The adverse impact of headway variability on bus transit ridership: Evidence from Bengaluru, India

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AUTHOR CONTRIBUTIONS: Study conception and design: Pinjari, A.R., Deepa, L., and Srinivasan, K.K.; Data curation: Deepa, L; Supervision of analysis: Pinjari, A.R.; Formal analysis, Validation, Investigation: Deepa, L., Pinjari, A.R., Nirmale, S., Srinivasan, K.K., and Biswas, M.; Model estimation software: Nirmale, S., Deepa, L.; Writing-Original draft preparation: Deepa, L., Pinjari, A.R.; Writing-reviewing and editing: Srinivasan, K.K., Biswas, M., and Nirmale, S. All authors reviewed the results and approved the final version of the manuscript.

DECLARATIONS: The other authors are not aware of any competing interests to declare.

ACKNOWLEDGEMENTS: The authors are grateful to BMTC for providing the bus ticket sales, schedules, and other operational data used in this study. The methodological work in this paper was carried out as part of a project titled "*A connected Intermediate Public Transport Mobility-as-a-service (IPTMaaS) solution for the last mile connectivity and multi-modal integration problems of public transit*" sponsored by the Ministry of Education (MoE) and the Department of Science and Technology (DST) of the Government of India. The empirical application was carried out as part of the project titled "*Collaborative Intelligent Transportation Systems Endeavour for Indian Cities – InTranSE Phase II*" funded by the Indian Ministry of Electronics and Information Technology (MeitY). The authors are thankful to Gautham Manoharan, Prateek Agarwal, and Yogalakshmi J. for assistance in various data processing tasks. Tarun Rambha provided helpful discussions on the computation of headway variability variables. Three anonymous reviewers provided useful comments on an earlier version of the manuscript.

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Abstract

This study examines the impact of bus service headway variability on bus transit ridership using direct demand models at different levels of spatial aggregation – route level and stop-route level – using transit demand and supply data from the city of Bengaluru, India. In addition, auxiliary models are developed to understand the determinants of service frequency and headway variability and to address the endogeneity of these service characteristics in the demand models. This is perhaps the first study in the public transit literature to compare and contrast the endogeneity and non-linearity effects of service frequency and headway variability in transit demand models using both a conceptual framework and empirical evidence from a large transit system. The empirical results offer evidence that variability in headways adversely impacts transit ridership and passenger-kilometres. The strength of the adverse effect increases with increasing variability. On the other hand, the influence of service frequency decreases with increasing frequency. Furthermore, it is shown conceptually and demonstrated empirically that ignoring the endogeneity of service variability results in an underestimation of its adverse effect on transit demand. On the other hand, the empirical results suggest that ignoring the endogeneity of service frequency would result in an overestimation of its beneficial effect. An important takeaway from these results and additional policy simulations is that transit agencies can potentially gain greater ridership and revenue by reducing headway variability rather than simply allocation more buses and crew to high-frequency routes.

Keywords: public transportation, transit ridership, headway variability, direct demand models, demand-supply endogeneity, public transit in Indian cities

1. INTRODUCTION

The reliability of headways is an important service attribute for both operators and users of bus transit systems. Headway at a given bus stop is defined as the time interval between the arrivals of consecutive buses at that stop (Tirachini *et al.*, 2021). Accordingly, the reliability of headways is viewed as the ability of a bus transit system to maintain regular headways by adhering to its schedule (Turnquist and Blume, 1980). In public transit literature, the variability of headways is considered as an important indicator of service reliability. This is because variation in headways causes buses to arrive late at stops and passengers to wait longer (Welding, 1957). Often, when headways vary considerably, and users are aware of a constant schedule adherence problem, they typically plan their travel to budget extra time to not miss their bus (Kittelson & Associates, 2013). In addition, headway variability can cause vehicle bunching, especially on frequent routes, forcing many passengers to ride in crowded buses (Cats *et al*., 2016). Further, overcrowded buses tend to deny boarding, aggravating wait times for passengers (Tirachini *et al*., 2013). In such situations, the transit operator may end up designating additional buses, drivers, and crew to maintain the scheduled headways leading to increased operating costs and reduced efficiency and productivity of the transit agency (Abkowitz *et al*., 1978). For instance, Cheranchery and Maitra (2021) identify improper fleet management (in terms of headway and crowding) as one of the major reasons for the increment in fares and heavy subsidy requirements. Therefore, as suggested in the literature (Diab, 2015), improving reliability is a win-win situation for both transit users and operators.

Although it is widely recognised that the unreliability of transit service (i.e., headway variability) leads to a loss in ridership, there is a scarcity of research that quantifies such impacts of transit ridership at various levels of spatial aggregation. While many studies consider headways (or service frequency) as an important variable to explain ridership, only a handful of studies in the literature examine the impact of variability in headways on ridership in transit systems (Kemp, 1981; Strathman *et al*., 1999; Kimpel *et al*., 2000). Besides, no literature exists on quantifying the impact of service unreliability on transit ridership in the cities of developing economies such as India, where a large share of travellers uses public transit systems. Furthermore, most studies in the literature ignore the non-linearity and endogeneity of the relationships between service unreliability and transit demand (more on this in Section 3).

In view of the above research gaps, the current study presents direct demand models to examine and quantify the impact of service reliability – as measured by variability of service headways – on bus transit ridership and passenger-kilometres (pkm) in the city of Bengaluru, India. Specifically, the study develops two route-level models – one for route-level boardings and another for route passenger-kilometres – and a stop-route-level boardings model. In formulating these models, the study investigates, using conceptual discussion and empirical analysis, the non-linearity and endogeneity of the relationships between service headway, headway variability, and transit demand. These explorations highlight the risk of underestimating the adverse effect of headway variability when endogeneity between headway variability and transit boardings is ignored. In this context, a two-stage residual inclusion (2SRI) method is used to address the endogeneity of service variables in transit demand models presented in this study. In addition, the study develops auxiliary models for service frequency and headway variability – to shed light on the factors influencing service quantity and quality in Bengaluru. All the empirical models in this study are estimated using bus transit demand and supply data from a large bus transit system in Bengaluru, India. The models developed in this study have practical applications for forecasting ridership in response to changes in transit service characteristics and catchment area attributes. The modeling approach, which includes a rich variable specification, non-linear specification, and endogeneity treatments, can be applied to investigate bus transit demand in other cities in India and across different countries.

In the rest of this paper, Sections 2.1 and 2.2 review the literature on the factors influencing headway variability in bus transit systems and the impact of headway variability on transit demand, respectively. Section 2.3 positions the current study in the context of research gaps identified in the earlier sections. Section 3 develops a conceptual framework with a tri-variate equations model to examine the relationships between service headway (i.e., service frequency), service variability (i.e., headway variability), and demand (i.e., ridership) in transit systems. Section 4 explains the econometric modeling framework employed in this study to translate the above conceptual framework into an empirical model. Section 5 describes the empirical data from Bengaluru and the dependent and independent variables employed in the proposed transit demand models. Section 6 presents the empirical models and discusses the findings and policy insights on the determinants of bus ridership and passenger-kilometres in Bengaluru, India. Finally, Section 7 summarizes the contributions of the study and identifies future research directions.

2 LITERATURE REVIEW AND CURRENT STUDY

2.1 Literature on the factors influencing service reliability in bus transit systems

There has been considerable research on the factors that influence bus service reliability. Sterman and Schofer (1976) published one of the first studies on bus service reliability. In their work, service reliability is calculated using the inverse of the standard deviation of point-to-point trip times. Using data from bus services in the Chicago area, they found that increasing the route length, the intensity of intersection control, traffic volumes, and bus passenger loadings significantly degraded service reliability. A study by Abkowitz and Engelstein (1983) found that, in addition to time-of-day and direction dummies, segment length, boarding and alighting numbers, on-street parking, and the number of signalised junctions had a substantial impact on the mean running time of a transit route. In their subsequent study (Abkowitz and Engelstein, 1984), the running time standard deviation was also regressed on mean running time using separate models for the AM peak, mid-day, and PM peak periods. Strathman and Hopper (1993) conducted an empirical study of the impact of internal factors such as route characteristics, schedule characteristics, driver experience, and operating characteristics, as well as external factors such as traffic congestion, traffic incidents, signal timing, on-street parking, and weather disruptions on the *on-time performance* of the fixed-route bus system in Portland, Oregon. El-Geneidy *et al*. (2011) used automated vehicle location (AVL) and automatic passenger counter (APC) data to predict the run time, schedule adherence, and reliability of a cross-town bus transit line at two scales: time-point segment level and route level. Finally, a recent paper by Tirachini *et al*. (2021) provides a comprehensive literature review of the factors that influence headway variability in transit systems. They synthesise the following as the key determinants of stop-level headway variability: headway discrepancy at the starting point of the route, operating conditions that vary due to variations in passenger demand (by days and time of day), route characteristics such as route length, number of stops, route service type, and the number of signalised intersections on the route.

Despite the plethora of studies discussed above, not much exists on understanding the factors that influence bus service reliability in Indian cities. To fill this gap, we use data from Bengaluru's bus transit system to investigate the impact of the above-discussed factors on headway variability in Bengaluru.

