in EMU
-0.010	-1.5	200.31	1.18	12.08	0.09
-0.010	-0.8	180.43	1.18	10.90	0.08
-0.010	-0.1	172.09	1.18	8.32	0.06
-0.005	-1.5	275.37	1.21	12.88	0.09
-0.005	-0.8	280.92	1.19	9.67	0.06
-0.005	-0.1	295.51	1.19	6.35	0.04
-0.001	-1.5	305.77	1.92	9.41	0.21
-0.001	-0.8	303.88	1.56	14.83	0.16
-0.001	-0.1	301.01	1.34	16.70	0.10

We have also applied the model to Corridor-2 described earlier. The simulations results for this corridor are presented in Table (5) and (6). The benefits to the RSP and the bus agency are similar to the ones presented for Corridor-1. There is an increase in the profits and mode shares of the transport operators. The obtained travel price and travel time of the collaborative service make it an attractive mode alternative for the travelers. Benefits to the travelers are reflected in the form of a reduction in travel time and an increase in the expected maximum utility. These results show that Model-3 can be applied to different corridors in a city for evaluating the benefits to RSP, bus agency, and travelers due to collaboration between RSP and bus agency for first/last mile connectivity to bus stops.

4.2 Simulation results for Stackelberg game with congestion-dependent travel times and mode shares based on the cross-nested logit model for mode choice (for two RSPs)

In this setting, we consider two RSPs (instead of a single RSP) and assume that the two RSPs are identical in every aspect (e.g., the prices charged for usual services are same for both RSPs). Therefore, we expect that the change in profits and other such quantities will be same for both the RSPs after collaboration. The values of various parameters used in the Stackelberg game are given in Table (1). In the no-collaboration case, both RSPs are assumed to be present in a nest with nesting parameter equal to 0.2. In the with-collaboration case, the assumed values of parameters of the nesting structure are: $\theta_1 = 0.2, \theta_2 = \theta_3 = 0.7, \theta_4 = 0.3, \theta_5 = \theta_6 = 0.5, \theta_7 = \theta_8 = 0.8$, $\lambda_{r_1 1} = \lambda_{r_2 1} = 0.8, \lambda_{r_1 2} = \lambda_{r_2 3} = 0.15, \lambda_{r_1 8} = \lambda_{r_2 7} = 0.05, \lambda_{r_1+b2} = \lambda_{r_2+b3} = 0.15, \lambda_{r_1+b4} = \lambda_{r_2+b4} = 0.5, \lambda_{r_1+b5} = \lambda_{r_2+b6} = 0.3, \lambda_{r_1+b7} = \lambda_{r_2+b8} = 0.05$ and $\lambda_{b5} = \lambda_{b6} = 0.5$. The values of n_1 and n_2 are equal to 10 and 15 respectively. The values of nesting structure parameters, n_1 and n_2 are same for both corridors.

4.2.1 Simulation results for Corridor-1

As shown in Table (7), the profits of RSP_1 and RSP_2 have increased after collaboration with the bus agency, although the values of profits are lower as compared to the earlier case of one RSP due to the presence of a competitor (i.e., the other RSP). Nevertheless, an increase in profit for both the RSPs encourages collaboration. As expected, the change in profits for both the RSPs are identical. It can also be observed that the mode shares of RSPs in no-collaboration case are lower as compared to the corresponding mode shares in no-collaboration case involving only one RSP. However, the effective shares of the RSPs have increased after collaboration. In this context, it is worth noting that the RSPs may be more willing to collaborate with the bus agency when their market shares are already low due to the presence of competitors in the market. The travel times of the usual services of RSPs have also decreased after collaboration. As expected, the decrease in travel times is identical for both the RSPs.

Table (7) also highlights the advantages to bus agency in the form of increased effective shares, profits and reduced travel times. It can be noted that the profit of the bus agency is slightly higher as compared to the with-collaboration case when only one RSP is present. This could be due to the competition at the follower's level in two RSPs case. Due to this competition the RSPs set a lower price for their first- and last-mile services. Then, the bus agency can set higher price for its part of the collaborative service without suffering a loss in market shares and thus is able to generate higher profit.

