A direct demand model for bus transit ridership in Bengaluru, India

L. Deepa [a]

Abdul Rawoof Pinjari [a,b, #]

Sangram Krishna Nirmale [a]

Karthik K. Srinivasan [c]

Tarun Rambha [a,b]

[a] Department of Civil Engineering, Indian Institute of Science (IISc), Bangalore 560012, India

[b] Centre for Infrastructure, Sustainable Transportation and Urban Planning (CiSTUP), Indian Institute of Science (IISc), Bangalore 560012, India

[c] Department of Civil Engineering, Indian Institute of Technology, Madras, Chennai 600036, India

[#] Corresponding author (abdul@iisc.ac.in)

ABSTRACT

This study formulates a disaggregate direct demand model of bus transit ridership while addressing the following substantive and methodological issues: (a) endogeneity and non-linearity of the influence of service frequency on ridership, (b) inter-route relationships such as competition and complementarity among routes within the bus transit network and with other transit networks (such as the metro/rail network), (c) relating spatially aggregated demand to disaggregate, stop-level catchment characteristics – although demand data are available only at an aggregation of stop-clusters, and (d) overlapping of catchment areas among closely spaced stops. The proposed model is applied to analyze bus transit ridership (boardings) during weekdays for morning peak period in Bengaluru, India. This study is among the first to develop a comprehensive direct demand model for forecasting bus transit ridership in an Indian city. Yet, the proposed conceptual and methodological framework and the findings from the study are general enough to be of use for transit planning in other cities of India and other countries. Transit agencies with spatially aggregate, fare-stage cluster-level ridership data can employ the proposed approach to examine the influence of disaggregate stop-level catchment characteristics on ridership. Additionally, transit agencies may utilise the proposed model to quantify bus ridership impacts of service network modifications, route alignments, and network connectivity/accessibility, while considering interactions with other transit networks. The empirical results suggest that while increasing service frequency increases ridership along low-frequency routes, the returns from increasing frequency diminish as current frequency levels increase. Further, it is shown that route-level passenger kilometres, a variable commonly available with transit agencies, serves effectively as an instrument for addressing endogeneity between route-level service frequency and stop-route-level ridership.

Keywords: public transportation, transit ridership, direct demand models, inter-route relationships, transit accessibility, demand-supply endogeneity

1. INTRODUCTION

1.1 Background

Ridership is a key measure of a transit system's performance – both as a metric for the extent of use of the system (i.e., transit demand) and as a determinant of the fare-box revenue. Therefore, ridership forecasting models are valuable tools for transit agencies to assess the possible impacts of their operational strategies and service improvements. Existing approaches to modeling transit demand can be grouped into two broad categories: (1) Regional travel demand model systems and (2) Direct demand models. Regional travel demand model systems are used to predict a region's spatial, temporal, and modal distribution of travel – as an outcome of travelers' trip frequency, departure time, destination, mode, and route choices – using aggregate, four-step models (McNally, 2007) or disaggregate, activity-based approaches (Pinjari and Bhat, 2011). While such regional models are very useful for city-scale transportation planning, they require extensive survey data on travel by all modes and can be time- and resource-intensive to develop and maintain. On the other hand, direct demand models can estimate transit ridership "directly" as a function of various factors influencing ridership, such as transit route- and stop-level service characteristics and accessibility. Once developed, such models are easy to apply for assessing the ridership impacts of changes to transit network or operational characteristics at a fine spatial/temporal scale, such as changes in periods of operation, route configuration, frequency/headways, and stop level amenities and accessibility (Banerjee *et al.*, 2021). Further, these models can be structured to utilize predictions of spatial-temporal distribution of travel from regional travel demand models. Therefore, even in cities with regional models, transit agencies benefit from using direct demand models of ridership.[1]

Not much research on transit ridership forecasting using direct demand models exists for cities in emerging economies such as India. Being highly populated and served by large-scale public transit systems, cities from such countries will benefit from methodologically sound and 7easy-to-use transit ridership forecasting tools. To address this gap, this paper develops a spatially disaggregate direct demand model for forecasting stop-level bus transit ridership (boardings) for Bengaluru, a major metropolitan city in India.

In the rest of this paper, Section 1.2 reviews literature on direct demand models for transit ridership forecasting. Section 1.3 positions the current study, including its substantive and methodological contributions. Section 2 describes the conceptual framework used in this study to model transit ridership. Section 3 describes the data sources and the variables constructed to develop the proposed model. Section 4 presents the econometric structure of the proposed model. Section 5 presents the empirical results and insights on the determinants of bus ridership in Bengaluru, validation of the empirical model, and a comparison with simpler specifications used in the literature. Section 6 summarizes the study and outlines directions for future research.

[1] To be sure, direct demand models are not free of disadvantages (Cervero *et al.,* 2010). It is not easy to accommodate competition between different modes since such models are not typically structured to consider the effect of pricing and travel times of competing modes. Similarly, it is not easy to account for substitution effects across different destinations. Besides, it is not possible to use direct demand models to assess the distributional impacts of changes in transit service, because these are aggregate models that do not consider demographic characteristics of individual travelers. Therefore, direct demand models should not be viewed as replacement of regional models. They are useful tools for supplementing the regional models for transit ridership analysis and forecasting.

1.2 Features of Direct Demand Models in the Literature

A summary of literature using direct demand models for bus transit ridership is presented in Table A.1 of Appendix A. This table identifies the various features of the models used, including the econometric structure, spatial and temporal resolution, variables to explain ridership, and whether the following methodological or substantial features have been considered – demand-supply endogeneity, non-linear effect of service characteristics on transit demand, inter-route relationships, the influence of accessibility/connectivity, and the use of detailed land-use variables. Some of these aspects are discussed next to identify the research gaps and to position the current study.

1.2.1 Spatial resolution of the model

Most previous studies have modeled ridership either at the route level, route-segment level, or stop level. Route-level models (*e.g.*, Kemp,1981; Hartgen and Horner, 1997) do not recognize the spatial variation of service characteristics, land-use and socio-demographics, and ridership within a route. To address this, some studies (Peng *et al.*, 1997; Kimpel *et al.*, 2000) model ridership at the spatial resolution of route segments defined by time-point stops or fare zones. In both route-level and segment-level models, the socio-demographic and land use characteristics as well as operational variables across stops within the respective route- or segment-catchment are assumed to be homogenous. However, the validity of this assumption is questionable, particularly along long routes that traverse heterogeneous sections of a city. Therefore, several studies modeled ridership at a stop level (Strathman *et al.,* 1997; Chu *et al.*, 2007; Estupinan and Rodrıguez, 2008; Cervero *et al.*, 2010; Dill *et al.*, 2013; Chakour and Eluru, 2016; Rahman *et al.*, 2019 and 2021; Berrebi *et al.*, 2021). Stop-level disaggregation allows the analyst to relate ridership at a transit stop to the infrastructure, amenities, population, employment, and land use in the catchment specific to that stop. To further characterize variation in ridership at a stop served by different routes, one may consider stop-route level models, where the unit of analysis is defined by the stop and the route serving the stop (Mucci and Erhardt, 2018). For example, if two different transit routes serve the same bus stop, they are treated as two distinct stop-route level observations. In such stop-route models, the route-level service attributes such as service frequency, the characteristics of the stop and its catchment area, and how the specific route interacts with other routes that pass through (or near) the stop may be used to explain ridership. Doing so helps in disaggregating stop-level ridership into ridership on different routes serving the stop, and in recognizing inter-route competition or complementarity.

Nevertheless, stop-route level ridership models are sparse in transit literature, and existing disaggregate-level studies (both stop-level and stop-route-level models) do not consider several methodological and substantive issues in modeling transit demand, as discussed next.

1.2.2 Spatial aggregation of demand

Most transit agencies store their ticket sales data (hence their ridership data) at a spatial aggregation coarser than the level of bus stops. Specifically, the data are stored at the same spatial aggregation as that of their fare stages or fare zones, where a fare stage is a cluster of a few consecutive stops on a route. This is because bus fares do not typically change from one stop to the immediate next stop; instead, they change from one fare stage to the next fare stage.

Previous studies either do not delve into this issue or combine the catchment characteristics of stops constituting a fare stage into an aggregate fare-stage level or a stop-cluster level (Berrebi *et al.*, 2021) and then relate these characteristics with ridership at the fare-stage level. However, to understand the influence of stop-level catchment area characteristics and stop-level amenities on ridership, it is important to relate the fare-stage level ridership to disaggregate stop-level attributes. Additionally, it would be useful to devise a method to disaggregate the aggregate fare-stage-level ridership into its component stop level, which would provide useful information to transit agencies while making stop-specific supply decisions.

1.2.3 Overlapping among stop-catchment areas

In stop-level and stop-route-level models, overlapping of the catchment areas of adjacent stops along a route can cause overestimation of the influence of land use and socio-demographic characteristics around the stops. To address this, some studies (*e.g.*, Chu *et al.*, 2007) first identify the neighboring stop buffers that overlap with the subject stop's buffer. Subsequently, they either subtract the overlapping areas from the subject stop buffer or divide the overlapping area among the corresponding stop buffers. Implementing such an *ad hoc* method for many stops becomes cumbersome when stops are closely spaced, and multiple stop buffers overlap. In this context, a study by Horner *et al.* (2010) introduced the idea of using Voronoi polygons to resolve the issue of catchment overlapping. However, this approach is yet to be widely used to construct variables describing stop-level catchments for transit ridership models.