2.2 Literature on the impact of service reliability on bus transit demand

In the context of demand forecasting, most studies that examine the influence of service reliability or travel time reliability on transit usage (Bates *et al*., 2001; Carrion and Levinson, 2012) employ discrete choice methods using disaggregate-level stated preference (SP) or revealed preference (RP) data. Specifically, they investigate the impact of service reliability on traveller choices, such as travel frequency (Carrel *et al*., 2013) and mode choice (Prashker, 1997; Prioni and Hensher, 2000; Bhat and Sardesai, 2006). Among these, relatively fewer studies use revealed preference (RP) data to investigate the impact of service reliability, perhaps due to the difficulty of measuring service (un)reliability faced by individual travellers (although the increasing availability of vehicle probe data might help fill this gap). Regardless of the type of data used – SP data, RP data, or a combination of both – most such studies use disaggregate (*i.e*., individual-level) travel choice models to explore the influence of service (un)reliability on travel choices in general and transit usage in particular. However, none of these studies goes all the way from building individual-level models to using such models for quantifying the effect of service variability on aggregate transit ridership. In fact, only a handful of studies (Kemp,1981; Strathman *et al.,* 1999; Kimpel *et al.,* 2000) examine the influence of service reliability on aggregate transit ridership using demand and supply data from a transit agency. Further, as mentioned earlier, we are not aware of studies quantifying the influence of service unreliability on transit ridership in Indian cities, where a large share of travellers uses public transit systems.

Another stream of literature uses the *direct demand modeling* approach, which employs revealed ridership data obtained from transit systems to estimate transit ridership "directly" as a function of numerous factors influencing it (see Deepa *et al*., 2022 for a recent review of such studies). A large body of this literature considers average headways on a route (i.e., route-level service frequency) as an important variable for explaining transit ridership. However, only a handful of direct demand modeling studies (Kemp, 1981; Strathman *et al*., 1999; Kimpel *et al*., 2000) consider the variability of headways as a determinant of transit ridership. Among these studies, Strathman *et al*. (1999) and Kimpel *et al*. (2000) show empirical evidence that variability in bus service (as measured by variability in headways) has a negative impact on transit boardings. Further, Kimpel *et al.* (2000) recognise possible endogeneity between headway variability and passenger ridership. Endogeneity arises because of the simultaneity between headway variability and transit ridership. That is, while the variability in headways has a negative influence on passenger boardings, high-ridership routes are typically associated with greater service variability. This is because high-ridership routes tend to have high variability in boarding and alighting, thereby associated with high variability in dwell times and headways (Tirachini *et al*. 2021). In demand models that ignore such a simultaneous relationship (or endogeneity) between headway variability and demand, the parameter estimates tend to be biased and cause distorted policy implications of the importance of headway variability on demand.

It is worth noting here that endogeneity may arise not only between service variability and transit demand but also between service frequency and transit demand. Recognising the endogeneity between transit demand, service frequency, and headway variability is essential to quantify the "true" effects of strategies aimed at service improvements. While some studies in the literature recognise the endogeneity between service frequency and transit demand (Berrebi *et al*., 2021; Deepa *et al*., 2022), and some studies address the endogeneity between service variability and transit demand (Kemp, 1981; Kimpel *et al*., 2000), only a few studies recognise both types of endogeneity. Besides, all these studies provide only empirical evidence of the repercussions of ignoring endogeneity, such as bias in parameter estimates and overestimation of the influence of service attributes, without providing theoretical reasons behind the nature of bias in parameter estimates.

Even among the few studies that investigate the influence of service variability on transit demand, none of them investigates the non-linearity of the relationship between service variability and transit ridership. However, it is likely that the influence of variability in headways might be much greater at higher variability levels than at lower variability levels. Considering such non-linear impacts in conjunction with endogeneity between transit demand and supply attributes can potentially offer a more nuanced understanding of the supply-demand relationships in transit systems.

Finally, most studies use transit boardings as a metric to measure transit demand for their investigation of the influence of service attributes on transit demand. However, many transit agencies use passenger-kilometres (pkm) as a performance metric to measure demand because pkm correlates much more with fare-box revenue than transit boardings. Therefore, in addition to models of transit boardings as a function of service attributes, it would be useful to develop models of transit pkm as a function of the same service attributes.

2.3 Current Study

In this section, we highlight the contributions of this study in light of the research gaps discussed above. First, we develop a direct demand model at the route level to examine the impact of service frequency (as a proxy for service quantity) and headway variability (as a proxy for service quality) on bus transit ridership in Bengaluru, India. This is perhaps the first attempt at developing a comprehensive route-level bus ridership model for Indian cities. In addition to the model of route-level passenger ridership (boardings), we develop a model of route-level passenger-kilometres (pkm). While the former model offers insights into the influence of various factors, such as route-level service frequency and headway variability, on route-level passenger boardings, the latter model can potentially be used to analyse the influence of these factors on pkm and revenue generated due to ridership.

Second, in addition to the above-mentioned route-level models, we build on a recently completed stop-route-level model of Deepa *et al*. (2022) to incorporate the influence of headway variability on stop-route-level transit boardings. By building models at both route level and stop level, the study demonstrates and quantifies the adverse effect of headway variability on transit ridership at different levels of spatial aggregation.[1]

Third, we explore the non-linearity and endogeneity of the influence of headway and its variability on transit demand. Although the study by Deepa *et al*. (2022) explored these aspects in the context of service frequency (or headway), they did not consider the role of headway variability in their analysis. Furthermore, in this study, we provide a conceptual discussion using a tri-variate structural equations framework for analysing transit demand, service frequency, and headway variability. Using this conceptual discussion and subsequent empirical analysis, we demonstrate in this study that the non-linear effect of headway variability and its endogeneity with transit demand manifest in ways that can be different from those of service frequency. More specifically, the adverse effect of headway variability increases with an increase in variability, whereas the influence of frequency decreases with increasing frequency. Further, we demonstrate using both conceptual discussion and empirical results that ignoring endogeneity between headway variability and transit demand will result in an underestimation of the adverse effect of headway variability. On the other hand, in the current empirical context, ignoring the endogeneity between service frequency and transit demand results in an overestimation of the influence of service frequency. From the standpoint of a transit agency, the above findings and relevant policy simulations demonstrate the importance of focusing on reducing service variability in their operations.

Fourth, to operationalise the tri-variate equations model (of transit demand, service frequency, and headway variability), in addition to the empirical models of transit demand, we develop models of service frequency and headway variability. These models shed light on the factors influencing route-level service quantity (frequency) and its quality (variability) in Bengaluru. Furthermore, these auxiliary models help in addressing the endogeneity of service characteristics used as explanatory variables in the demand models. Recognising the endogeneity between transit demand, service frequency, and headway variability is essential to

[1] The ridership model of Deepa *et al*. (2022) was at the stop-route-level considering a cluster of stops on a specific route. That is, they modelled boardings at a cluster of stops (called fare-stage cluster) on a given route as a function of both stop-level and route-level characteristics. Such a disaggregate offers the ability to analyse the influence of stop-level amenities and the effect of network interactions on stop-level boardings. While route-level models do not offer the resolution to examine changes in stop-level ridership, they can provide insights into metrics of interest to transit agencies such as route-level boardings, route-level passenger-kilometres (pkm) and revenue.

quantify the "true" effects of strategies aimed at service reliability improvements, which we discuss next.

3. BUS SERVICE HEADWAY, SERVICE VARIABILITY, AND DEMAND

3.1 Endogeneity between service headway, its variability, and transit demand

The general form of a direct demand function at a route-level may be written as below:

$$D_r = \beta_0 + \beta_1 F_r + \beta_2 H_r + \boldsymbol{\alpha}'\boldsymbol{Q_r} + \varepsilon_r \quad (1)$$

In the above equation, D_r is the route-level demand (ridership) for route r, which is modeled as a function of service frequency (F_r), headway variability (H_r), and a vector ($\boldsymbol{Q_r}$) of other factors influencing ridership. Further, β_0 is a constant and ε_r is a normally distributed error term with zero mean and standard deviation σ. Next, consider the following structural equation for route-level service frequency (F_r) as a function of demand (D_r) and other factors ($\boldsymbol{S_r}$):

$$F_r = c_0 + c_1 D_r + \boldsymbol{\mu}'\boldsymbol{S_r} + \eta_r \quad (2)$$

Similarly, to model headway variability, we use the coefficient of variation (CV) of headway, which is a preferred metric among the several headway variability indicators suggested in the transit literature (Osuna and Newell, 1972; Abkowitz *et al.*, 1978; Strathman *et al.*, 2002; Kittelson & Associates, 2013; Chen *et al.*, 2009; Soza-Parra *et al.*, 2021). The headway CV of a route is defined as the ratio of the standard deviations of all stop-level headways on the route (across several days) to the mean of the headways. The structural equation for headway CV for route r is a function of the demand (D_r) and other factors ($\boldsymbol{P_r}$) influencing service quality as follows:

$$H_r = d_0 + d_1 D_r + \boldsymbol{\zeta}'\boldsymbol{P_r} + \nu_r \quad (3)$$

In Equations (2) and (3), c_0 and d_0 are constants and η_r and ν_r are the normally distributed error terms. Together, Equations (1) through (3) form a tri-variate system of structural equations for route-level transit demand, service quantity (frequency), and service quality (CV of headways).