Table 7: Comparison of profits (Rs./traveler), effective shares (%) and travel times (hr.) of RSP_1, RSP_2 and bus agency for Corridor-1

	$-\beta_p$	$-\beta_t$	Profit (N.C.)	Profit (W.C.)	Share (N.C.)	Effective share (W.C.)	Travel time (N.C.)	Travel time (W.C.)
RSP_1	-0.010	-1.5	16.78	24.04	2.87	21.20	0.67	0.67
	-0.010	-0.8	7.52	14.61	1.29	15.07	0.67	0.67
	-0.010	-0.1	3.29	8.57	0.56	10.22	0.67	0.67
	-0.005	-1.5	86.38	87.31	14.78	26.12	0.73	0.72
	-0.005	-0.8	48.59	56.94	8.32	20.78	0.68	0.68
	-0.005	-0.1	24.34	33.95	4.17	15.15	0.67	0.67
	-0.001	-1.5	149.33	150.74	25.55	31.03	1.26	1.06
	-0.001	-0.8	128.93	131.67	22.06	30.17	1.02	0.89
	-0.001	-0.1	96.09	107.35	16.44	26.80	0.80	0.77
RSP_2	-0.010	-1.5	16.78	24.04	2.87	21.20	0.67	0.67
	-0.010	-0.8	7.52	14.61	1.29	15.07	0.67	0.67
	-0.010	-0.1	3.29	8.57	0.56	10.22	0.67	0.67
	-0.005	-1.5	86.38	87.31	14.78	26.12	0.73	0.72
	-0.005	-0.8	48.59	56.94	8.32	20.78	0.68	0.68
	-0.005	-0.1	24.34	33.95	4.17	15.15	0.67	0.67
	-0.001	-1.5	149.33	150.74	25.55	31.03	1.26	1.06
	-0.001	-0.8	128.93	131.67	22.06	30.17	1.02	0.89
	-0.001	-0.1	96.09	107.35	16.44	26.80	0.80	0.77
Bus agency	-0.010	-1.5	186.88	223.64	91.73	94.95	1.88	1.88
	-0.010	-0.8	196.18	220.51	96.29	97.43	1.88	1.88
	-0.010	-0.1	200.44	216.44	98.38	98.77	1.88	1.88
	-0.005	-1.5	107.44	160.37	52.73	66.90	1.99	1.92
	-0.005	-0.8	149.56	186.79	73.41	80.55	1.90	1.89
	-0.005	-0.1	176.60	201.18	86.68	89.69	1.89	1.88
	-0.001	-1.5	14.93	59.63	7.33	22.73	2.77	2.37
	-0.001	-0.8	40.72	98.40	19.99	38.76	2.43	2.11
	-0.001	-0.1	82.24	132.66	40.37	54.58	2.09	1.97

Table 8: RSP_1+bus and RSP_2+bus collaborative services attributes (prices (Rs.), travel times (hr.), shares (%)) and change in EMU of travelers for Corridor-1

$-\beta_p$	$-\beta_t$	Travel price of RSP_1 + bus	Travel price of RSP_2 + bus	Travel time of RSP_1 + bus	Travel time of RSP_2 + bus	Share of RSP_1 + bus	Share of RSP_2 + bus	Change in EMU
-0.010	-1.5	400.60	400.60	1.18	1.18	19.47	19.47	0.44
-0.010	-0.8	395.87	395.87	1.17	1.17	14.20	14.20	0.31
-0.010	-0.1	391.54	391.54	1.17	1.17	9.81	9.81	0.21
-0.005	-1.5	492.63	492.63	1.24	1.24	15.53	15.53	0.41
-0.005	-0.8	472.30	472.30	1.19	1.19	14.63	14.63	0.33
-0.005	-0.1	455.25	455.25	1.17	1.17	11.93	11.93	0.26
-0.001	-1.5	557.10	557.10	1.81	1.81	8.59	8.59	0.54
-0.001	-0.8	557.14	557.14	1.51	1.51	12.53	12.53	0.43
-0.001	-0.1	557.10	557.10	1.32	1.32	13.84	13.84	0.32

It can be observed from Table (8) that the prices of collaborative services are less as compared to RSPs' usual services. Also, the increase in expected maximum utility is more as compared to the earlier case of one RSP because now the travelers have two collaborative travel modes available to them. Both of these travel modes provide better services to travelers in terms of travel time and travel cost.