1.2.4 Demand-supply endogeneity

Many transit ridership modeling studies do not recognize the endogeneity between transit demand (*e.g.*, boardings) and supply characteristics (*e.g.*, service frequency). While ridership levels in a transit system depend on the extent of supply, the supply provided along different routes, in turn, depends on anticipated ridership, thus causing endogeneity. Likewise, increased service frequency during peak hours and cut schedules during off-peak hours are manifestations of the feedback effect of anticipated demand on supply. Ridership models that ignore such endogeneity tend to overestimate the influence of supply enhancements on transit demand (Voith, 1991; Fitzroy and Smith, 1999; Holmgren, 2007; Rahman *et al.*, 2019). To be sure, some route-level models accommodate endogeneity between route-level transit demand and route-level service frequency (Kyte *et al.*, 1988; Peng *et al.*, 1997). However, doing so for stop-level models is a challenge because multiple routes can serve the same stop, and it is not easy to incorporate the endogeneity of the route-level service characteristics of all such routes (because demand manifests at the stop level whereas service frequency is a route-level attribute). Although, considering stop-level headway instead of route-level frequency as a supply variable makes it easier to capture supply-demand endogeneity at the stop level, route-level frequency can be directly modified by transit agencies, and thus it would be easier for them to understand the influence of the latter on transit demand. In this context, modeling ridership at a stop-route level makes it easier to consider endogeneity of service frequency of a specific route as its effect manifests at the stops served by the route.

1.2.5 Non-linear effect of service attributes

Most studies in the literature assume that service frequency has a linear effect on transit demand. However, in many service systems including transit systems, the effect of supply on

demand is likely to be non-linear. Although bus stops with a higher potential for demand are likely to benefit from a higher frequency of buses, the expected increase in boardings due to an increase in service frequency typically exhibits a diminishing trend beyond an optimum level of frequency (Berrebi *et al.*, 2021). Ignoring such non-linear effects can potentially lead to a loss in model fit, decreased prediction accuracy, and distorted assessment of the influence of service frequency changes. To the authors' knowledge, Kyte *et al.* (1988), Rahman *et al.*, (2019), and Berrebi *et al.* (2021) are the only transit ridership studies that consider non-linear effects of service frequency.

1.2.6 Inter-route relationships in transit networks

Most stop-level or stop-route-level studies do not comprehensively characterize inter-route relationships in transit networks. In this context, one may define the following three different types of inter-route relationships (Peng *et al.*, 1997; Pendyala and Ubaka, 2000; Chu *et al.*, 2007): (a) independence (if the routes do not interact because they do not go through the subject stop's catchment area), (b) complementarity (if the routes intersect in the subject stop's catchment area to increase accessibility of the subject stop to more destinations through transfer boardings), and (c) competition effects (if the routes go through the subject stop's catchment area but they are parallel to each other and mostly offer accessibility to a similar set of destinations). Part of the difficulty in accommodating such network-level interactions at a stop level is because, although interactions occur among routes, their competing and complementary effects on ridership are manifested at the stop level. It is not easy to separate these effects by the different routes or directions that a stop serves, if ridership is modeled at the stop level without differentiating the routes serving the stop. However, when ridership is modeled at a stop-route level, it becomes possible to capture network interactions of a specific route (defined by path, direction, service type, and service characteristics) with other routes and the manifestations of such interactions at a specific stop.[2]

Further, except for a few studies (Peng *et al.*, 1997; Polzin *et al.*, 2011; Mucci and Erhardt, 2018), the transfer boarding potential due to passengers arriving from or (heading to) a stop's upstream or (downstream) stops is not considered in transit ridership modeling. Peng *et al.* (1997) incorporate inter-route transfers by considering alightings from complementary routes. Mucci and Erhardt (2018) follow a similar approach of including alightings from bus/rail passengers within walking distance of a stop-route while modeling boarding at that stop. However, this approach is restrictive since alightings from complementary routes must be predicted before modeling boarding at the subject stop. Besides, such variables might be endogenous to the subject-stop boardings.

1.3 Current Research

In view of the research gaps discussed above, this study develops a conceptual and methodological framework to model stop-route-level bus transit ridership (boardings), while considering the following aspects: (a) spatial aggregation of demand data (typically aggregated

[2] Route-level or route-segment models (Peng *et al.*, 1997) that incorporate inter-route relationships cannot recognize that the interactions may change from one stop to the other. Some stop-level models (Chu *et al.*, 2007) incorporate inter-relationships using measures of accessibility to opportunities as explanatory variables. Other models (Rahman *et al.*, 2019) include the number of neighbouring stops of other routes within the subject stop's catchment area, without characterizing the type of interdependency.

to a stop-cluster level) and relating it to disaggregate, stop-level catchment characteristics, (b) the influence of inter-route interactions such as competition and complementarity within the bus network and other transit networks (such as Metro rail), (c) the influence of connectivity from the subject bus stop to the rest of the city via the transit network, (d) demand-supply endogeneity, and (e) non-linear impacts of service frequency on demand. This framework is applied to develop a direct demand model of bus transit ridership (boardings) in the city of Bengaluru, India, for the AM peak period.

Since observed ridership data are typically available at a fare-stage cluster level, the framework considers the unit of analysis as a fare-stage cluster and route combination. To relate the fare-stage cluster boardings to disaggregate stop-level boardings (on a given route), the proposed model takes the form of a non-linear regression that accommodates a log-sum variable that "collects" information from all stops belonging to a fare-stage cluster into a composite explanatory variable. Further, overlapping among disaggregate stop-catchment areas is resolved by constructing Voronoi polygons of the stops using geospatial tools.

A rich variety of variables are included in the model, such as trip generating characteristics of the catchment area of a fare-stage stop cluster (fed into the model at the disaggregate level of constituent stops in the cluster), accessibility/connectivity of transit stops via the bus (and Metro) network to rest of the study region with and without transfers, stop and service typology, and frequency of routes serving the subject cluster. Importantly, explanatory variables are included to account for complementarity and competition among the routes passing through the stop-route combination for which boarding is modeled. Inter-route relationships and their manifestation at the stop-route level are considered through variables such as the frequency of service of competing and complementary routes.

The model also corrects for endogeneity between route-level service frequency and stop-route-level ridership (demand-supply endogeneity); thanks to the disaggregate, stop-route level at which the ridership is modeled. Specifically, endogeneity is addressed using the two-stage residual inclusion (2SRI) approach, where the residual from an auxiliary route-level frequency model (supply model) is used as an explanatory variable – along with the observed service frequency and other variables – in the stop-route-level ridership model (demand model). In this context, it is demonstrated that passenger-kilometers served along a route – a variable commonly available with transit agencies – effectively serves as an instrumental variable (IV) in the route level frequency model. Further, the non-linear effect of route-level service frequency on ridership is incorporated through a piecewise linear (*i.e.*, spline) specification.

To the authors' knowledge, this is perhaps the first comprehensive ridership modeling study that simultaneously addresses all the above-mentioned methodological and substantive issues. The proposed model can be used by transit agencies to quantify bus ridership impacts of service network modifications, route alignments, and network connectivity/accessibility, while considering interactions with other transit networks. Further, transit agencies with spatially aggregate, fare-stage cluster-level ridership data can employ the proposed approach to examine the influence of disaggregate stop-level catchment characteristics on ridership.

2. CONCEPTUAL FRAMEWORK

The conceptual framework of the proposed model is presented in Figure 1. As shown in the figure, the demand for ridership at a stop on a given route is influenced by two broad set of

factors: (1) external factors and (2) internal factors. The external factors include socio-demographics and land use of the study area, which the transit agency cannot control (at least in the short term). The internal factors include spatial and temporal coverage of the network and operational service characteristics, which the transit agency can control. Interactions among the external and internal factors manifest in the form of inter-route relationships (competing or complementary effects) and accessibility (or connectivity).

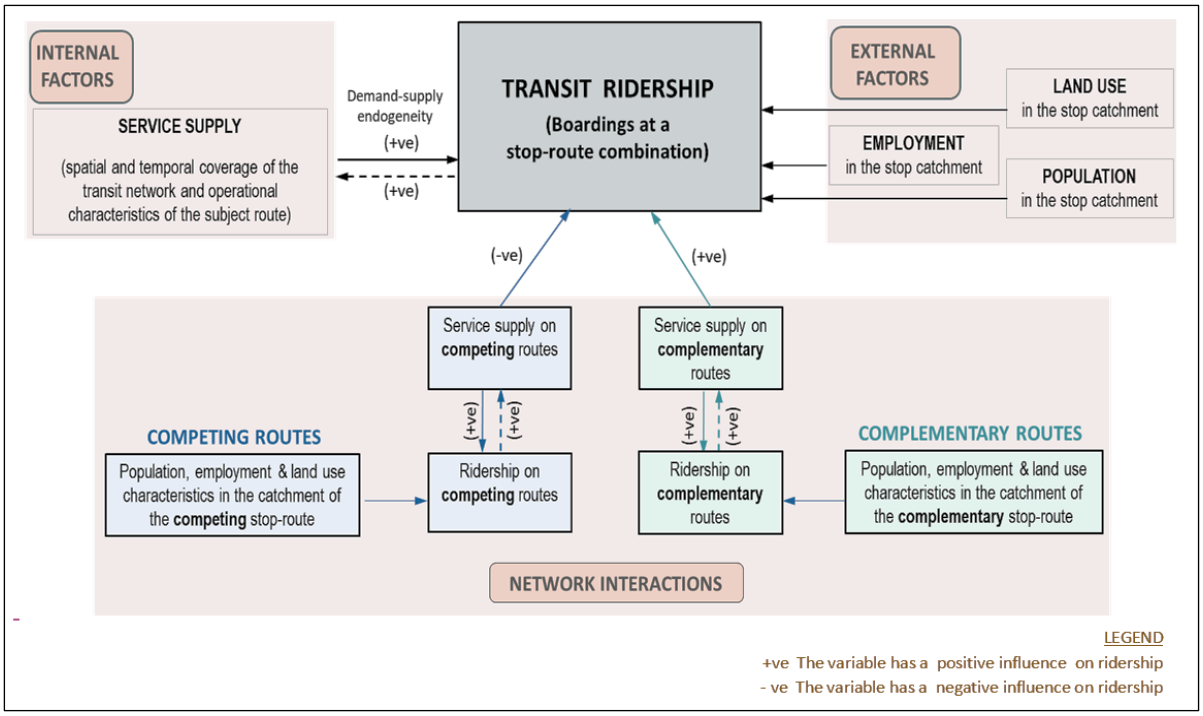

Figure 1 Conceptual framework of the proposed model

The study uses transit boardings to represent (and model) transit ridership. Since boardings represent transit trip generation, understanding the influence of internal and external factors on transit boardings can help identify ways to increase transit ridership. Prior to discussing the influence of internal and external factors on transit boardings, the next section discusses the issue of spatial aggregation of transit ridership data, which has a bearing on the conceptualization of the variables representing the internal and external factors.