Endogeneity occurs because of simultaneity between demand and supply variables. As can be observed from Equation (1), supply variables enter the demand equation, but the demand also influences the supply variables in Equations (2) and (3).[2] This is because transit agencies

[2] An explanation for the selection of endogenous explanatory variables in Equations (2) and (3) is in order here. In theory, it may be argued that simultaneity exists among all three endogenous variables – demand (D_r), service

run buses at high frequency on routes with high anticipated demand. And high variability in headways can be anticipated on high-demand routes. Such demand-supply simultaneity causes the supply variables in the demand equation to be correlated with the error term of the demand equation, a classic manifestation of endogeneity. To see this, substitute the right-hand side of Equation (1) for D_r into Equations (2) and (3) to arrive at the following reduced form equations:

$$F_r = \frac{1}{(1-c_1\beta_1)} \left(c_0 + c_1\beta_0 + c_1\beta_2 H_r + c_1\boldsymbol{\alpha}'\boldsymbol{Q_r} + c_1\varepsilon_r + \boldsymbol{\mu}'\boldsymbol{S_r} + \eta_r\right) \tag{4}$$

$$H_r = \frac{1}{(1-d_1\beta_2)} \left(d_0 + d_1\beta_0 + d_1\beta_1 F_r + d_1\boldsymbol{\alpha}'\boldsymbol{Q_r} + d_1\varepsilon_r + \boldsymbol{\zeta}'\boldsymbol{P_r} + \nu_r\right) \tag{5}$$

Assuming that the covariance between ε_r and η_r and that between ε_r and ν_r is zero, it is easy to see from the above equations that the sign of the covariance between F_r and ε_r depends on the sign of $E[F_r\ \varepsilon_r] = \frac{c_1}{(1-c_1\beta_1)} E[\varepsilon_r\ \varepsilon_r]$. Similarly, the sign of the covariance between H_r and ε_r depends on the sign of $E[H_r\ \varepsilon_r] = \frac{d_1}{(1-d_1\beta_2)} E[\varepsilon_r\ \varepsilon_r]$. Regardless of the sign of the covariances, it is clear that demand-supply simultaneity causes service frequency and headway reliability variables to be correlated with the error term (ε_r) in the demand equation. As a result, the exogeneity assumption that $E[F_r\ \varepsilon_r] = 0$ and $E[H_r\ \varepsilon_r] = 0$ is not satisfied for the supply variables. Therefore, a demand model that ignores endogeneity due to demand-supply simultaneity can potentially result in biased estimation and distorted policy implications.

frequency (F_r), and CV of headways (H_r) – and, therefore, service frequency and CV of headways should appear as endogenous explanatory variables in the equations of each other, in addition to demand entering both Equations (2) and (3). This is because an increase in headway variability on a route might require a transit agency to increase service frequency to serve the same demand, and an increase in service frequency might change headway variability. However, we did not include service frequency in Equation (3) and CV of headways in Equation (2) for the following reasons. First, the metric we used for service variability is the coefficient of variation (CV) of headways, which is the ratio of standard deviation in headways to average headway. While the variance or standard deviation of headways likely depends on the magnitude of headways, the CV of headways is less likely to depend on the magnitude of headways or frequency (We verified this empirically in our study). Also, it is assumed that transit agencies generally set their service schedule (i.e., service frequency) without considering service variability. Second, while it may be interesting to relax these assumptions, implementing such a theoretical model as an identified econometric model typically requires good instrumental variables (IVs) as proxies for demand and service variability in the right side of Equation (2) and for demand and service frequency in the right side of Equation (3). The IVs, as discussed in Section 4, should be sufficiently correlated with the suspected endogenous variable, and they should only indirectly influence the dependent variable through the suspected endogenous variable. It is not easy to find IVs that satisfy such criteria. In the absence of such IVs, it is a common practice to exclude (for identification reasons) some endogenous explanatory variables from a few of the equations. Therefore, in this study, given the focus on the endogeneity of service frequency and service variability in the demand equation of Equation (1), only demand enters the right side of Equations (2) and (3). We did not consider the secondary endogeneity effects due to potential simultaneity between service frequency and the CV of headways in Equations (2) and (3).

3.2 Deriving the direction of bias

Previous studies indicate that even low levels of endogeneity can produce biased parameter estimates and distorted interpretations of the effects of endogenous variables (Zaefarian *et al.,* 2017). In this context, it is useful to not only establish the presence of endogeneity but also examine the direction of bias. Therefore, we present a detailed technical discussion on the direction of bias for the coefficients on service frequency and headway variability in Appendix A. Here, we provide a summary of the implications from the discussion in Appendix A.

Interestingly, the theoretical findings suggest that the repercussions of ignoring endogeneity between headway variability and demand are likely to be different from those of service frequency. That is, ignoring the endogeneity of headway variability in our transit ridership models would always result in an underestimation of its adverse effect. On the other hand, ignoring the endogeneity of service frequency may result in either an overestimation or an underestimation of the benefits of increasing service frequency, depending on the magnitude of empirical parameter estimates of the models. Although the latter has been empirically demonstrated in several previous direct demand modeling studies, we are not aware of studies that discuss the repercussions (and the direction of the bias) due to ignoring endogeneity between headway variability and demand. The contrasting repercussions due to the endogeneity of frequency and headway variability are corroborated by our empirical analysis later in the paper. Next, we discuss the econometric structure of the demand model and discuss the approach used to correct for the endogeneity of service frequency and headway variability.

4. ECONOMETRIC MODEL STRUCTURE

According to public transit literature (Deepa et al., 2022), the demand for ridership is influenced by two broad sets of factors: (1) external factors and (2) internal factors. The external factors include socio-demographics and land use of the catchment area, which is not in the control of the transit agency (at least in the short term). The internal factors include variables that explain service quantity (spatial and temporal coverage of the network and service frequency) and service quality (variability of headways), both of which are under the transit agency's control. Interactions between external and internal elements may also exist, which manifest in the form of inter-route relationships (competing or complementing effects) and accessibility (or connectivity). To examine the influence of these factors at an aggregate route level, we propose a route-level transit boarding model, as discussed next.

4.1 Specification of the demand model

As shown in the model structure in Eq. (6) below, the total boarding (B_r) of a route r is modeled using a log-linear regression approach where the dependent variable of interest is the natural logarithm of the total route-level boardings $ln(B_r)$.

$$ln(B_r) \ = \ \beta_0 + \boldsymbol{\alpha}'\boldsymbol{X}_r + \beta_f Freq_r + \beta_h HCV_r + \boldsymbol{\gamma}'\boldsymbol{Z}_r + \lambda\, \hat{\eta}_r + \zeta \hat{v}_r + \varepsilon_r \qquad (6)$$

The total boarding on a route r is modeled as a function of the following sets of variables: (a) trip-generating characteristics $(\boldsymbol{X}_r)$ such as population, employment and built environment characteristics of the route catchment area, (b) operational characteristics of the subject route, such as route-level hourly service frequency $(Freq_r)$ and service variability (headway CV across all stops on the route, denoted by HCV_r), and (c) variables measuring the influence of inter-route relationships $(\boldsymbol{Z}_r)$ within the bus transit network and with the metro network. Further, to address the endogeneity bias between demand and supply variables, we use predicted residuals, $\hat{\eta}_r$ and $\hat{v}_r$ from the auxiliary supply models for route-level service frequency and headway variability, respectively (more on this in Section 4.4). Lastly, the error term ε_r is assumed to be normally distributed with zero mean and standard deviation σ_1.

To understand how headway variability impacts revenue, we also model route passenger-kilometres in addition to route boardings. This is because total passenger-kilometres along a route directly influence the fare-box revenues from that route. The route-level passenger-kilometres model also employs a log-linear regression approach where the dependent variable of interest is the natural logarithm of the total passenger-kilometres ($ln(P_r)$). The empirical model specification is similar to that of the route boarding model in Eq. (6) with the exception that there is no auxiliary model for service frequency. As a result, while the route-level passenger-kilometres model addresses endogeneity between demand and headway variability, it does not address the plausible endogeneity between demand and service frequency. This is because of the difficulty in finding a good instrument variable (IV) for addressing potential endogeneity between passenger-kilometres and service frequency. Nevertheless, the model is relevant from a policy perspective since it can be used to understand and quantify the influence of service reliability on passenger-kilometres and revenue.

4.2 Endogeneity correction using auxiliary supply models

In this research, a two-stage residual inclusion (2SRI) technique (Terza *et al*., 2008), which is akin to the Two-Stage Least Squares (2SLS) estimation, is used to address endogeneity bias. Following this approach, we propose two auxiliary supply models – a route-level service frequency model and a route-level headway CV model – both of which are estimated as a function of factors that influence these supply variables. It is essential, however, that the auxiliary models for the suspected endogenous variables (frequency or headway coefficient of variation, as the case may be) contain at least one instrument variable (IV). If at least one IV is present in the auxiliary regression model for a suspected endogenous variable, the analyst can use the predicted residual from the auxiliary regression model as an additional regressor in the demand equation to isolate the component that may be correlated with the error term. Specifically, such a 2SRI approach divides the variation in the suspected endogenous variable into two parts, one of which is exogenous to the demand variable and the other which is correlated with the error term in the demand equation (ε_r). For instance, in the route-level transit boarding model in Eq. (6), we include the predicted residual from the auxiliary supply model of route-level service frequency ($\hat{\eta}_r$) and the predicted residual from the auxiliary supply model of route-level headway CV ($\hat{\nu}_r$) to correct for the endogeneity of service frequency and headway CV, respectively. In addition to correcting for potential endogeneity, this method offers a test for the presence of endogeneity *via* a t-test (against zero) on the coefficient of the predicted residual (Hausman, 1978).

It is important that a variable used as an IV should meet two criteria (Labrecque and Swanson, 2018). The IV must first be sufficiently correlated to the suspected endogenous variable, a requirement known as the *relevance condition*. This condition may be verified by looking at how strongly the IV loads in the auxiliary regression for the endogenous variable. Second, the IV should not have a direct effect on the primary outcome variable (ridership). Instead, it should have only an indirect influence *via* the endogenous variable. This condition is called the *exclusion restriction*. Although it is difficult to verify this second condition, the analyst can utilize theoretical knowledge to assess if this requirement is met (Labrecque and Swanson, 2018).