Note that although price and time sensitivity values are not available due to a lack of empirical data on travelers' preferences, the obtained results are promising. There is an increase in profits and effective mode shares of the RSPs and the bus agency as well as a reduction in travel time for all pairs of price and time sensitivity values under consideration. It shows the robustness of the model to change in price and time sensitivity values.

4.2.2 Simulation results for Corridor-2

Table 9: Comparison of profits (Rs./traveler), effective shares (%) and travel times (hr.) of RSP_1, RSP_2 and bus agency for Corridor-2

	$-\beta_p$	$-\beta_t$	Profit (N.C.)	Profit (W.C.)	Share (N.C.)	Effective share (W.C.)	Travel time (N.C.)	Travel time (W.C.)
RSP_1	-0.010	-1.5	6.17	11.27	1.27	11.07	0.65	0.65
	-0.010	-0.8	3.23	7.58	0.67	8.64	0.65	0.65
	-0.010	-0.1	1.67	5.21	0.35	6.62	0.65	0.65
	-0.005	-1.5	46.12	52.91	9.52	19.47	0.67	0.67
	-0.005	-0.8	27.62	36.53	5.70	15.64	0.66	0.66
	-0.005	-0.1	15.61	24.43	3.22	12.32	0.65	0.65
	-0.001	-1.5	118.37	115.13	24.43	29.74	1.17	0.97
	-0.001	-0.8	100.16	96.93	20.67	28.95	0.94	0.82
	-0.001	-0.1	76.67	78.96	15.82	26.75	0.77	0.73
RSP_2	-0.010	-1.5	6.17	11.27	1.27	11.07	0.65	0.65
	-0.010	-0.8	3.23	7.58	0.67	8.64	0.65	0.65
	-0.010	-0.1	1.67	5.21	0.35	6.62	0.65	0.65
	-0.005	-1.5	46.12	52.91	9.52	19.47	0.67	0.67
	-0.005	-0.8	27.62	36.53	5.70	15.64	0.66	0.66
	-0.005	-0.1	15.61	24.43	3.22	12.32	0.65	0.65
	-0.001	-1.5	118.37	115.13	24.43	29.74	1.17	0.97
	-0.001	-0.8	100.16	96.93	20.67	28.95	0.94	0.82
	-0.001	-0.1	76.67	78.96	15.82	26.75	0.77	0.73
Bus agency	-0.010	-1.5	13.26	39.99	95.88	96.79	1.60	1.60
	-0.010	-0.8	13.53	34.97	97.84	98.27	1.60	1.60
	-0.010	-0.1	13.68	30.42	98.88	99.08	1.60	1.60
	-0.005	-1.5	9.30	41.47	67.22	74.33	1.64	1.63
	-0.005	-0.8	11.12	40.72	80.37	84.33	1.62	1.61
	-0.005	-0.1	12.30	38.13	88.91	90.98	1.61	1.61
	-0.001	-1.5	1.39	25.29	10.04	25.02	2.48	2.08
	-0.001	-0.8	3.30	38.42	23.88	41.62	2.11	1.82
	-0.001	-0.1	5.77	45.93	41.73	56.32	1.82	1.69

Table 10: RSP_1+bus and RSP_2+bus collaborative services attributes (prices (Rs.), travel times (hr.), shares (%)) and change in EMU of travelers for Corridor-2