2.1 Dependent Variable: Fare-stage Cluster-level Transit Boardings

In many transit networks, bus stops on any route are typically grouped into fare-stage clusters comprising of at least two consecutive bus stops, as shown in Figure 2. The fares vary from one fare-stage cluster to the other cluster but do not change across the stops belonging to a fare-stage cluster. For the same reason, the ticket sales data in many transit systems are stored at a spatial aggregation of fare-stage clusters (or fare zones). For example, for each ticket sold, the available data may include the timestamp of its sale, fare-stage identifiers (IDs) of passenger boarding and alighting stops, and route ID. Therefore, the specific stops at which a passenger boarded and alighted are unknown, but the corresponding fare-stage clusters are known. A practical challenge with such data is to relate the aggregate, fare-stage-level demand data to disaggregate, stop-level supply and catchment area characteristics, which this study aims to address (more on this is in Section 4).

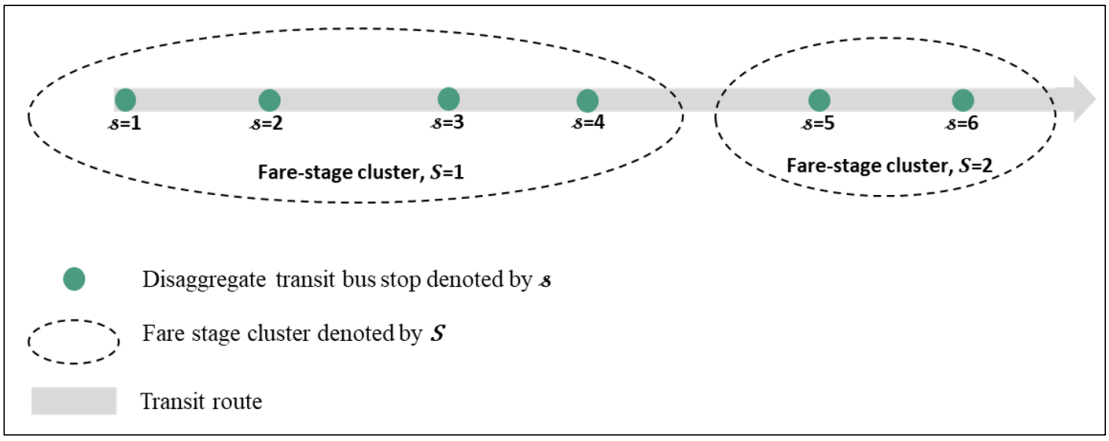

Fig 2 Schematic representation of fare-stage cluster construction

2.2 Influence of External Factors (Trip Generating Variables) on Transit Boardings

Most transit ridership studies use some metrics of the population (and its socio-demographics), employment, and land-use within the catchment of a bus stop to explain ridership at that stop. Such variables may be viewed as trip-generating variables, since people who live in, work at, and visit the catchment area of the stop are likely to use the transit system at that stop.

In dense transit networks, where bus stops on a route are closely spaced, it can be difficult to determine the catchment area of each stop. This study uses the concept of Voronoi-based decomposition (Okabe *et al.*, 2000) of the spatial area around a group of stops on a route to identify mutually exclusive catchment areas. Specifically, viewing the geographical area as a plane, the *Voronoi polygon* of a subject bus stop contains all points that are closer to the subject bus stop (based on Euclidean distance) than other bus stops on the same route. To limit the catchment area, one can consider a pre-determined maximum distance from the bus stops for creating such polygons. It is implicitly assumed that travelers prefer the stop closest to their home/activity locations, if all else is the same across the stops on a route. As described in Horner *et al.* (2010) (also, see Okabe *et al.*, 2000), this approach is computationally simple and easy to implement, because Voronoi tessellation results in convex polygons (Okabe *et al.*, 2000) for which geospatial operations such as intersection and clipping are computationally efficient.

Figure 3 shows a schematic of the Voronoi-based decomposition for a route used in this study. Specifically, a one-kilometer space around all bus stops on the route is divided into stop-specific Voronoi polygons. Subsequently, geospatial data belonging to each of the polygons – such as census block-level population, parcel-level land-use and employment – are used to create trip-generating variables for the stop-specific catchment area. Such stop-level Voronoi polygon variables may also be aggregated to form fare-stage cluster-level catchment area variables. In this study, the variables describing the external factors (i.e., trip generating variables) are considered at the disaggregate stop level. As discussed in the next section, the variables describing internal factors such as service frequency of the route and the interactions of the subject route with other routes, and those quantifying accessibility downstream of the subject stop-route combination are aggregated to the fare-stage cluster level.

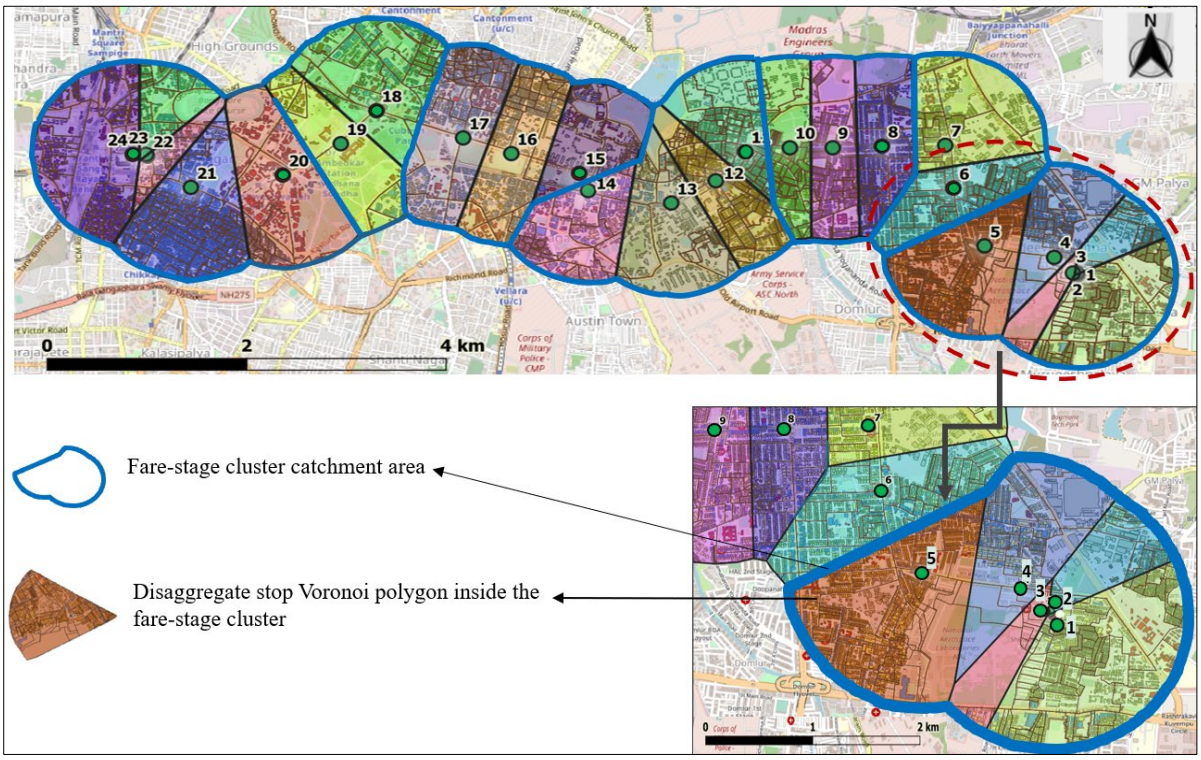

Fig 3 Fare-stage cluster construction using Voronoi polygons that avoid stop-catchment overlaps

Note: The direction of the route shown in the figure is from right (East) to left (West).

2.3 Influence of Route-level Service Frequency

The most important of the internal factors affecting ridership at a stop-route combination is the route-level service frequency. To incorporate the non-linear effect (*i.e.*, diminishing marginal effect) of the frequency variable, we use a spline specification, where "kinks" are introduced at service frequency values of 3 buses/hour and 6 buses/hour using three spline variables labeled *Freq_1to3, Freq_3to6,* and *Freq_6plus*.

$$
\begin{aligned}
Freq\_1to3 &= service\ frequency && if \quad service\ frequency < 3 \\
&= 0 && if \quad service\ frequency \geq 3 \\
Freq\_3to6 &= 0 && if \quad service\ frequency < 3 \\
&= service\ frequency - 3 && if \quad 3 \leq service\ frequency < 6 \\
&= 3 && if \quad service\ frequency \geq 6 \\
Freq\_6plus &= 0 && if \quad service\ frequency < 6 \\
&= service\ frequency - 6 && if \quad service\ frequency \geq 6
\end{aligned}
\tag{1}
$$

Piecewise linear specifications, as in the above example, help in recognizing that the marginal effect of increasing frequency depends on the current frequency. For example, the coefficient of the *Freq_3to6* variable informs the marginal effect of increasing frequency anywhere from 3 buses/hour to 5 buses/hour.