The 2SRI estimation approach, being a limited information approach, is less efficient than simultaneous estimation methods such as the Three-Stage Least Square (3SLS). However, achieving unbiased estimates with the 2SRI method only requires that the IVs and disturbance terms within each equation are uncorrelated. On the other hand, the typical approach to 3SLS estimation assumes that the same set of IVs is valid for every equation. Further, the 3SLS estimator is consistent only if all IVs are uncorrelated with all disturbance terms and all equations are free of misspecification (Schmidt, 1990; Zellner and Theil, 1962). Considering the practicality of satisfying these conditions, the 2SRI approach was chosen in this study, albeit it comes with some loss of efficiency.

4.2.1 Auxiliary supply model of service frequency

The auxiliary supply model for route-level service frequency is given by:

$$Freq_r = \delta_0 + \delta_1 Pkm_r + \boldsymbol{\varphi}'\boldsymbol{Y}_{fr} + \eta_r \tag{7}$$

where $Freq_r$ is the service frequency on route r, Pkm_r is the route passenger-kilometers (total distance in kilometers travelled by all bus passengers on the route), and $\boldsymbol{Y}_{fr}$ is a vector of other route-level variables such as the total population within one kilometer buffer of the route and the total employment within the buffer. δ_1 is the coefficient on route passenger-kilometers, and $\boldsymbol{\varphi}$ is the coefficient vector on $\boldsymbol{Y}_{fr}$. The error component η_r is assumed to be normally distributed with zero mean and standard deviation μ.

Once estimated, the predicted residual, $\hat{\eta}_r = Freq_r - \widehat{Freq}_r | (Pkm_r, \boldsymbol{Y}_{fr})$ is included as an additional explanatory variable in the route-level boarding model in Eq. (6). As discussed in Deepa *et al.* (2022), route passenger-kilometres (Pkm_r) is the IV employed in the model to correct for endogeneity between ridership and service frequency. Because the anticipated passenger-kilometers on a route influence the bus service frequency set for that route, the route-level Pkm_r meets the relevance criterion necessary for a good IV. Further, it can be reasoned that passenger-kilometers on a route do not directly influence boardings on that route, whereas the number of boardings on the route determines its passenger-kilometers.

4.2.2 Auxiliary supply model of headway variability

The auxiliary supply model for route-level headway coefficient of variation (CV) is given by:

$$HCV_r = \theta_0 + \theta_1 BCV_r + \boldsymbol{\omega}'\boldsymbol{Y}_{hr} + \nu_r \tag{8}$$

In the above equation, HCV_r is the headway CV on route r, BCV_r is the coefficient of variation of boardings on route r, and $\boldsymbol{Y}_{hr}$ is a vector of other route-level variables that influence the headway CV. These variables include, for example, route characteristics such as route length, average speed on the route as a proxy for traffic conditions on the route, and route service type (ordinary, express, etc.). θ_1 is the coefficient on boarding variation (BCV_r) and $\boldsymbol{\omega}$ is the coefficient vector on $\boldsymbol{Y}_{hr}$. ν_r is the error term assumed to be normally distributed with zero mean and standard deviation τ. Subsequently, the residual from the prediction $\hat{\eta}_r = HCV_r - \widehat{HCV}_r | (BCV_r, \boldsymbol{Y}_{hr})$ is included in the route-level boarding model in Eq. (6) as an additional explanatory variable.

Note that we use variation in route-level boarding (BCV_r) as an IV to correct for endogeneity between ridership and headway CV. The relevance condition is met in this case because an increase in passenger boarding variation can lead to longer dwell times at bus stops, resulting in increased headway variability. Also, to justify the exclusion restriction, it may be reasoned that demand variability does not directly influence demand as much as demand affects demand variability.

4.3 Model Estimation

The ordinary least squares (OLS) approach can be utilized to estimate the parameters of the route-level boarding model and route-level passenger-kilometers model, as well as the auxiliary supply models. Including the residuals from the auxiliary supply models as additional explanatory variables makes the OLS estimation consistent (as discussed in Section 3, the estimation would not be consistent otherwise).

In this study, several variable specifications and functional forms were examined to generate the empirical specifications of the demand models and supply models. The final specifications were arrived at after removing statistically insignificant variables using t-tests and behavioural interpretation considerations. Since the empirical models were estimated at the route level with data from only 530 routes in the city, following the guidance of Ortuzar and Willumsen (2011), we retained explanatory variables that were significant even at the 80% confidence level if the variables had an intuitive influence and were found in the earlier literature as influential on transit demand.

5. EMPIRICAL STUDY AREA, DATA, AND VARIABLES

5.1 Study area and data sources

Our study area is located in Bengaluru, one of India's fastest-growing cities, with a population of more than 13 million people, a bus transit system operated by Bengaluru Metropolitan Transport Corporation (BMTC) with more than 6,000 buses, and a small metro system of about 42 kilometres running along two metro lines. The Bruhat Bengaluru Mahanagara Palike (BBMP) region, which defines our study area, forms the core of Bengaluru city. The following bus transit data sources from BMTC were utilised for this study: (a) ticket sales records during the working weekdays of October 2019 from the Electronic Ticket Machines (ETM) used in the BMTC buses, (b) GPS data on bus arrival times at bus stops from the vehicle monitoring units on these buses, (c) transit network data that comprises bus routes, bus stop locations, stop-to-stop distances extracted using Open Street Maps, and (d) bus service information, including schedule and operations data for October 2019. We also utilised the metro rail station locations for the city's green and purple lines, which span a total distance of 42 kilometres. The socio-demographic data used in this study was derived from four sources: (a) census block-level population data, (b) census ward-level employment data, (c) parcel-level land-use classification data of all parcels in Bengaluru, available as colour-coded raster image maps, and (d) building-level data (plinth area and height of the building) of 1.4 million buildings in the study region.

The demand data for the estimation of route-level models comes from a sample of 530 routes (about 40% of all routes in operation during the AM peak time). The demand data for the estimation of stop-level models comes from 1821 fare-stage clusters of stops on these routes. In the route-level boarding and passenger-kilometres models, the variables describing external and internal factors are considered at the route-catchment level. In the following sections, we will provide a brief overview of the explanatory variables used in both these demand models. For further details regarding the data cleaning, data processing, and generation of these variables, the readers may refer to Deepa *et al.* (2022).

5.2 Operational characteristic representing supply quantity

The route-level service frequency, which acts as a proxy for explaining the quantity of supply, is an important service variable influencing ridership. We employ a spline specification to incorporate its non-linear effects. To do so, "kinks" are added at service frequency values of 3 buses/hour and 6 buses/hour using three spline variables labelled *Freq 1to3*, *Freq 3to6*, and *Freq 6plus* (9). Such piecewise linear specifications help in recognising that the marginal effect of increasing frequency is dependent on the present frequency.

$$
\begin{aligned}
Freq\_1to3 &= service\ frequency && if\quad service\ frequency < 3\\
&= 3 && if\quad service\ frequency \geq 3\\
Freq\_3to6 &= 0 && if\quad service\ frequency < 3\\
&= service\ frequency - 3 && if\ \ 3 \leq service\ frequency < 6\\
&= 3 && if\quad service\ frequency \geq 6\\
Freq\_6plus &= 0 && if\quad service\ frequency < 6\\
&= service\ frequency - 6 && if\quad service\ frequency \geq 6
\end{aligned}
\tag{9}
$$

To address the endogeneity of this variable, as discussed before, we include in the demand function the predicted residual from an auxiliary supply equation for service frequency.

5.3 Operational characteristic representing supply quality

To examine the impact of headway variability on demand, we use the headway coefficient of variation (CV) as an explanatory variable in the demand model. The headway CV is the ratio of the standard deviation of headways to the mean headway. For the stop-level model, the headway CV is calculated at the fare-stage cluster level by considering the headway data points for all the stops belonging to that cluster (over all working days across two weeks). For the route-level model, the route-level headway CV is calculated by considering the headway data points for all stops belonging to that route (over all working days across two weeks). An alternative method is to calculate the headway CV for all individual fare-stage clusters belonging to the route and then average the values across all clusters. Both methods yielded comparable results. To account for the non-linear impacts of headway variability, we employ a spline specification. Specifically, a "kink" is inserted at a headway CV value of 1.5, resulting in two spline variables designated *Hcov 1.5* and *Hcov 1.5plus*, as shown in Eq.(10). To account for the endogeneity effects, we incorporate in the demand function the predicted residual from the previously described auxiliary supply equation for headway CV.

$$\begin{aligned}
HCV_1.5 \quad &= headway\ CV & &if \quad headway\ CV < 1.5 \\
&= 1.5 & &if \quad headway\ CV \geq 1.5 \\
HCV_1.5plus \quad &= 0 & &if \quad headway\ CV < 1.5 \\
&= headway\ CV - 1.5 & &if \quad headway\ CV \geq 1.5
\end{aligned} \tag{10}$$

5.4 Trip-generating variables

The trip-generating variables within the route catchment include population (and its socio-demographics), employment, and land use. This is because people who live, work, or visit the route catchment are more likely to use the transit system.