$-\beta_p$	$-\beta_t$	Travel price of RSP_1 + bus	Travel price of RSP_2 + bus	Travel time of RSP_1 + bus	Travel time of RSP_2 + bus	Share of RSP_1 + bus	Share of RSP_2 + bus	Change in EMU
-0.010	-1.5	236.39	236.39	1.18	1.18	10.10	10.10	0.21
-0.010	-0.8	233.69	233.69	1.18	1.18	8.12	8.12	0.17
-0.010	-0.1	232.47	232.47	1.18	1.18	6.35	6.35	0.13
-0.005	-1.5	306.17	306.17	1.21	1.21	11.85	11.85	0.28
-0.005	-0.8	300.15	300.15	1.19	1.19	11.04	11.04	0.24
-0.005	-0.1	292.27	292.27	1.19	1.19	9.70	9.70	0.21
-0.001	-1.5	306.22	306.22	1.74	1.74	8.29	8.29	0.54
-0.001	-0.8	306.22	306.22	1.46	1.46	12.40	12.40	0.42
-0.001	-0.1	306.20	306.20	1.31	1.31	14.49	14.49	0.33

Table (9) and (10) provide the simulation results for this corridor. The increase in profit and effective mode shares of the RSPs and the bus agency due to collaboration holds true for this corridor as well. As in the case of Corridor-1, a reduction in travel time (due to lower congestion) and a lower travel price of the collaborative service (than the direct RSP service) benefit the travelers. The expected maximum utility of the travelers also increases due to the introduction of collaborative service. These results suggest that the overall findings are similar across different corridors and that the formulated model can be applied to different travel corridors in a city.

5 Determination of Nash equilibrium in a game that allows RSPs to make a decision regarding collaboration

In the previous section it was assumed that both RSPs collaborate with bus agency. However, it may be of interest to analyze the case where each RSP has a choice regarding the decision to collaborate with bus agency. Such a decision of an RSP would not only affect that RSP but the other RSP as well as bus agency. This entanglement hints that we have to formulate this case as a game and search for Nash equilibrium if it exists. We have used Corridor-1 for the analysis of this game.

We will assume the following setting: There are two RSPs. Each RSP can decide whether to collaborate with the bus agency or not. If an RSP offers to collaborate, the bus agency has to accept that offer. Let us consider a strategic form game, $\Gamma =< N, S_{i\{i\in N\}}, U_{i\{i\in N\}} >$ where, N is the set of players, S_i is the strategy set of player i and U_i is the payoff of player i. We have, N={RSP_1, RSP_2} and S_i={C, NC} where C denotes collaboration offer and NC denotes no-collaboration offer by i^{th} RSP. The payoff of i^{th} RSP, U_i is defined as the difference between the profit of i^{th} RSP after both the RSPs have made their offers and no-collaboration case profit. To understand the game and the payoffs, let us examine the following cases:

1. **None of the RSPs collaborate:** In such a case, the change in profit for both the RSPs

will be zero.

2. **Only one RSP collaborates:** Let us consider the case when only RSP_1 decides to collaborate with the bus agency. In such a case the nesting structure is as follows: Nest-1 has both RSP_1 and RSP_2. RSP_1+bus is present in nest-2 and nest-3 with RSP_1 and RSP_2 respectively. RSP_1+bus is also present in nest-4 with bus. The parameters of the nesting structure are: $\theta_1 = 0.2, \theta_2 = 0.7, \theta_3 = 0.8, \theta_4 = 0.5, \lambda_{r_1 1} = \lambda_{r_2 1} = 0.9, \lambda_{r_1 2} = \lambda_{r_2 3} = 0.1, \lambda_{r_1+b4} = 0.8, \lambda_{r_1+b2} = 0.15$ and $\lambda_{r_1+b3} = 0.05$. Table (11) gives the change in profits of RSP_1 and RSP_2. It can be seen that the profit of RSP_1 increases after collaboration while that of RSP_2 decreases. This happens because RSP_1 is collaborating with the bus agency to increase its effective share and profit while RSP_2 sees a decrease in the share of its usual service due to the introduction of a new travel mode (RSP_1+bus) and therefore the corresponding profit decreases. Note that since the two RSPs have been assumed identical, therefore the results will be symmetric when RSP_2 collaborates with bus agency and RSP_1 does not.

3. **Both RSPs collaborate:** This case has been discussed in Section-4.2.1.