As discussed earlier, the route-level service frequency and stop-route-level ridership are likely to be endogenous to each other. This issue is discussed in detail in Section 4.

2.4 Influence of Stop-specific Facilities and Route Service Type

Ridership levels across bus stops can be different due to differences in connectivity with the rest of the network and stop-level facilities/infrastructure. In Bengaluru, for example, each bus stop may be categorized into one of the following categories: a simple bus stop, station, depot,

or Traffic Transit Management Centre (TTMC). The latter three types of stops tend to attract greater ridership – even after accounting for differences in connectivity with the rest of the network – due to better stop-level infrastructure and amenities. Particularly, a TTMC is a major hub that provides connectivity to a large number of destinations and includes services such as vending stalls, ATMs, park and ride facilities, and sitting areas. A bus station is a large hub connected to several bus routes. Bus depots allow for overnight parking and can hold a greater number of buses than a conventional bus stop. Therefore, to account for differences in stop-level infrastructure compared to a simple bus stop, we use a dummy variable to identify if a stop is designated as a TTMC, or bus station, or bus depot. Further, even across simple bus stops, stop-level facilities such as the presence of a shelter, lighting, and walking access to the stops can potentially influence ridership.

At the route level, there are likely to be differences across different service types – ordinary, and air-conditioned (AC) services – due to differences in comfort, fare, and travel times. The bus service types in Bengaluru are: (a) Ordinary (non-AC), (b) Vajra (express AC service), and (c) Vayu Vajra (express AC service to the airport) routes. Different service types passing through the same set of stops are categorized as different routes. Of course, such disaggregation leads to competition effects among the routes, which are discussed next.

2.5 Network Interactions within the Bus Transit Network

To analyze ridership for a stop-route combination, one must also consider the service characteristics of other routes serving the subject stop or other nearby stops. Based on the network structure, other routes can either compete for ridership or complement ridership to the route being considered.

To identify the routes that interact with (either compete with or complement) a subject stop-route combination, all other routes that have a stop in the catchment area of the subject stop are first identified. These *interacting routes* are further divided into the following four categories based on the number and location of the stops common to subject route.

- *Fully competing routes*: An interacting route that completely overlaps with all downstream stops of the subject route is called a *fully competing route*. Such routes compete for riders going to the same downstream destinations, as shown in Figure 4(a). This definition holds regardless of the interactions upstream of the subject stop.
- *Partly competing routes*: An interacting route that does not completely overlap with the subject route (downstream to the subject stop) but has a greater number of overlapping downstream stops than the number of non-overlapping downstream stops is called a *partly competing route*, as shown in Figure 4(b). In such cases, the competition effect is moderated because the competition is only for riders traveling to the overlapping stops. In addition, an interacting route that overlaps with a subset of the downstream stops of the subject route and does not offer any new destinations is also called a *partly competing route.* See Figure 4(c) for an example.
- *Fully complementary routes*: An interacting route that does not overlap with the subject route except at the subject stop, as in Figure 4(d), is called a *fully complementary route*. Such routes bring riders from upstream who might transfer at the subject stop-route.

- *Partly complementary routes*: An interacting route that does not have any downstream overlap with the subject route but has at least one common stop upstream of the subject stop is called a *partly complementary route*. Although such *partly complementary routes* bring transfer riders from upstream, the overlapping upstream stops act as alternate transfer locations to moderate the complementarity effect at the subject stop. See Figure 4(e) for an example.

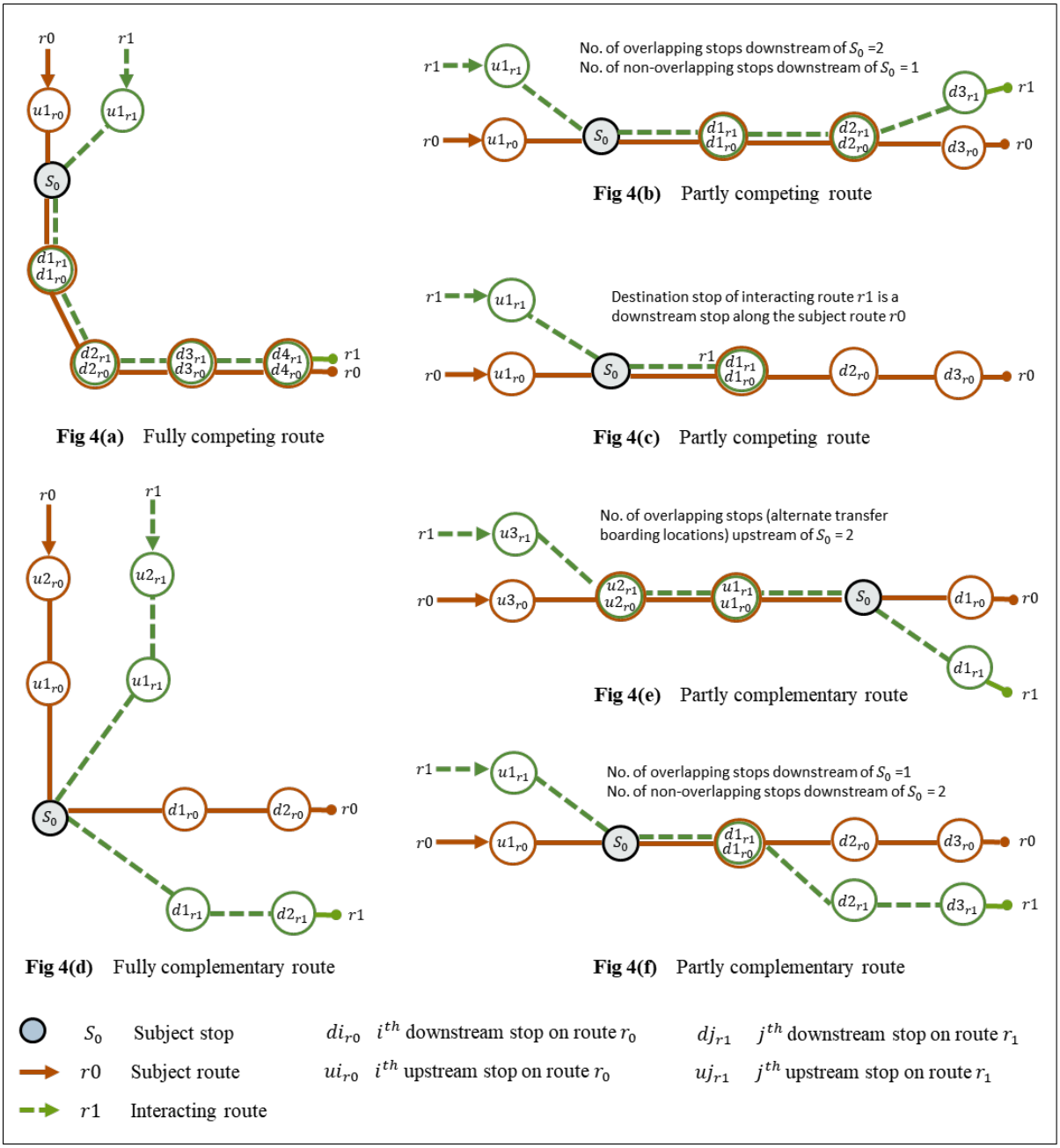

Fig 4 Schematic representation showing network interactions

Regardless of the upstream overlaps, an interacting route that does not go to the same destination as that of the subject route but has a smaller number of overlapping downstream stops than the number of non-overlapping downstream stops is also called a *partly complementary route.* This is because the number of new downstream destinations accessible to travelers transferring from the interacting route to the subject route (complementary effect)

at the subject stop is greater than the number of overlapping downstream destinations (competition effect). See Figure 4(f) for an example.

For each stop-route combination, we created four variables – each of which aggregated (summed) the frequency of one of the above four types of interacting routes. As a result, the following four variables were created for each stop-route combination: (1) total frequency of all fully competing routes, (2) total frequency of all partly competing routes, (3) total frequency of all fully complementary routes, (4) total frequency of all partly complementary routes. In addition, we created a fifth variable that counts the number of upstream overlaps that partly complementary routes make with the subject route. This fifth variable captures competition effects with the subject stop-route combination due to alternate transfer boarding locations upstream of the subject route.

Next, the above five variables created at the disaggregate, stop-route level were aggregated to characterize the interactions at a fare-stage cluster level. To do so, we explored three different types of aggregation – (1) summation of all stop-level interactions (of each type) to the fare-stage cluster level, (2) average of all stop-level interactions to the fare-stage cluster level, and (3) variables of the stop (within the fare-stage cluster) with the maximum number of interactions as a proxy for the network interactions at the fare-stage cluster level. In this context, for all interacting routes except fully complementary routes, summing the frequencies across all stops of a fare-stage cluster will likely overcount the interaction effects. This is because the interacting routes (other than fully complementary routes) will likely go through several or all stops in the cluster. So, either an average or a maximum value might better explain such interactions at a fare-stage cluster level. However, one can better capture the fare-stage level effect of fully complementary routes as a sum of frequencies of such routes across all stops within the cluster. This is because fully complementary routes interact with the subject route at only one stop of the fare-stage cluster (by definition), which makes the fully complementary routes at one stop different from those at another stop. Finally, the fifth variable in the above list (no. of upstream overlaps with partly complementary routes), was constructed with respect to the first stop of each fare-stage cluster.