5.5 Stop-specific facilities and route service type

Large bus stops such as a Traffic Transit Management Centre (TTMC), a bus station, or a bus depot tend to draw higher ridership owing to better stop-level facilities and amenities even after accounting for variations in connection with the rest of the network. Therefore, in the route-level models, the total number of such major stops (TTMC, bus station, or bus depot) along the subject route is computed and utilised as an explanatory variable to account for variations in stop-level infrastructure. In addition, because of differences in comfort, pricing, and journey durations, different service types are classified as separate routes in both route-level and stop-level demand models. Ordinary (non-AC) services, Vajra (express AC) services, and Vayu Vajra (express AC to the airport) services are the three bus service types in BMTC's operations. Of course, such disaggregation creates competition among the routes, which we discuss next.

5.6 Inter-route relationships

For analysing ridership on a subject route, it is important to assess if other routes that intersect/interact with it compete for passengers or complement it (Banerjee *et al.*, 2021). In this study, any route with at least one stop within a 0.5 km radius of any stops of the subject route is called an interacting route for the subject route. Such interacting routes are divided into the following four categories based on the extent of overlap with the subject route: (1) *fully competing routes*, (2) *fully complementary routes*, (3) *partly competing routes*, and (4) *partly complementary routes*. To do so, for each subject route, the percentage overlap for each interacting route is measured as a percentage of subject route stops that overlap with the interacting route. The interacting routes in the *fully competing* category go through all stops on the subject route and, therefore, compete for riders on the subject route. On the other hand, *fully complementary routes* intersect with the subject route only at a single stop. Such complementary routes bring riders from upstream of the intersecting stop, some of whom may transfer to the subject route. For interacting routes that intersect at multiple (but not all) stops of the subject route, they are labelled *partly competing routes* if they overlap at more than 50% of the stops on the subject route. If the overlap is at multiple but less than 50% of the subject route stops, then the interacting route is called a *partly complementary route*. For such routes, the percentage of non-overlap is measured as the percentage of stops on the subject route that do not overlap with the interacting route. Subsequently, to represent the competition and complementarity effects, the following four variables were created for each subject route: (1) total sum of frequencies of all fully competing routes, (2) total sum of frequencies of all fully complementary routes, (3) weighted sum of frequencies of all partly competing routes (weighted by percentage overlap of each such route), and (4) weighted sum of frequencies of all partly complementary routes (weighted by percentage non-overlap of each such route).

6. EMPIRICAL MODEL RESULTS

6.1 Empirical findings from the route-level demand models

The empirical parameter estimates of the route-level boarding model and route-level pkm model are presented in Tables 1 and 2, respectively. The behavioural interpretations and policy implications of both the demand models are discussed together next.

<u>*6.1.1 Influence of service frequency*</u>

Service frequency has a strong positive influence on route-level transit boardings and pkm, as demonstrated by many empirical studies (Kyte *et al.*, 1988; Estupinan and Rodrıguez, 2008; Chakour and Eluru, 2016; Mucci and Erhardt, 2018; Deepa *et al.*, 2022).

TABLE 1 Estimation results of route-level boarding model $[ln(total\ route\ boardings)]$

Explanatory Variables	Parameter	t-stat
Constant	3.2396	10.36
Socio-demographic variables $(\boldsymbol{Z_r})$		
o ln (Population in the route catchment)	0.0613	1.72
o ln (Employment in the route catchment)	0.0197	1.03
o ln (Commercial floor area in the route catchment)	-0.0252	-1.88
o ln (Industrial floor area in the route catchment)	--	--
o ln (Public service floor area in the route catchment)	--	--
Operational characteristics of the subject route $(\boldsymbol{F_r})$		
o Service frequency expressed as a piecewise linear (spline) function:		
Observed Frequency (1-3 buses per hour)	0.8362	17.11
Observed Frequency (above 3 buses per hour)	0.1503	5.58
o Residual $(\hat{\eta}_r)$ from the auxiliary route service frequency model	-0.4439	-14.46
o Headway variability expressed as a piecewise linear (spline) function:		
Headway coefficient of variation less than 1.5	-0.6685	-4.72
Headway coefficient of variation greater than equal to 1.5	-1.9394	-7.97
o Residual $(\hat{\vartheta}_r)$ from the auxiliary route headway CV model	0.4027	2.61
o Bus service type is *Ordinary*	--	--
o Number of *bus station*, or *bus depot* or *TTMC* along the subject route	0.0513	3.14
Network interactions of the subject route $(\boldsymbol{X_r})$		
Network interactions within BMTC		
o Total sum of frequencies of all fully competing routes	-0.0391	-1.58
o Weighted sum of frequencies of all partly competing routes	-0.2218	-1.44
o Total sum of frequencies of all fully complementary routes	0.1010	2.85
o Weighted sum of frequencies of all partly complementary routes	--	--
Network interactions with Metro		
o Presence of competing Metro stops in the route catchment	-0.1401	-1.86
o Presence of complementary Metro stops in the route catchment	0.1157	1.66
Standard error (σ) of ε_r	0.6099	26.81
Goodness-of-fit metrics		
Number of observations (fare-stage clusters)		530
Number of parameters in the model		17
Adjusted R Square		0.7157

-- Not statistically significant even at 80% confidence level and removed from the final specification

TABLE 2 Estimation results of route-level passenger-kilometre model [$ln(total\ route\ pkm)$]

Explanatory Variables	**Parameter**	**t-stat**
Constant	3.8104	8.07
Socio-demographic variables $(\boldsymbol{Z_r})$		
o ln (Population in the route catchment)	0.3189	6.05
o ln (Employment in the route catchment)	0.0586	2.01
o ln (Commercial floor area in the route catchment)	-0.1080	-5.51
o ln (Industrial floor area in the route catchment)	--	--
o ln (Public service floor area in the route catchment)	--	--
Operational characteristics of the subject route $(\boldsymbol{F_r})$		
o Observed service frequency (buses per hour)	0.1878	6.65
o Headway variability expressed as a piecewise linear (spline) function:		
Headway coefficient of variation less than 1.5	-0.9102	-4.18
Headway coefficient of variation greater than equal to 1.5	-2.9934	-6.19
o Residual $(\hat{\vartheta}_r)$ from the auxiliary route headway CV model	0.4117	2.06
o The bus service type is *Ordinary*	--	--
o Number of *bus station*, or *bus depot* or *TTMC* along the subject route	0.1879	6.27
Network interactions of the subject route $(\boldsymbol{X_r})$		
Network interactions within BMTC		
o Total sum of frequencies of all fully competing routes	-0.0490	-1.47
o Weighted sum of frequencies of all partly competing routes	-0.3978	-1.65
o Total sum of frequencies of all fully complementary routes	0.1473	2.72
o Weighted sum of frequencies of all partly complementary routes	--	--
Network interactions with Metro		
o The presence of competing Metro stops in the route catchment	-0.4775	-4.00
o The presence of complementary Metro stops in the route catchment	0.3451	3.06
The standard error (σ) of ε_r	1.0071	28.73
Goodness-of-fit metrics		
Number of observations (fare-stage clusters)		530
Number of parameters in the model		16
Adjusted R Square		0.6205

-- Not statistically significant even at 80% confidence level and removed from the final specification

Further, as can be observed from the parameter estimates of the service frequency variable in Table 1, the influence of frequency on ridership is not linear. As discussed in Berrebi *et al.* (2021) and Deepa *et al.* (2022), the marginal gains in ridership due to adding an extra bus trip on a route decrease with increasing frequency levels. However, we did not observe such non-linearity in the effect of service frequency on route-level pkm.

6.1.2 Influence of headway variability

The coefficients of headway variability variables in both ridership and pkm models reveal important findings. First, as evidenced by the significant and negative coefficients in the route-level ridership model, an increase in headway variability leads to a decrease in ridership. Similarly, the route-level pkm model estimation results suggest that service unreliability has a negative impact on passenger-kilometres (and, therefore, fare-box revenues). This is one of the few empirical studies in the literature that demonstrates the detrimental effect of headway variability on bus transit ridership and pkm (and revenues).

Second, in both ridership and pkm models, the non-linear effect of headway variability is evident. Specifically, the influence of headway variability depends on whether the coefficient of variation of headways is below or above 1.5. The parameter estimates indicate that the adverse effect of headway variability (on ridership and pkm) increases with an increase in variability. These results suggest that service irregularity has a significant negative impact on transit boardings and pkm (and revenue), with a bigger penalty for routes with higher variability. It is worth noting here that the increasing non-linear effect of headway variability is in contrast to the non-linear trend in the effect of service frequency, whereas transit ridership increases albeit at a decreasing rate with an increase in service frequency. From a policy standpoint, these findings suggest that transit agencies may gain more in terms of ridership and pkm (and revenue) if they focus on reducing headway variability rather than merely adding extra buses on high-frequency routes.

6.1.3 Influence of endogeneity of service frequency and headway variability on demand

In the transit ridership model (Table 1), to correct for endogeneity between demand and supply, we included, as additional explanatory variables, predicted residuals from the auxiliary supply models estimated for service frequency and headway coefficient of variation. The auxiliary models, which will be discussed in more detail later, include instrument variables – passenger-kilometers as an instrument in the frequency model and the coefficient of variation in the total route boardings as an instrument in the headway variability model. The reader is referred to Deepa *et al*. (2022) and the references therein for a detailed discussion of the use of instrument variables to control for endogeneity between demand and supply variables.