Now that we have analyzed the component cases, we can go to the game Γ defined earlier. Table (12) gives the payoff matrix of RSP_1 and RSP_2 for $(-0.010, -1.5)$ pair of price and time sensitivity values. The first entry in each block is the change in profit of RSP_1 and second entry is the change in profit of RSP_2. It can be easily seen that (C,C) is the Nash equilibrium of the game, e.g., if RSP_1 unilaterally deviates from this strategy profile then it will incur a negative change in profit equal to -2.10 Rs./traveler. If RSP_2 unilaterally deviates from this strategy profile it will also incur the same change in profit. This equilibrium can be verified for other pairs of price and time sensitivity values as well. This result shows that the proposed model enforces the collaboration between RSPs and bus agency when the RSPs have a choice regarding collaboration. Any deviation from the enforced equilibrium causes RSPs to incur a loss. This is one of our important findings with policy implications.

Table 11: Comparison of change in profits (Rs./traveler) of RSPs when RSP_1 collaborates with the bus agency and RSP_2 does not

$-\beta_p$	$-\beta_t$	Profit of RSP_1 (N.C.)	Profit of RSP_1 (After collaboration offer)	Change in profit of RSP_1	Profit of RSP_2 (N.C.)	Profit of RSP_2 (After no-collaboration offer)	Change in profit of RSP_2
-0.010	-1.5	16.78	30.81	14.03	16.78	14.68	-2.10
-0.010	-0.8	7.52	16.61	9.09	7.52	6.83	-0.69
-0.010	-0.1	3.29	8.30	5.01	3.29	3.09	-0.20
-0.005	-1.5	86.38	107.36	20.98	86.38	79.36	-7.02
-0.005	-0.8	48.59	65.87	17.28	48.59	45.94	-2.65
-0.005	-0.1	24.34	35.56	11.22	24.34	23.61	-0.73
-0.001	-1.5	149.33	163.23	13.90	149.33	141.85	-7.48
-0.001	-0.8	128.93	150.82	21.89	128.93	117.81	-11.12
-0.001	-0.1	96.09	123.55	27.46	96.09	89.27	-6.82

Table 12: Payoff (Rs./traveler) matrix of RSP_1 and RSP_2 for $(-0.010, -1.5)$ pair of price and time sensitivity values

		RSP_2 NC	RSP_2 C
RSP_1	NC	0,0	-2.10,14.03
	C	14.03,-2.10	7.26,7.26

6 Simulation results for alternate assumptions on vehicle rebalancing costs

The models discussed in the earlier sections assume that after an operator takes the travelers from a start node to an end node, the same vehicle needs to return to the start node without any traveler and the associated cost is borne only by the operator. However, in real world scenarios, the operators do not need to rebalance their vehicles by using such a procedure, e.g, the RSPs receive other trip requests from the travelers which can be served without returning back to the start node. Therefore, the rebalancing costs get reduced in such cases. The models proposed in this research paper do not consider a spatio-temporal distribution of demand and therefore can not represent real-world scenarios. However, to test the robustness of these models we have simulated them on Corridor-1 after reducing the rebalancing costs of all the mode alternatives to half of their values (an assumption). Under such an assumption, the change in profits of RSPs and bus agency have been presented in Table (13) for Model-3 and Model-4 (for Model-4 only the change in profits of RSP_1 have been reported as the two RSPs are identical). It can be seen that there is no major decline in the profits of both operators even if the rebalancing costs are reduced to half of their values except for the following pairs of sensitivity values (for RSPs): $(-0.001, -1.5)$ and $(-0.001, -0.8)$ at which the VoTs are Rs. 1500/hr and Rs. 800/hr respectively, which are quite high. This analysis does not replace the need for working with a real-sized network considering spatio-temporal distribution of demand. It serves to examine the robustness of our findings on the benefits of collaboration in the form of increased profits for the transport operators (RSPs and bus agency), improved connectivity at more affordable prices than using an RSP alone for the entire travel, and decreased travel times for travelers.