2.6 Network Interactions with the Metro Transit Network

To conceptualize interactions between subject stop-routes in the fare-stage cluster and the metro transit network, we identified if a metro station lies within the 1 km catchment area of the subject stop. For each such interacting metro station, we classified it as fully competing, partly competing, fully complementary, or partly complementary with at least one of the stops in the subject fare-stage cluster (using the same principles discussed in the earlier section). Bengaluru only has two metro lines, and hence we worked with categorical variables that identified the type of interaction between the metro station and the stop-route combination (instead of using the metro service frequency variables).

2.7 Accessibility (or Connectivity) Provided by the Transit System

The influence of the interplay between the external and internal factors on ridership at a stop-route may be considered through variables that represent accessibility (or downstream connectivity) provided by the transit network from the corresponding stop to various activity locations in the city. Specifically, stops or routes that offer access or connectivity to a larger number of destinations/activity opportunities within a shorter time duration (preferably without

transfers) are likely to attract greater levels of ridership (Chu *et al.*, 2007; Cervero *et al.*, 2010; Polzin *et al.*, 2011). In this study, the total number of downstream stops directly reachable (without transfers) from the subject fare-stage cluster is employed as a surrogate to quantify the directly accessible activity opportunities. In addition to such directly accessible destinations, a traveler at the subject stop (or fare stage cluster) can access additional destinations by transfering to routes that intersect the subject route at any of its downstream stops. However, it is unlikely that riders tolerate transfers beyond a certain threshold. Therefore, a variable was created to measure the total number of additional stops accessible from the fare-stage cluster by making one transfer to bus routes intersecting the subject route at its downstream stops (without double-counting the same stops accessible by multiple routes). Such downstream transfer routes could be in the metro network as well. Hence, we created another variable to measure the total number downstream stops along the subject route from where one can transfer to the Metro line to reach additional destinations (Metro stations).

Notably, while the variables discussed in the earlier two sections describe the effects of origin-end interactions between the subject route and other routes of bus and metro networks, the accessibility variables discussed above capture the effect of downstream interactions. Nevertheless, to reduce the computational burden and the amount of data processing work, the above-discussed variables were constructed only for the first stop of each fare-stage cluster in the data considering that, in Bengaluru's transit network, the first stop in any cluster is invariably the major stop (among other stops) in terms of its stop-level amenities, network interactions effects, connectivity with the remaining network, *etc*.

3 DATA DESCRIPTION AND EMPIRICAL DATA-SETUP

3.1 Study Area

Bengaluru is one of the rapidly growing cities of India and has a population of more than 13 million served by an extensive bus transit network with about 6,000 buses. The study area is the Bruhat Bengaluru Mahanagara Palike (BBMP) region of Bengaluru, which is shown in Figure A.1 of Appendix A. Given the density of the transit network, bus ridership at a stop is likely to be influenced by inter-route relationships among the various routes going through or nearby the stop. Further, two metro lines (of a total of 42 km) run along east-west and north-south corridors. Since various bus routes intersect with or run parallel to the metro lines, it becomes important to consider interactions with the metro network as well.

3.2 Data Sources and Data Processing

The bus transit ridership and service data for this research come from the following sources: (a) ticket sale records in the working weekdays of October 2019 from the Electronic Ticket Machines (ETM) installed in the buses of the Bengaluru Metropolitan Transport Corporation (BMTC), (b) GPS bus location data from the vehicle monitoring units on these buses, (c) transit network data, including bus routes, bus stop locations (and their sequence) on each route, stop-to-stop distances extracted using Open Street Maps, and metro rail stop locations, and (d) bus service information, including schedule and operations data (route-level service frequency, etc.) for October 2019. The sociodemographic and land use data used in this research come from the following sources: (a) Census ward-level population and employment data, (b) Census block-level population data, and (c) parcel level land-use classification data of all parcels in the

city of Bengaluru, available in the form of color-coded raster image maps, (d) building-level data (plinth area and height of the building) of 1.4 million buildings in the study region.

Table A.1 in Appendix A lists the variables considered in this study, along with their descriptive statistics. A description of the procedures used to create the variables follows next.

3.2.1 Dependent variable

As discussed earlier, the ETM ticket sales data are stored at a spatial aggregation of fare-stage clusters. Specifically, for each ticket sold, the available data include the timestamp of its sale, fare-stage identifiers (IDs) of passenger boarding and alighting stops, route ID, schedule ID, and vehicle number. For this study, fare-stage cluster level boardings data were extracted for the AM peak period (8 AM to 11 AM) for each working weekday in October 2019. The data were averaged across the working weekdays to create the dependent variable – daily AM peak period fare-stage cluster level boardings (this variable was natural log-transformed).

The demand data used for this study comes from a total of 3,915 fare-stage clusters (that comprise a total of 12,301 bus stops), covering a sample of 436 routes (~ 35% of all routes in operation during the AM peak period). Among these, ridership data of 2,955 fare-stage clusters were used to estimate the model parameters. The data of the remaining 960 fare-stage clusters (~25%) were set aside to validate the models.

3.2.2 Independent variables

Several independent variables were created following the framework described in Section 2. As discussed earlier, the population data was already available at a census-block level. This data was aggregated to stop-level Voronoi polygons for all stops on each route considered in this analysis. To create other independent variables, the parcel-level land-use maps in raster image format were converted into vector shapefiles using image processing techniques to identify the land-use type based on the colour coding of the parcels. The resulting information was merged with building-level data to designate land-use types for each building. Subsequently, the building plinth area and height were used to approximate the floor area of each building (assuming 3m/floor height for residential buildings, 5m/floor height for industrial, commercial, and institutional buildings, as per codes of Karnataka state building byelaws). Subsequently, the building-level floor area (combined with land-use type information) was aggregated to Census-block-level floor area variables, one for each of the three land-use types – commercial, industrial, and public service.

Since the employment information was available only at the ward level, the above-discussed block-level floor area information was used to disaggregate the ward level employment to the block level based on the proportion in which the ward-level floor space (for commercial, industrial, or public service land-uses) was distributed across the Census blocks. Similar to the population data, the block-level land-use (floor area) and employment data were aggregated to the stop-specific Voronoi polygons.

4 ECONOMETRIC MODEL STRUCTURE

4.1 Specification of the Ridership Model

Denote the fare-stage cluster (S) and route (r) combination for which the transit boardings are modeled as Sr. Such a fare-stage cluster and route combination (Sr) is the unit of analysis for

our model. Further, as depicted in Figure 2, the disaggregate bus stops belonging to a fare-stage cluster on route r are denoted as $\mathscr{s}r$ ($\mathscr{s}r = 1,2, \ldots, N_{Sr}$, where N_{Sr} is the number of bus stops in the fare-stage cluster Sr). Let the total number of boardings across all stops ($\mathscr{s}r$) in the fare-stage cluster Sr be denoted by B_{Sr}. In this study $ln(B_{Sr})$ – the natural logarithm of total boardings for a fare-stage cluster and route combination (Sr) – is the primary dependent variable. This variable is modeled using a non-linear regression approach, as a function of explanatory variables at three different levels of spatial aggregation – stop level, fare-stage cluster level, and route level – as shown in the equation below:

$$ln(B_{Sr}) = \beta_0 + \theta\, ln\left\{\sum_{\mathscr{s}r=1}^{N_{Sr}} exp(\boldsymbol{\gamma}'\boldsymbol{Z}_{\mathscr{s}r})\right\} + \boldsymbol{\alpha}'\boldsymbol{X}_{Sr} + \boldsymbol{\beta}'\boldsymbol{F}_r + \lambda\, \hat{\eta}_r + \varepsilon_{Sr} \qquad (2)$$

The stop-level explanatory variables include the population, employment, and land-use characteristics of the stop-level catchment area (Voronoi polygon) and are represented by the vector $\boldsymbol{Z}_{\mathscr{s}r}$ ($\boldsymbol{\gamma}$ is the corresponding coefficient vector). Since the stop-level variables are at a finer spatial aggregation than that of the dependent variable, they are aggregated using a *logsum* term $ln\left\{\sum_{\mathscr{s}r=1}^{N_{Sr}} exp(\boldsymbol{\gamma}'\boldsymbol{Z}_{\mathscr{s}r})\right\}$ which helps carry information from the disaggregate, stop level to the aggregate, fare-stage cluster level. This empirical strategy allows transit agencies with only fare-stage cluster-level ridership data to examine the influence of stop-level catchment area variables on transit ridership. Another advantage of using this approach is that once the fare-stage boarding prediction is made, it can be disaggregated into the constituent stop-level boardings using a simple logit expression. Specifically, once the total boardings ($\hat{B}_{Sr}$) at a fare-stage cluster on a route are predicted, the boardings (B_{jr}) at a stop j on that route r may be estimated as $\hat{B}_{jr} = \hat{B}_{Sr} \times \frac{\boldsymbol{exp}(\boldsymbol{\gamma}'\boldsymbol{Z}_{jr})}{\sum_{\mathscr{s}r=1}^{N_{Sr}} \boldsymbol{exp}(\boldsymbol{\gamma}'\boldsymbol{Z}_{\mathscr{s}r})}$.