Results from Table 1 show that the coefficient on predicted residual ($\hat{\eta}_r$) from the auxiliary model of route-level service frequency is statistically significant (at a 95% level of confidence), implying the presence of endogeneity between service frequency and ridership (Hausman, 1978). When a model was estimated without this predicted residual, the parameter estimates and t-statistics of the piecewise-linear service frequency variables were higher than those reported in Table 1. These results corroborate the discussion in Section 3 that the magnitude of the effect of service frequency would be overestimated (*i.e.*, biased away from zero) if the endogeneity between demand and frequency is not considered. Similarly, the estimated coefficient on the predicted residual from the auxiliary model of headway coefficient of variation ($\hat{\vartheta}_r$ in Table 1) indicates endogeneity between headway variability and ridership. However, unlike in the case of the frequency variable, the magnitude of the coefficients of headway variability variables was weaker in a model that did not correct for endogeneity than those from the model reported in Table 1. These results also corroborate the discussion in Section 3 that the effect of service variability would be underestimated (*i.e.*, biased toward zero) if the endogeneity between demand and frequency is not recognized. Further, while not shown in the paper for brevity, the data fit of the models that ignored endogeneity of service frequency and service variability was inferior to that of the model in Table 1 that recognizes endogeneity with respect to both these variables.

For the model reported in Table 2 with pkm as the dependent variable, we did not address the endogeneity of the service frequency variable because of the unavailability of suitable instrument variables to do so. Therefore, it is likely that the model overestimates the influence of service frequency on route-level pkm. Due to the unavailability of instrument variables, future research should consider other approaches, such as simultaneous equations modeling of pkm and frequency, to address endogeneity between these two variables. On the other hand, the endogeneity of the headway variability variable (coefficient of variation of headways) has been addressed in a similar way to that of the ridership model. Without doing so, similar to the findings of the transit ridership model, the effect of headway variability was underestimated.

6.1.4 Influence of other factors

Although the primary goal of this study is to examine the impact of headway and its variability on demand, we summarize here the impact of other significant factors. The parameter estimates for the catchment area population and employment variables in the route-level demand models (Tables 1 and 2) are in line with previous findings that sizeable population and employment along a route are important determinants for creating demand for transit along that route (Kimpel *et al.*, 2000; Chu *et al.*, 2007; Cervero *et al.*, 2010). On the other hand, a high frequency of buses on competing routes passing through the route catchment reduces boardings on the subject route. This is because some riders would choose the competing routes to go to the same destinations offered by the subject route. For the same reason, a larger percentage overlap of partly competing routes with the subject route also reduces boardings on the subject route. On the contrary, a high frequency of buses on complementary routes passing through the route catchment increases boardings on the subject route since these routes bring transfer riders to the subject route. In addition to the above-mentioned interactions within the bus network, the existence of competing metro stations in the catchment area of a route has a negative impact on bus ridership. However, the existence of fully complementing metro stations in the catchment area of a route seems to promote bus ridership by feeding transfer riders from the metro network to the bus network. Even after accounting for trip-generating variables, network interactions, connectivity, and service frequency effects, the difference in boardings between major stops and simple stops suggests that better stop-specific facilities and higher-than-usual network interactions contribute to greater ridership at major stops such bus stations, bus depots, and TTMCs. Finally, ordinary service routes have larger ridership than the Vajra (express) and Vayu Vajra (express AC) services, which is likely due to the higher ticket price and limited range of destinations associated with the latter services.

6.2 Empirical findings from the auxiliary service frequency model

Table 3 presents the estimation results of the auxiliary regression model for route-level service frequency. In this model, as in Deepa *et al*. (2022), route-level passenger-kilometers (pkm) is used an instrumental variable (IV) for addressing the endogeneity of the service frequency variable in the transit ridership model (of Table 1).

TABLE 3 Estimation results of the auxiliary route-level service frequency model

Explanatory variables	Parameter	t-stat
Constant	-0.0836	-0.26
Total passenger-kilometres (in thousands of km) along the route	0.6158	22.17
ln (Population in the route catchment)	0.1832	4.03
ln (Employment in the route catchment)	0.0479	1.80
ln (Commercial floor area in the route catchment)	-0.0646	-3.01
ln (Industrial floor area in the route catchment)	-0.0208	-1.99
Ln(Public service floor area in the route catchment)	-0.0202	-1.64
The standard error (υ) of the error term (η_r)	0.9038	50.28
Goodness-of-fit metrics		
Number of observations (routes)		530
Number of parameters		8
Adjusted R-square		0.7285

As can be seen from Table 3, route-level pkm has a strong influence on the service frequency variable, indicating that it meets the relevance criteria for an IV. This is expected since many transit agencies strive to maximise fare-box revenue by increasing supply along routes that can serve a large passenger base and generate high pkm. However, another criterion necessary for the pkm variable to be a good IV is that it has no direct influence on ridership. This is not verifiable and must rely on theoretical judgment. In this regard, as discussed in Deepa *et al*. (2022), it may be reasoned that increasing passenger boardings on a route always leads to an increase in passenger-kilometres on the corresponding route but increasing passenger-kilometres need not necessarily increase its boardings. This is so because, in the latter case, it is possible that only the current riders' journey lengths increase without the addition of new riders. Bus frequency is higher on routes that travel through densely populated areas than on other routes. Similarly, routes that traverse through locations with higher employment have higher service frequency. Transit agencies often provide more supply along corridors with anticipated higher trip generation; hence these effects are expected. Routes that travel through regions with higher industrial, commercial, or public service activity (measured using the industrial floor area, commercial floor area, and public service floor area, respectively) have lower service frequency after controlling for population and employment. This is perhaps because such areas are not often visited by many individuals or because such locations may be supplied by employer-provided shuttle services (hence public transit is in short supply).

6.3 Empirical findings from the auxiliary model for headway variability

Table 4 presents the estimation results of the auxiliary regression model for route-level headway variability (CV of headways). Variability in route-level passenger demand (*i.e*., coefficient of variation of route-level boardings) is used as an instrumental variable (IV) in this model to address endogeneity between headway coefficient of variation and transit demand. The corresponding parameter estimate in Table 4 suggests that routes with high variability in demand are also associated with high headway variability. This is expected since higher variability in boarding and alighting might lead to longer bus dwell times at stops, causing a divergence from scheduled arrival timings downstream (Abkowitz *et al*., 1978). Further, it is reasonable to assume that the demand variability does not directly influence demand. Therefore, the coefficient of variation of route-level boardings satisfies the criteria to serve as an IV to address endogeneity between route-level service variability and transit demand.

Further, from the empirical results, it is apparent that longer routes and ordinary service types suffer greater service variability than other routes. Importantly, this shows that providing shorter routes (or those with fewer intermediate bus stops) during peak hours would be helpful. In this context, splitting lengthy routes into shorter routes or into express and regular routes might be an alternative technique for improving headway reliability. It is, however, necessary to consider the additional waiting time or transfer time faced by passengers due to splitting lengthy routes, which may outweigh the potential benefits of increased reliability. Therefore, further research is needed to evaluate such strategies before policy-level conclusions can be drawn. Finally, as expected, routes that experience higher congestion (as represented by the average traffic speed on the route) are associated with higher service variability. Finally, although we did not consider service frequency and headway variability as endogenous to each other in our theoretical model, we explored in the empirical model if service frequency has an influence on the CV of headways or vice versa. Neither of these effects were found to be statistically significant.

TABLE 4 Estimation results of the auxiliary route-level headway variability model

Explanatory variables	Parameter	t-stat
Constant	0.1684	1.42
Coefficient of variation in the total route boardings	0.2211	4.06
ln(length of the route in kms)	0.2780	12.19
The route service type is *Ordinary*	0.0701	1.61
The average speed (kmph) of the route as a proxy for the traffic state	-0.0253	-4.99
The standard error (υ) of the error term (η_r)	0.3333	36.72
Goodness-of-fit metrics		
Number of observations (routes)		530
Number of parameters		5
Adjusted R-square		0.1727

6.4 Empirical findings from the stop-route-level demand model

The earlier section discussed empirical findings from the route-level demand and supply models. These models highlighted the strong negative effect of headway variability on transit demand at the route level. To examine the influence of headway variability on transit demand at other levels of aggregation, we extended the stop-route-level transit boarding models of Deepa *et al.* (2022) to include the influence of the headway coefficient of variation (CV) variable, along with addressing its endogeneity using the instrumental variables approach discussed earlier. Since such an empirical model without the headway CV variable was discussed in detail in Deepa *et al.* (2022), the empirical results of the model are not discussed in detail here. However, the estimation results of the stop-level boarding model and the auxiliary stop-level headway variability model are reported in Table B.1 and Table B.2, respectively in Appendix B. Since the auxiliary service frequency model is at the route level, the model reported in Table 3 was utilized.

From the estimation results of the stop-route-level boardings model, we see empirical evidence of the negative impact of headway variability on boardings at the stop-route level too. Further, as with the route-level models discussed earlier, there is non-linearity in the effect of headway variability in that the strength of the effect increases with an increase in the coefficient of variation. In addition, there is also endogeneity between stop-level headway coefficient of variation and ridership, which was addressed using the residual obtained from the auxiliary model for headway CV. As in the route-level model, ignoring endogeneity resulted in an underestimation of the strength of headway variability effect on stop-route-level boardings.

The estimation results of the auxiliary model for stop-route cluster level model for headway variability suggest that locations with high variability in boardings and alighting, locations further downstream of a route, presence of signalised intersection, etc., are associated with higher variability in headways. Also, stops on longer routes, routes going through congested corridors, and ordinary routes are more likely to have higher variability in the headways. These results are similar to the findings reported in earlier studies.

6.5 Policy analysis

Recall from the discussion in Section 6.1 that the influence of service frequency (or headways) on transit ridership decreases with increasing frequency, whereas the influence of headway variability increases with increasing variability. To understand the policy implications of such contrasting non-linear trends in the effects of service quantity (frequency) and service quality (headway variability), we used the route-level demand models from Section 6.1 to evaluate the following (Figure 2) policy scenarios for a sample of high-frequency routes: (a) decreasing headway variability (standard deviation) by 10% from the current variability levels and (b) increasing service frequency by 10% from the current frequency levels.