Table 13: Comparison of change in profits (Rs./traveler) of RSPs and bus agency when the rebalancing costs of all the mode alternatives have been reduced to half of their values

$-\beta_p$	$-\beta_t$	Change in profit of RSP for Model-3	Change in profit of bus agency for Model-3	Change in profit of RSP_1 for Model-4	Change in profit of bus agency for Model-4
-0.010	-1.5	24.82	6.87	13.77	9.71
-0.010	-0.8	18.40	1.61	13.76	3.49
-0.010	-0.1	11.00	0.04	10.94	1.03
-0.005	-1.5	6.46	22.82	-1.46	42.42
-0.005	-0.8	15.72	9.83	9.89	21.90
-0.005	-0.1	11.77	3.71	13.53	9.03
-0.001	-1.5	-10.63	19.72	-7.26	38.85
-0.001	-0.8	-6.25	24.05	-7.82	45.38
-0.001	-0.1	5.62	15.57	1.72	34.21

7 Key ideas for the extension of Model-4 to a general network

Model-4 enables the bus agency and the two RSPs to play the Stackelberg game for setting the price of collaborative service. However, Model-4 is limited to a stylized four-node network. In real-world situations, the cities have an extensive road network with several bus routes. Such networks have many origin-destination (O-D) pairs with different associated demands. Model-4 cannot be directly applied to such real-world networks in its present form because it is applicable to only one O-D pair having one bus route. However, it has been formulated in such a way that extending it to a general network is possible with a few modifications. As mentioned earlier, these modifications comprise future work and therefore, this section discusses only some key ideas for the same.

Consider a general city-scale network with several O-D pairs and bus routes. The network has the presence of following modes (which are the same as considered earlier): 1. rideshare service, 2. bus, 3. personal vehicle, and 4. collaborative service. Note that the travelers who want to travel between any O-D pair may not have access to each of these modes, e.g., the bus service will be available only at those nodes which are actual bus stops or which are present within the walkable distance from the bus stop nodes. Similarly, for the collaborative service, the RSPs may decide to provide first- and last-mile connectivity only at nodes that are close to the bus stops. The modal split information at any O-D pair is critical to set the optimal price of collaborative service offered by the RSPs and the bus agency and therefore such variation in the mode availability should be included in the overall framework. With this aspect of mode availability, the objectives of various agents are as follows:

- *Bus agency*: The objective of the bus agency is to maximize its profit by collaborating with the RSPs. The profit expression of the bus agency for an O-D pair is similar to (17), if the collaborative service is also available for that O-D pair. Otherwise, the term corresponding to the collaborative service profit is not present in the profit expression. However, there are several such O-D pairs now at which the bus agency is providing its service. Therefore, each of these O-D pairs will have a similar profit expression and the total profit is given by the sum of these expressions. The travel prices for the part of the collaborative service provided by the bus agency at various O-D pairs are its decision variables. The bus agency anticipates the strategies of the RSPs (travel prices for the part of the collaborative service offered by the RSPs) to decide its own strategies.
- *RSPs*: The RSPs have the same objective of profit maximization as that of the bus agency. The RSPs provide direct end-to-end connectivity and/or first- and last-mile connectivity to the nearest bus stop for various O-D pairs. The profit expression of the RSPs for an O-D pair will be similar to (18), if they are also providing the first- and mile-connectivity for that O-D pair. Otherwise, the term corresponding to the collaborative service profit is not present in the profit expression. The total profit of the RSPs is given by the sum of such expressions across all O-D pairs. The decision variables of the RSPs are travel prices for their part of the collaborative service at various O-D pairs. The RSPs optimize these prices after observing the prices set by the bus agency.
- *Travelers*: The travelers choose modes that maximize the utility accrued to them in such choice situations. While choosing a mode, they consider the dependence of travel times on the mode shares and the traffic volumes present on links.