The next set of variables is a vector $\boldsymbol{X}_{Sr}$ of fare-stage cluster-level variables, with $\boldsymbol{\alpha}$ as the corresponding vector of coefficients. $\boldsymbol{X}_{Sr}$ includes a variety of internal factors influencing ridership, such as the variables quantifying competition and complementarity interactions (within the bus network) in the fare-stage cluster (as described in Section 2.5), variables for interactions with the metro network (described in Section 2.6), and those quantifying the accessibility provided by the transit system for travelers from the fare-stage cluster (described in Section 2.7). In addition, variables identifying stop-level infrastructure (*e.g.*, if at least one of the stops is a major stop) and those describing the service type of the route (*e.g.*, ordinary vs. express service) may be included here.

Note here that some of the fare-stage cluster-level variables in the vector $\boldsymbol{X}_{Sr}$, such as those quantifying interactions within the bus network and with the metro network, and those quantifying accessibility, may also be constructed and specified at a disaggregate, stop level as part of the *logsum* term via the vector $\boldsymbol{Z}_{\mathscr{s}r}$. We explored both the possibilities – fare-stage cluster-level specification and stop-level specification – in the empirical analysis and found that the model with fare-stage cluster-level specification offered better fit and more intuitive explanation. This may be because the influence of network interactions and accessibility variables is better captured at the aggregate, fare-stage cluster level than the stop level. For example, downstream accessibility of individual stops in a fare-stage cluster in not likely to be different from that of the cluster.

The route-level variables include a vector $\boldsymbol{F}_r$ of piecewise-linear variables of the route-level frequency (with the corresponding coefficient vector $\boldsymbol{\beta}$), specified as described in Section 2.3. An alternative specification is to use a linear specification of frequency by directly including the frequency variable (denoted as $Freq_r$). Such a model would be a special case of the model with piecewise-linear variables ($\boldsymbol{F}_r$) of frequency. In any case, as discussed earlier, route-level service frequency is likely to be endogenous to the demand variable (ridership). Therefore, along with the frequency variables, the residual term $\hat{\eta}_r$ obtained from an auxiliary model for the route-level frequency is used to correct for the endogeneity between fare-stage cluster-level boardings and route-level frequency, as discussed in the next section.

Finally, for completing the model specification, the error terms ε_{Sr} in Equation (2) is assumed to be normally distributed with zero mean and standard deviation σ.

4.2 Endogeneity Correction

To correct for endogeneity between frequency and ridership, we use the two-stage residual inclusion (2SRI) approach, also known as the control function method (see Wooldridge, 2010; and Terza *et al.*, 2018). Specifically, we estimate an auxiliary model for route-level frequency using linear regression, as below:

$$Freq_r = \delta_0 + \delta_1 Pkm_r + \boldsymbol{\varphi}'\boldsymbol{Y}_r + \eta_r \quad (3)$$

In the above equation, $Freq_r$ is the service frequency on route r, Pkm_r is the passenger-kilometers traveled on the route, and $\boldsymbol{Y}_r$ is a vector of other route-level variables that the transit agency might consider while setting the frequency of the route ($\boldsymbol{\varphi}$ is the corresponding coefficient vector). These variables include, for example, the total population living along one-km buffer of the route and the total employment within the buffer. η_r is the error term and is assumed to be normally distributed with zero mean and standard deviation υ.

Econometrically speaking, endogeneity arises because the error term (η_r) representing the unobserved factors influencing frequency and the error term (ε_{Sr}) representing the unobserved factors influencing ridership may be correlated due to common unobserved factors influencing both demand and supply. To reduce such correlation, we first predict the route-level frequency from the auxiliary regression model of Equation (3) as $\widehat{Freq}_r|(Pkm_r, \boldsymbol{Y}_r) = \hat{\delta}_0 + \hat{\delta}_1 Pkm_r + \hat{\boldsymbol{\varphi}}'\boldsymbol{Y}_r$. Subsequently, the residual from the prediction $\hat{\eta}_r = Freq_r - \widehat{Freq}_r|(Pkm_r, \boldsymbol{Y}_r)$ is included as an additional explanatory variable in the ridership model as shown in Equation (2).

An alternative approach to address endogeneity is to use predicted frequency $\widehat{Freq}_r$ as an explanatory variable in the place of the original frequency = ($Freq_r$) in the ridership model of Equation (2). This two-stage prediction inclusion (2SPI) approach is akin to the 2SLS approach commonly used for linear models. However, we prefer the 2SRI approach since it allows us to work with the original frequency variable (instead of the predicted frequency variable) to specify non-linear effects of frequency through $\boldsymbol{F}_r$ while also employing $\hat{\eta}_r$ to address endogeneity. Besides, it has been shown that the 2SRI approach results in consistent parameter estimates while the 2SPS approach might not (Terza.*et al.*, 2018).

Regardless of the method – 2SRI or 2SPI – used to address endogeneity, it is important that the auxiliary model for the suspected endogenous variable (frequency, in this case) includes at least one instrumental variable (IV) that satisfies two conditions (Labrecque and

Swanson, 2018).[3] First, the IV should be sufficiently correlated to the suspected endogenous variable (frequency). This condition, called the relevance condition, can be verified by examining how strongly the IV loads in the auxiliary regression for the endogenous variable. Second, the IV does not directly influence the outcome variable of interest (ridership), but only indirectly through its effect on the endogenous variable. This condition, called the exclusion restriction, cannot be verified but the analyst can use theoretical knowledge to justify that the condition is not violated (Labrecque and Swanson, 2018). In this study, we use the route-level passenger-kilometers (Pkm_r) as an IV to help correct for endogeneity between ridership and frequency. This is because anticipated passenger-kilometers on a route influences the frequency of buses set on that route (relevance condition). At the same time, passenger-kilometers on a route do not directly influence boardings at individual stops of that route.

4.3 Model Estimation

The ordinary least squares (OLS) method can be used to estimate the parameters of the auxiliary model for route-level frequency. However, the OLS method cannot be used for the ridership model estimation since the parameters associated with the *logsum* term are non-linearly related to the dependent variable. The resulting non-linear regression model may be estimated using the maximum likelihood estimation (MLE) technique. In this study, the likelihood function of the proposed model was coded in the Gauss mathematical modeling platform. To develop the empirical specification of the transit boardings model, several variable specifications and functional forms were explored; the final specification was built by removing the statistically insignificant variables (at a 95% confidence level) using a systematic process of statistical tests and intuitive considerations.

5 EMPIRICAL MODEL RESULTS

The parameter estimates of the ridership model and the frequency model are presented in Tables 1 and 2, respectively. The findings from each of these models are discussed next.

5.1 Empirical Findings from the Ridership (Demand) Model

5.1.1 Influence of stop-level trip generating variables (external factors)

The parameter estimates of the stop-level catchment area population and employment variables have a positive sign with high t-statistic values. These results are in line with an established finding that catchments with large population and employment tend to generate high transit ridership (Kain and Liu, 1999; Taylor *et al*. 2009; Cervero *et al.,* 2010; Dill *et al.,* 2013). Interestingly, the parameter estimates associated with land-use variables such as floor area allocated to commercial, industrial, and public service sectors have a negative sign. This is plausible because these land-use types are typically good predictors of trip attraction (alighting) than trip generation (boarding) in the AM peak period. As discussed in Rahman *et al.* (2019), such land-use variables are more likely to have a positive influence on the number of boardings

[3] Using an IV in the auxiliary regression model partitions the variation in the suspected endogenous variable ($Freq_r$) into two parts – one that is exogenous to the demand variable ($\widehat{Freq}_r$), and another that is correlated with the error term of the demand equation ($\hat{\eta}_r$). Therefore, using the residual $\hat{\eta}_r$ as an additional regressor in the demand equation helps in extracting out (from ε_{Sr}) the part that could be correlated with η_r. This approach helps in correcting for endogeneity and as a test for endogeneity via a t-test on the coefficient of $\hat{\eta}_r$ (Hausman, 1978).

during the evening periods. Another plausible reason is that once the total employment in the catchment area is controlled for, the floor area represents the amount of space available per person. It is likely that stops in locations with less dense areas (based on the floor area) generate fewer transit riders than those with denser areas.

Note that the dependent variable is at a coarser spatial aggregation than the stop-level trip generating variables specified here. Nevertheless, the model yields intuitive results and demonstrates the feasibility of relating fare-stage cluster-level boardings to stop-level demographic and land-use data. As discussed in Section 4, one can use a logit function to disaggregate the predicted fare-stage cluster level boardings from such a model to the stop level. In this study, since boarding data was not available at the stop level, we could not validate the accuracy of the predictions obtained from such disaggregation. However, as expected, the coefficient on the dummy variable identifying the fare-stage stop in the cluster suggests that such stops attract higher boarding than other stops in the fare-stage cluster, *ceteris paribus*.

5.1.2 Influence of network interactions within the BMTC network

The next set of variables in the model capture the influence of the network structure on fare-stage cluster-level ridership. Specifically, as hypothesized earlier, a high frequency of buses on competing routes going through the catchment reduce boardings on the subject route. This is because, at any fare-stage cluster of a subject route, as the service frequency of competing routes increase, some riders would use those routes to go to the same destinations offered by the subject route. On the other hand, a high frequency of buses on complementary routes going through the fare-stage cluster catchment area increase boardings on the subject route. This is because complementary routes bring riders who transfer to the subject route at the subject fare-stage cluster. In this context, as expected, the results suggest that fully complementary routes tend to bring more riders to the subject route (at the subject fare-stage cluster) than partly complementary routes. Further, the number of upstream overlaps between the subject route and other interacting routes have a negative influence on ridership in the subject fare-stage cluster. This is because overlapping stops upstream of the subject fare-stage cluster offer alternate boarding locations for passengers transferring to the subject route.