The first policy scenario implies an average of 1-minute reduction in the standard deviation of headway variability for the routes used in this analysis (route-specific reductions in standard deviation ranged from 0.5 minutes to 1.6 minutes, which averaged to 1 minute across all routes). The second policy strategy implies an average increase of 0.9 buses/hour across the routes used in this analysis (route-specific increases in frequency ranges from 0.3 buses/hour to 1.22 buses/hour, which averaged to 0.9 buses/hour across all routes). Figures 1(a) and 1(b) show the changes in ridership and pkm, respectively, due to these strategies. As can be observed from these figures, the benefits of reducing headway variability are larger than those of increasing frequency on all these routes. These results corroborate the discussion in Section 6.1 that transit agencies might gain greater ridership and pkm (and revenue) if they reduce headway variability rather than merely adding extra buses to high-frequency routes. Of course, similar findings may not necessarily hold for low-frequency routes, because such routes might benefit more from increasing service frequency than high-frequency routes would.

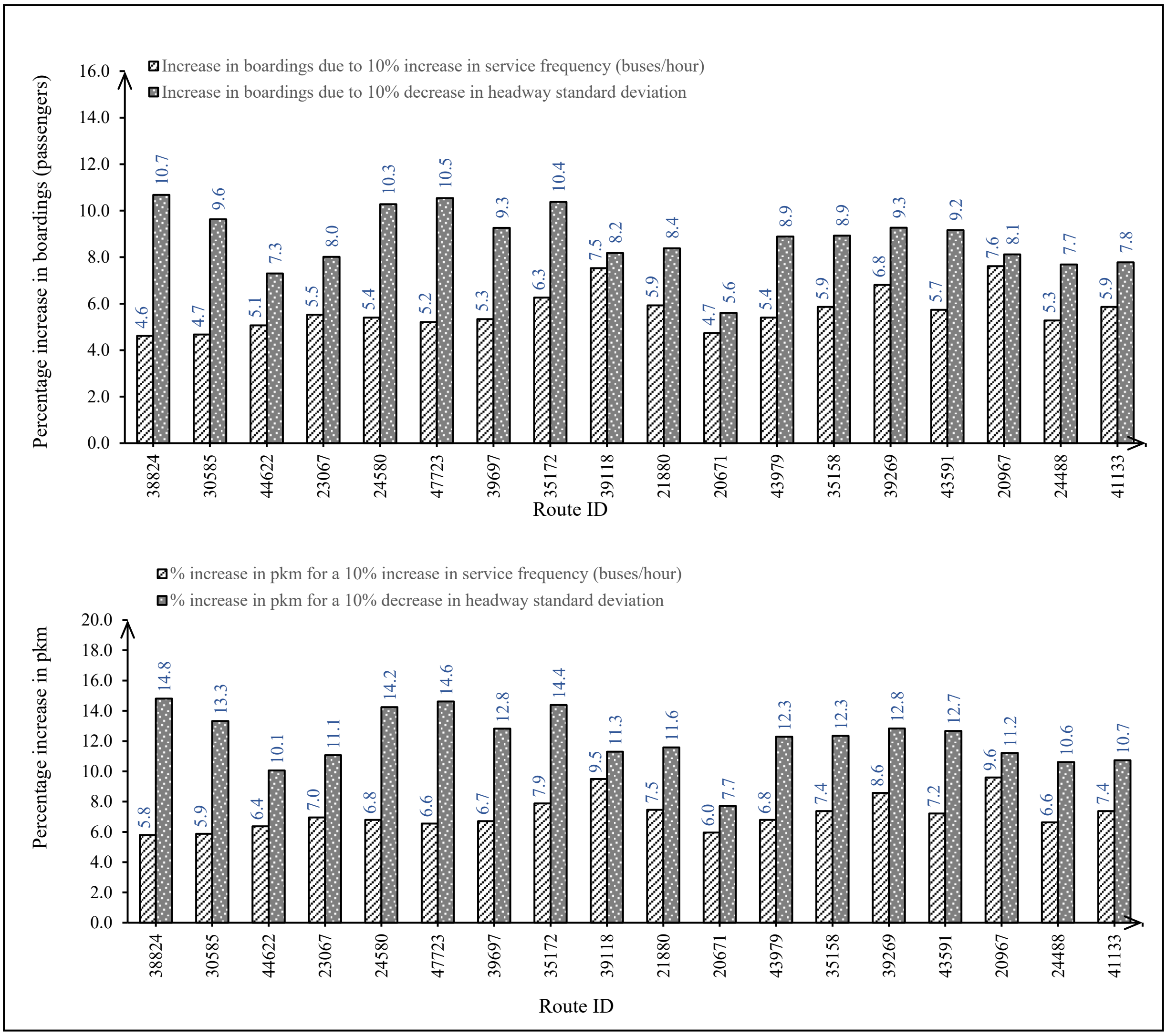

Figure 2 Ridership and pkm benefits of reducing headway variability vs. increasing service frequency

7. CONCLUSIONS AND FUTURE WORK

This study is perhaps among the first in the public transit literature to examine and quantify the influence of service quantity (service frequency) and service quality (headway variability) on bus transit demand, while considering issues of endogeneity and non-linearity of the effects of such service characteristics on bus transit ridership and revenue. To do so, we develop direct demand models of bus transit demand at different levels of spatial aggregation to examine the influence of service frequency and headway variability (coefficient of variation of headways) on transit demand. Specifically, two route-level models – one for route-level ridership (boardings) and another for route passenger-kilometres – and a stop-route-level ridership model are developed. In addition, auxiliary models are developed for service frequency and headway variability to examine the factors influencing service supply characteristics and also to address the endogeneity of these service variables in transit demand models. The two-stage residual inclusion (2SRI) method has been used to address the endogeneity of service variables in transit demand models. Empirical models are estimated using bus transit demand and supply data from a large bus transit system in Bengaluru, India.

The following are the specific contributions of this study. First, this study makes a significant contribution to the public transit literature by quantifying the influence of service quantity (service frequency) and service quality (headway variability) on bus transit demand. Specifically, using a conceptual discussion based on a tri-variate structural equations model (of transit demand, service frequency, and headway variability), we show how demand-supply simultaneity results in endogeneity between supply and demand. Further, by deriving the direction of bias, we show theoretically that the endogeneity of service frequency and headway variability with ridership might manifest in opposite ways. That is, ignoring endogeneity between headway variability and transit demand would always result in an underestimation of the influence of headway variability. On the other hand, ignoring endogeneity between service frequency and transit demand may result in an overestimation or underestimation of the influence of service frequency, depending on the magnitude of a few parameter estimates.

Second, using empirical data from the BMTC, we present empirical models of route-level transit ridership and passenger kilometres. Using these models, we provide empirical evidence that service unreliability (as measured by the coefficient of variation in headways) has a strong adverse impact on transit ridership at both the route level and stop-route level. It has an adverse impact on total route passenger-kilometres also, a metric that directly influences fare-box income.

Third, perhaps for the first time in the transit literature, we offer empirical evidence that the non-linear effects of headway variability and its endogeneity with transit demand manifest in ways that are opposite to those of service frequency. More specifically, the adverse effect of headway variability increases with an increase in variability, whereas the influence of frequency decreases with increasing frequency.

Fourth, the empirical results corroborate our conceptual model that ignoring endogeneity between headway variability and transit demand can lead to an underestimation of the adverse effect of headway variability. On the other hand, ignoring the endogeneity between service frequency and transit demand resulted in an overestimation of its influence. These findings, as well as subsequent policy simulations, indicate that transit agencies can potentially gain greater ridership and revenue if they can lower headway variability rather than by merely adding more buses to high-frequency routes. This result warrants research on and implementation of approaches to reduce headway variability in bus transit systems, such as bus-bunching control techniques and bus priority lanes.

Fifth, the auxiliary supply models we estimated shed light on the factors influencing route-level service quantity (frequency) and service quality (headway variability) in Bengaluru. These insights contribute to understanding the determinants of bus transit service characteristics.

Sixth, in addition to the route-level transit demand and supply models, we estimated stop-route-level transit demand models that offer empirical evidence similar to that from the route-level models, such as: (a) the negative impact of headway variability on stop-level boardings, (b) the non-linearity in the effect of headway variability on transit demand, and (c) the endogeneity between stop-level headway variability and ridership, which resulted in an underestimation of the impact of headway variability on stop-route-level boardings.

The current empirical study focused on analysing bus transit demand in Bengaluru. However, the conceptual framework, modeling approach, and some of the empirical findings have broader applicability. Specifically, the models developed in this study can be used as descriptive tools for understanding the determinants of bus transit demand in other cities in India and other countries (if the empirical parameters are estimated using data from those cities). By incorporating the models into forecasting exercises, transit planners and policymakers can assess the ridership and revenue impacts of *what-if* scenarios, such as changes in service frequency, strategies to control headway variability, etc. The endogeneity and nonlinearity relationships that we have recognized between service frequency, headway variability, and transit demand are important model specification issues applicable to other transit contexts. In this context, the theoretical finding that ignoring the endogeneity between headway variability and transit ridership would lead to underestimation of the adverse impact of headway variability on transit ridership applies to such models in other geographic contexts. Further, based on the conceptual discussions and empirical findings of this study, it is likely that bus transit systems in general can gain greater ridership and revenue if they can control headway variability rather than by merely adding more buses to high-frequency routes. Of course, it would be useful to apply the proposed models to other cities to examine the extent of generalizability of findings and to allow for comparisons and benchmarking across various contexts.