The city-scale network has several O-D pairs with different modal availability. In the stylized Model-4, the mode shares and the travel times were dependent on each other. This led to a set of fixed-point equations which were solved numerically to obtain the equilibrium mode shares and travel times. A similar dependence holds true for the general network also. Before discussing the procedure for calculating the travelers' choices in the case of a general network with various modes, we will discuss a simpler case. Assume that only a single mode is available to the travelers at all the O-D pairs. Therefore, there is no choice available to the travelers as far as the modes are concerned. However, for each O-D pair, the travelers may have several paths available to them in order to reach their destination. A traveler is interested in choosing the path with minimum travel time. But the travel times on the links depend on the traffic flows on the links themselves. Therefore, the traffic network equilibrium methodologies have to be used in such a case. These methodologies help to obtain the link flows for the network which satisfy “Wardrop’s equilibrium”. This notion of equilibrium can be stated as follows: At equilibrium, all used paths have equal and minimal travel time. Therefore, if a traveler unilaterally deviates from his/her path, he/she will incur a greater travel time. Since we have considered multiple modes (available at each O-D pair), the travel times for which depend on the path chosen by the travelers, the mode shares and the traffic network equilibrium become dependent on each other. To obtain the equilibrium mode shares, the mode shares and the traffic network equilibrium have to be calculated alternately until the convergence in mode shares is reached. Implementing and calculating the equilibria in this whole set-up could be complex and therefore an agent-based simulation can also be developed.

8 Conclusion

The paper presents models to evaluate the benefits of mobility as a service (MaaS) provided by the RSPs collaborating with the bus agency. The RSPs provide the first- and last-mile services while the bus agency provides the long-haul service. In addition, the RSPs continue to provide their usual end-to-end service. We studied a sequence of models of increasing complexity culminating in Model-4 which is a Stackelberg game. In this model, the bus agency is the lead player. Keeping in mind the extant situation in several cities world-wide, this model considers two competing RSPs as the followers. The bus agency as well as the two RSPs optimize their prices for the collaborative service while considering travelers’ response to travel prices and travel times. The travelers’ response is represented using random utility-based discrete choice models which consider the influence of travel prices and travel times to provide the probability associated with choosing a mode or, in other words, the mode share. The travel times are in turn specified to depend on the mode shares and thus take into account traffic congestion.

The proposed framework was applied to a stylized network representing a major travel corridor in Bengaluru, India. The framework was also tested for applicability to another travel corridor in the same city. In the absence of empirical data on travelers’ preferences, several numerical simulations were conducted to represent a range of travelers’ sensitivities to travel times and travel prices. In addition, different cross-nested choice modeling structures were considered to represent flexible substitution patterns among the mode choice alternatives available to the travelers. Furthermore, an alternate assumption was examined to reflect different rebalancing costs (due to empty miles) faced by the RSPs.

Our findings suggest that collaboration increases market shares and profits for all the collaborating players - the bus agency and the two RSPs. The travelers see more attractive mode alternatives as a consequence of the collaboration. Due to this, there is an increased use of the bus agency’s service which is an indicator of the reduction in the use of RSPs’ direct service and personal vehicles. This not only reduces congestion and travel times for the travelers but also has positive environmental implications. All of these hold under a wide variety of price and time sensitivities. Thus, the Stackelberg game framework suggests a win-win situation for all stakeholders.

We next addressed the question of whether the RSPs collaborating with the bus agency is self-enforcing. To study this, we considered a game where the actions of the RSPs were either to collaborate or not to collaborate with the bus agency in providing this service. Numerical simulations suggest that for a wide variety of price and time sensitivities, the situation when both RSPs collaborate with the bus agency is a Nash equilibrium. This has important policy implications because it demonstrates that collaboration between RSPs and bus agency is a viable option despite the strategic nature of participating entities.

In our models, we assumed that an empty vehicle returns back to the start node after dropping off the travelers at the end node and the associated cost is incurred by the operator only. But in real-world scenarios, the operators get other trip requests due to which such rebalancing of vehicles is not required in most of the cases. Therefore, we also simulated the case where rebalancing costs were reduced to half of their values for every operator and observed that there was no major decline in the profits of all the operators at possible price and time sensitivity values.

All the above results are highly encouraging, though on a stylized four-node network. Therefore, we have also provided key ideas to extend the model to a general network. We are currently extending this work to a general network with the more realistic spatio-temporal distribution of demand. This requires an integration of the traffic network equilibrium model into the Stackelberg game framework, because for any pair of origin and destination nodes, there are multiple paths to choose from in addition to travel modes. Additional features that are being studied are diurnal demand patterns, vehicle rebalancing procedures, and the associated supply side constraints.

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