5.1.3 Influence of interactions with the Metro network

Similar to the inter-route relationships within the bus network, the interactions with the metro network influence bus transit boardings at the fare-stage cluster. Specifically, as can be observed from the parameter estimates, the presence of competing metro stations in the fare-stage cluster reduces bus ridership (due to cannibalization of the market share). On the other hand, presence of fully complementary metro stations in the fare-stage cluster catchment increases bus ridership. This is because sections of metro lines that share a complimentary alignment with the bus network feed transfer riders from the metro network to bus transit. However, the presence of partly complimentary metro stations did not show a significant effect on bus ridership in the fare-stage cluster. Most studies in bus transit literature do not consider the competing and complementing effects of the metro/rail network on bus transit ridership in such a detailed way. A few studies (e.g., Cervero et al., 2010; Rahman et al., 2019) use simple dummy variables for the presence of metro/rail stations. Mucci and Erhardt (2018), in addition to a dummy for a major terminal, use light-rail boardings and alightings in the bus ridership model.

TABLE 1 Estimation results of the transit ridership (boardings) model

Explanatory Variables	Parameter	t-stat
Constant	-2.9437	-10.60
Stop level variables $(\boldsymbol{Z}_{sr})$		
o ln (Population in stop-specific catchment/Voronoi polygon)	0.6960	8.05
o ln (Employment in stop-specific catchment/Voronoi polygon)	0.2517	6.96
o ln (Commercial floor area in stop specific Voronoi polygon)	-0.1388	-3.91
o ln (Industrial floor area in stop specific Voronoi polygon)	-0.0282	-1.86
o ln (Public service floor area in stop specific Voronoi polygon)	-0.3190	-7.12
o Dummy variable for the stage stop (first stop) in the fare-stage cluster	0.5628	3.58
Fare-stage cluster level variables $(\boldsymbol{X}_{Sr})$		
Network interactions within BMTC		
o Service frequency of fully and partly competing routes in the fare-stage cluster catchment	-0.0011	-2.66
o Service frequency of fully complementary routes in the cluster catchment	0.0090	3.88
o Service frequency of partly complementary routes in the cluster catchment	0.0003	4.83
o ln (Number of upstream overlaps that the subject route makes with interacting routes)	-0.0966	-7.87
Network interactions with Metro		
o Presence of fully competing metro stops in the fare-stage cluster	-0.2225	-3.03
o Presence of partly competing metro stops in the fare-stage cluster	-0.2582	-4.49
o Presence of fully complementary metro stops in the fare-stage cluster	0.2989	3.65
o Presence of partly complementary metro stops in the fare-stage cluster	--	--
Connectivity variables as surrogates for accessibility by transit		
o ln (No. of downstream stops accessed directly from the fare-stage cluster)	0.2862	11.38
o ln (No. of downstream stops accessible via one transfer from the subject route)	0.0240	1.96
o Number of downstream stops along the subject route from where one can transfer to a metro line to reach destinations different from those on the subject route	0.0091	2.76
Stop infrastructure and route service type		
o Any bus stop in the fare-stage cluster is *bus station*, *bus depot*, or *TTMC*	0.5365	7.48
o Bus route service type is *Ordinary*	0.6217	13.11
Route level variables $(\boldsymbol{F}_r)$		
o Service frequency of the subject route expressed as a piecewise linear (spline) function:		
Observed Frequency (1-3 buses per hour)	0.6905	29.25
Observed Frequency (3-6 buses per hour)	0.1310	5.10
Observed Frequency (above 6 buses per hour)	0.0657	3.86
o Residual $\hat{\eta}_r$ (difference of observed and estimated service frequencies)	-0.0497	-2.89
Coefficient on the *logsum* variable (θ)	0.6247	9.71
Standard error (σ) of ε_{Sr}	0.9615	76.63
Goodness-of-fit metrics		
Number of observations (fare-stage clusters)		2955
Number of parameters in the model		25
Log-Likelihood at convergence		-4077.00
Adjusted Rho Square with respect to a null model		0.4282

-- not statistically significant even at 90% confidence level and removed from the final specification

5.1.4 Influence of accessibility provided by the transit system

The next set of variables quantify accessibility (or downstream connectivity) provided by the transit system from the subject fare-stage cluster. As one would expect, fare-stage clusters with a greater number of directly accessible destinations are more likely to attract higher ridership, *ceteris paribus*. Specifically, for a 1% increase in the number of destinations directly accessible from the fare-stage cluster without making a transfer, the cluster-level boarding increases by 0.31%. At the same time, for a 1% increase in the number of destinations accessible from the fare-stage cluster by making one transfer, the cluster-level boarding increases only by 0.02%. As expected, although the additional connectivity via transfers can help increase ridership, the demand elasticity in this case is not as much as that of direct connectivity. This is presumably because of the increase in travel time and uncertainty associated with to transfers. Nevertheless, the potential benefits of additional accessibility due to transfer opportunities should not be ignored in designing transit routes and schedules because it is generally easier to increase the number of destinations accessible from a subject stop by increasing transfer opportunities than extending the subject route to connect to all those destinations directly. For example, just one more downstream stop at which a traveler can transfer to another route can potentially increase the number of accessible destinations substantially. The challenge, however, lies in enabling easy transfers with short transfer times and seamless transfer facilities.

The above-discussed benefits of additional downstream connectivity through transfers are relevant for transfers from the subject route to the metro network as well. As can be observed from the parameter estimates, fare-stage clusters on routes with greater downstream connectivity to Metro stations from where passengers can reach additional destinations have higher ridership (than other fare-stage clusters, *ceteris paribus*). Therefore, the proposed variable specification can be used to quantify the potential benefits of downstream connectivity to the metro network as well (in addition to the effect of origin-end interactions with the metro network in the catchment area of the cluster, as discussed in Section 5.1.3).

5.1.5 Influence of stop-level facilities and route service type

The coefficient of the dummy variable identifying if at least one stop in the fare-stage cluster is a major stop (*e.g.*, a bus station, depot, or TTMC) suggests that such stops attract higher boarding than other stops *ceteris paribus*. The difference in boardings between major stops and simple stops – even after including trip-generating variables, network interactions, connectivity, and service frequency effects – suggests that better stop-specific facilities and higher than usual network interactions contribute to greater ridership at the major stops.

Next, the parameter estimate of the dummy variable for the service type (of the subject route) indicates that ordinary service routes tend to have higher ridership than the Vajra (express) or Vayu Vajra (express AC) services. This is expected because the latter services are more expensive and serve a limited set of destinations. For example, the Vayu Vajra service is primarily for travelers to/from the airport.

5.1.6 Influence of service frequency

As highlighted in many empirical studies (Kyte *et al.*, 1988; Estupinan and Rodrıguez, 2008; Deb and Filippini, 2013; Chakour and Eluru, 2016; Mucci and Erhardt, 2018), service frequency has a strong influence on transit boardings. Further, as can be observed from the parameter estimates of the piecewise-linear variables of service frequency, the marginal effect

of increase in service frequency diminishes at high-frequency levels (Berrebi *et al.*, 2021). For two fare-stage clusters with different levels of current frequency (while all else is same), adding an extra service is expected to yield more additional boardings for the fare-stage cluster with current frequency in the 1 to 3 buses per hour category than that in higher frequency categories.

In addition to the non-linear effects of service frequency, we corrected for the endogeneity of observed service frequency using the residual ($\hat{\eta}_r$) from the frequency model as an additional regressor in the transit ridership model. As can be observed, the coefficient of the residual is statistically significant (at a 95% level of confidence), indicating the presence of endogeneity in the empirical dataset (Hausman, 1978). Without including this residual, the effect of the piecewise-linear frequency variables was stronger (the t-statistics and the corresponding parameter estimates were higher in magnitude than what is reported in the table). These observations highlight the need to address the endogeneity of service frequency.

5.2 Empirical Findings from the Frequency (Supply) Model

The parameter estimates of the auxiliary regression model for route-level frequency are reported in Table 2. As can be observed from this table, routes going through highly populated corridors have a higher frequency of buses (than other routes). Similarly, routes going through locations with high employment tend to have high service frequency. This is expected because transit agencies typically provide greater supply along corridors with a potential for higher transit trip generation. At the same time, after controlling for population and employment, routes going through corridors with large industrial activity or those with large commercial activity (as represented by industrial floor area and commercial floor area, respectively) tend to have less service frequency. While the influence of commercial floor area is not significant, the negative influence of industrial floor area is perhaps because not many people travel to industrial activity locations. The total passenger-kilometers on a route has a strong influence on service frequency. This is an expected result, as many transit agencies aim to provide high supply along routes that can serve a large passenger base and increase fare-box revenue (passenger-kilometers tends to be highly correlated with fare-box revenue). The strength of influence of the route-level passenger-kilometers variable on the frequency variable also indicates that it satisfies the relevance condition required for an IV. However, as discussed earlier, the assumption of the exogeneity condition (that passenger-kilometers do not directly affect ridership) is not verifiable but can only be justified using the analyst's judgment. In this context, while it is easy to see that increasing passenger boardings at a stop-route necessarily increases passenger-kilometers on the corresponding route, it is not necessary that increasing passenger-kilometers leads to an increase in the stop-route-level boardings (i.e., only the trip length of the current riders might increase, instead of attracting new boardings). Hence, it is reasonable to assume that route-level passenger-kilometers do not directly affect stop-level boardings. Besides, route-level passenger-kilometers is a variable commonly available with many transit agencies. Therefore, route-level passenger-kilometers serves effectively as an IV for correcting the endogeneity of frequency in a boardings-based transit demand model.[4]

[4] The route-level service frequency model was estimated on a total of 1254 routes, whereas the ridership model discussed earlier was estimated using data from only 436 routes. The use of additional data from 818 routes strengthens the case for correcting endogeneity using residuals obtained from such an empirical model.