Some shortcomings of the study pave the way for further research. First, we did not address the endogeneity of the service frequency variable in the route-level pkm model due to the unavailability of suitable instrumental variables. Future research should explore alternative approaches to address endogeneity. Second, from the standpoint of transit practitioners, it would be worthwhile to use the models from this study to examine the benefits of different headway management strategies (such as bus holding, frequency optimisation, limited-stop services, and traffic signal priority) on transit ridership and revenue. Doing so will require not only the demand models developed in the study but also bus service (supply) simulation models to evaluate the effect of headway management strategies on headway variability. Such an integrated tool to simulate bus service operations (to estimate the headways and their variability) and forecast demand can be valuable for transit agencies. The authors are currently pursuing these directions. Third, the current study does not consider the role of information availability (such as the expected arrival time of buses through passenger Apps). The availability of accurate information can potentially moderate the effect of headway variability on transit ridership. Considering the role of information availability (and its reliability) on transit ridership is an important topic for future exploration. Another direction to explore is to examine the effect of headway variability on the ridership of different socio-demographic segments and that of captive users and choice users.

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APPENDIX A: Nature of Bias due to Endogeneity in Transit Ridership Models

Consider the following classic linear regression equation: $\boldsymbol{Y} = \boldsymbol{X}'\boldsymbol{\beta} + \boldsymbol{\varepsilon}$.

The OLS estimator of $\boldsymbol{\beta}$ for this equation is $\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y}$, which can also be written as $\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'(\boldsymbol{X}'\boldsymbol{\beta} + \boldsymbol{\varepsilon})$. To understand the direction of bias in estimation, one can apply the expectation operator to both sides of this expression as below:

$$E[\widehat{\boldsymbol{\beta}}] = \boldsymbol{\beta} + E[(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{\varepsilon}] \tag{A1}$$

As can be observed from this expression for $E[\widehat{\boldsymbol{\beta}}]$ whether the magnitude of $\boldsymbol{\beta}$ is underestimated or overestimated depends on the sign of the true parameter $\boldsymbol{\beta}$ and the sign of the correlation between $\boldsymbol{X}$ and $\boldsymbol{\varepsilon}$. With these preliminaries, we next discuss the anticipated direction of bias in estimating the coefficients of frequency (F_r) and headway variability (H_r).

3.2.1 Direction of bias in the coefficient on service frequency

Consider the true coefficient β_1 of the service frequency variable in Equation (1). Also, stack the service frequency data (F_r) for all the routes into a single vector $\boldsymbol{F}$ and assume for now that frequency is the only endogenous variable in the model. Next, using Equation (A1), one can express the OLS estimator of β_1 as:

$$E[\widehat{\beta_1}] = \beta_1 + E[(\boldsymbol{F}'\boldsymbol{F})^{-1}\boldsymbol{F}'\boldsymbol{\varepsilon}] \tag{A2}$$

In the above expression, the sign of β_1 (true coefficient of frequency in the demand model) can be expected to be positive since high-frequency routes can be expected to have high demand, *ceteris paribus*. The sign of the bias term $E[(\boldsymbol{F}'\boldsymbol{F})^{-1}\boldsymbol{F}'\boldsymbol{\varepsilon}]$ is the same as the sign of $E[\boldsymbol{F}'\boldsymbol{\varepsilon}]$, which depends on the sign of $\frac{c_1}{(1-c_1\beta_1)}$ (see Equation (4) and subsequent discussion). Since the sign of c_1 can be expected to be positive because higher-demand routes tend to have greater frequency, the bias term would be positive if $0 < c_1\beta_1 < 1$ and negative if $c_1\beta_1 > 1$. That is, ignoring the endogeneity of service frequency can be expected to result in an overestimation of the effect of service frequency in the demand model if $0 < c_1\beta_1 < 1$ and underestimation if $c_1\beta_1 > 1$. Whether the benefits of increasing bus frequency are underestimated or overestimated depends on the empirical values of the parameter estimates of c_1 and β_1.

3.2.2 Direction of bias in the coefficient on headway variability

Similar to the above discussion, one can express the OLS estimator for the coefficient β_2 of headway variability (H_r) as below, assuming, for now, that headway variability is the only endogenous variable in the model:

$$E\left[\widehat{\beta_2}\right] = \beta_2 + E[(\boldsymbol{H'H})^{-1}\boldsymbol{H'\varepsilon}] \tag{A3}$$

Here, one can expect the sign of β_2 (true coefficient of headway variability) to be negative since routes with greater service variability can be expected to have lower demand, *ceteris paribus*. The bias term $E[(\boldsymbol{H'H})^{-1}\boldsymbol{H'\varepsilon}]$, however, can always be expected to be positive. This is because the sign of the term $E[\boldsymbol{H'\varepsilon}]$ depends on $\frac{d_1}{(1-d_1\beta_2)}$ and one can expect d_1 would always be positive (because higher demand results in greater headway variability) and β_2 would always be negative (because greater headway variability results in lower demand). Therefore, using an OLS estimator would always lead to underestimation of the adverse effects of headway variability if endogeneity is ignored.

APPENDIX B

TABLE B.1 Estimation results of the fare-stage cluster boardings model [$ln(total\ cluster\ boardings)$]

Explanatory Variables	Parameter	t-stat
Constant	-2.0874	-5.40
Stop level variables $(\boldsymbol{Z}_{sr})$		
o ln (Population in stop-specific catchment/Voronoi polygon)	0.8782	8.20
o ln (Employment in stop-specific catchment/Voronoi polygon)	0.2933	6.37
o ln (Commercial floor area in stop-specific Voronoi polygon)	-0.1751	-3.68
o ln (Industrial floor area in stop-specific Voronoi polygon)	-0.0636	-3.23
o ln (Public service floor area in stop-specific Voronoi polygon)	-0.3393	-6.38
Fare-stage cluster-level variables $(\boldsymbol{X}_{Sr})$		
<u>*Headway variability expressed as coefficient of variation*</u>		
o Coefficient of variation < 1.5	-0.7866	-3.88
o Coefficient of variation ≥ 1.5	-0.8004	-2.78
o Residual $(\hat{\vartheta}_{Sr})$ from the auxiliary headway CV model at fare-stage cluster level	0.7866	3.28
<u>*Network interactions within BMTC*</u>		
o Service frequency of fully and partly competing routes in the cluster catchment	-0.0014	-3.26
o Service frequency of fully complementary routes in the cluster catchment	0.0132	3.71
o Service frequency of partly complementary routes in the cluster catchment	0.0003	4.55
o ln (No. of upstream overlaps that the subject route makes with interacting routes)	-0.1022	-6.57
<u>*Network interactions with Metro*</u>		
o Presence of fully-competing metro stops in the fare-stage cluster	-0.5301	-5.07
o Presence of partly-competing metro stops in the fare-stage cluster	-0.4166	-5.59
o Presence of fully-complementary Metro stops in the fare-stage cluster	0.1565	1.54
o Presence of partly-complementary Metro stops in the fare-stage cluster	--	--
<u>*Connectivity variables as surrogates for accessibility by transit*</u>		
o ln (No. of downstream stops accessed directly from the fare-stage cluster)	0.0900	2.68
o ln (No. of downstream stops accessible via one transfer from the subject route)	0.0308	2.09
<u>*Other variables*</u>		
o Any bus stop in the fare-stage cluster is *bus station*, *bus depot*, or *TTMC*	0.5507	6.29
o The fare-stage cluster constitutes last few (15%) stops of the route *(dummy)*	-1.0138	-11.31
o Bus route service type is *Ordinary*	0.6787	10.37
Route level variables $(\boldsymbol{F}_r)$		
o Service frequency of the subject route expressed as piecewise linear (spline) function:		
Observed Frequency (1-3 buses per hour)	0.6787	20.01
Observed Frequency (3-6 buses per hour)	0.1548	5.26
Observed Frequency (above 6 buses per hour)	0.0762	4.08
o Residual $(\hat{\eta}_r)$ from the auxiliary model of route-level service frequency	-0.0581	-3.06
Coefficient on the *logsum* variable (θ)	0.6273	10.88
Standard error (σ) of ε_{Sr}	0.9273	60.25
Goodness-of-fit metrics		
Number of observations (fare-stage clusters)		1821
Number of parameters in the model		28
Log-likelihood at convergence		-2446.52
Log-likelihood of constants only model (2 parameters)		-3219.84
Adjusted Rho Square with respect to a constants-only model		0.2400

-- Not statistically significant even at 80% confidence level and removed from the final specification

TABLE B.2 Estimation results of fare-stage cluster-level headway variability model

Explanatory Variables	Parameter	t-stat
Constant	0.2884	4.22
Fare-stage cluster-level variables		
o Coefficient of variation in the number of boardings at the fare-stage cluster	0.0775	3.66
o Coefficient of variation in the number of alightings at the fare-stage cluster	0.2322	9.83
o Number of scheduled intermediate stops to the fare-stage stop in a cluster	0.0021	4.60
o The presence of a signalised intersection in the fare-stage cluster (*dummy*)	0.0241	1.78
Route level variables		
o ln(length of the route in kms)	0.0574	3.18
o The average speed (kmph) of the route as a proxy for the traffic state	-0.0023	-2.44
o The route service type is *Ordinary*	0.0307	1.37
Standard error (σ)	0.2673	90.72
Goodness-of-fit metrics		
Number of observations (fare-stage clusters)		1821
Number of parameters in the model		10
Adjusted R-square		0.3563