TABLE 2 Estimation results of the route-level frequency model

Explanatory variables	Parameter	t-stat
Constant	0.0304	0.25
Total passenger kilometers (in thousands of km) along the route *(Instrumental variable)*	0.7368	191.26
ln (Population in route catchment) #	0.0554	3.15
ln (Employment in route catchment)	0.0267	1.80
ln (Industrial floor area in route catchment)	-0.0131	-2.36
ln (Commercial floor area in route catchment)	-0.0236	-1.91
Standard error (υ) of the error term (η_r)	0.5462	136.85
Goodness-of-fit metrics		
Number of observations (routes)		1254
Number of parameters		7
Adjusted R-square		0.7993

route catchment area is an aggregation of all stop-level Voronoi polygons on the route

5.3 Model Validation and Comparisons

To assess the predictive accuracy of the final empirical model reported in Table 1, a hold-out sample of 960 fare-stage clusters was used. The Root Mean Square Error (RMSE) of the model predictions (fare-stage cluster boardings) against observed boardings in the validation data is 8.15. The corresponding root mean square percent error (RMSPE)[5] is 78%. When compared to the RMSPE values reported in the literature (for example, an RMSPE of 409% reported for the route-stop model in Mucci and Erhardt, 2018), the proposed model appears to offer better accuracy. However, since our model predictions are at an aggregated fare-stage cluster level and Mucci and Erhardt's (2008) model predictions are at a stop-level, such comparisons may not be fair. Unfortunately, there are not many stop-route level models in the literature that report validation results and predictive assessments. Therefore, to assess the value of the substantive enhancements proposed in our transit ridership model, we compared the goodness-of-fit and predictive accuracy measures for different specifications in both the estimation and validation samples used for this study. Specifically, for each empirical specification tested in the study, we compared the log-likelihood value at convergence, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values. In addition, we compared the RMSE and RMSPE values for the validation dataset. Table 3 reports these values. As can be observed from the AIC and BIC values in the table, the proposed specification (Specification 1) provides the best fit among all the empirical specifications explored in this study. Further, while not shown in the table, the log-likelihood ratio (LLR) test between the final empirical specification and various other specifications that impose restrictions favors the proposed specification that accounts for endogeneity and non-linear effects of service frequency, network interactions, and accessibility variables. Removing any of these features resulted in a deteriorated fit in both estimation and validation samples. Including each of these features resulted in better predictive accuracy in both estimation and validation samples, as evidenced by the RMSE and RMSPE values. All these results highlight the benefits of the features included in the proposed model.

[5] $RMSPE = \sqrt{\frac{\sum_{i=1}^{N}\left(\left|\frac{\hat{y}_i - y_i}{y_i}\right| \times 100\right)^2}{N}}$, where $\hat{y}_i$ is the predicted boarding and y_i is the observed boarding.

TABLE 3 Goodness-of-fit and validation metrics of different model specifications

Model specification	No. of parameters	Goodness-of-fit in estimation sample (N=2955)			Goodness-of-fit in validation sample (N=960)				
		Log-Likelihood	AIC value	BIC value	Log-Likelihood	AIC value	BIC value	RMSE	RMSPE
Specification 1: Final specification accounting for endogeneity and non-linear effects of service frequency, accessibility, network interactions within BMTC and with Metro, and stop-level catchment characteristics	25	-4077.0	8203.9	8353.8	-567.7	1185.4	1307.2	8.15	*78%*
Specification 2 (without accessibility variables, all else is same as Specification 1)	22	-4149.8	8343.5	8475.3	-609.4	1262.9	1370.0	11.15	*141%*
Specification 3 (without network interaction variables, all else is same as Specification 1)	18	-4160.4	8356.8	8464.6	-660.1	1356.3	1443.9	9.59	*96%*
Specification 4 (without stop-level catchment characteristics, all else is same as Specification 1)	18	-4260.0	8556.0	8663.9	-674.0	1384.1	1471.7	13.18	*156%*
Specification 5 (without endogeneity and non-linear effects of service frequency)	22	-4337.1	8718.2	8850.0	-658.3	1360.7	1467.9	8.94	*85%*

Figure A.2 in Appendix A presents visualizations of observed and predicted boardings for the same validation sample. The top-left and top-right panels in the figure show the spatials patterns of observed and predicted boardings, respectively. The bottom-left panel shows spatial patterns in the prediction errors and the bottom-right panel shows a scatter plot between observed and predicted boardings. All these plots suggest that the proposed model is able to capture the spatial patterns of boardings observed in the validation data reasonably well.

5.4 Elasticities and Marginal Effects

We used the ridership model estimated in this study to compute elasticities of ridership with respect to the continuous explanatory variables. For discrete variables such as dummies representing network interactions, we computed marginal effects. For the service frequency variable, we computed the elasticity and the marginal effect of increasing frequency by 1. The elasticities and marginal effects are reported in Table 4. As can be observed from this table, and reported by other studies (for example, Mucci and Erhardt, 2018; Dill *et al.,* 2013), population residing in the catchment area is the most influential trip generating variable for boardings with an elasticity value of 0.436. The marginal effects of the dummy variables indicate that the presence of a competing metro stop nearby a fare-stage cluster reduces its bus ridership by 20.4%. On the other hand, the presence of a complementary metro stop nearby increases its ridership by 34.8%. In the context of service frequency, as discussed earlier, increasing the frequency by an additional bus per hour shows highest gains in ridership (83%) on routes with a current frequency of less than 3 buses per hour. This is equivalent to an elasticity of 0.77. High frequency routes, on the other hand, show a much lower gain in ridership (only 6.8%) from increasing service frequency. These trends as well as the elasticity values are in line with those reported in the literature (Kyte et al., 1988; Berrebi et al., 2021).

TABLE 4 Elasticities and marginal effects of bus ridership to explanatory variables

Explanatory Variables	Elasticity	Marginal effect
Stop level variables $(\boldsymbol{Z}_{sr})$		
ln (Population in stop-specific catchment/Voronoi polygon)	0.436	NA
ln (Employment in stop-specific catchment/Voronoi polygon)	0.157	NA
ln (Commercial floor area in stop specific Voronoi polygon)	-0.087	NA
ln (Industrial floor area in stop specific Voronoi polygon)	-0.018	NA
ln (Public service floor area in stop specific Voronoi polygon)	-0.199	NA
Fare-stage cluster level variables $(\boldsymbol{X}_{Sr})$		
Network interactions within BMTC		
Frequency of fully & partly competing routes in fare-stage cluster catchment	-0.021	NA
Service frequency of fully complementary routes in the cluster catchment	0.018	NA
Service frequency of partly complementary routes in the cluster catchment	0.049	NA
ln (No. of upstream overlaps that the subject route makes with interacting routes)	-0.096	NA
Network interactions with Metro		
Presence of fully or partly competing metro stops in the fare-stage cluster	NA	-20.4%
Presence of fully complementary metro stops in the fare-stage cluster	NA	34.8%
Connectivity variables as surrogates for accessibility by transit		
ln (No. of downstream stops accessed directly from the fare-stage cluster)	0.290	NA
ln (No. of downstream stops accessible via one transfer from the subject route)	0.024	NA
No. of downstream stops along the subject route from where one can transfer to a metro line to reach destinations different from those on the subject route	0.025	NA
Service frequency of the subject route $(\boldsymbol{F}_r)$		
Observed Frequency (1-3 buses per hour)	0.770	83.0%
Observed Frequency (3-6 buses per hour)	0.595	13.4%
Observed Frequency (above 6 buses per hour)	0.556	6.8%

Note: The table reports elasticities for continuous variables, marginal effects for dummy variables, and both elasticity and marginal effects for the frequency variable. NA: Not applicable.

6 CONCLUSIONS AND FUTURE WORK

This study is perhaps among the first to develop a comprehensive direct demand model for forecasting bus transit ridership in an Indian city. In doing so, the study addresses the following substantive and methodological issues associated with modeling transit demand: (a) spatial aggregation of demand data and relating spatially-aggregated demand to disaggregate, stop-level catchment characteristics, (b) the influence of inter-route interactions such as competition and complementarity for transit riders both within the bus network and with other transit

networks (such as Metro rail), (c) the influence of connectivity from the subject bus stop to the rest of the city via the bus and metro transit networks, (d) demand-supply endogeneity, and (e) non-linear impacts of service frequency on demand.

The proposed model is developed for a disaggregate stop-route level while considering boardings data that are typically aggregated at a fare-stage cluster (of stops) level. The model takes the form of a non-linear regression that accommodates a *logsum* variable that "collects" information from all stops belonging to a fare stage into a composite variable at the fare-stage cluster level. Doing so helps relate the fare-stage-level boardings to disaggregate, stop-level boardings (on a given route). Further, overlapping among disaggregate stop catchment areas is resolved by constructing Voronoi polygons of the stops using geospatial tools (as opposed to *ad hoc* subtraction of overlapping buffer areas).

A rich variety of explanatory variables are conceptualized and included in the empirical model. These include: (a) variables measuring the influence of inter-route relationships on bus ridership (from bus and metro network), (b) trip generating characteristics of the catchment area of a fare-stage cluster (specified at the disaggregate, stop-catchment level), (c) accessibility/connectivity of transit stops via the bus (and metro) transit network to rest of the study region with and without transfers, stop and service typology, and (d) the frequency along the route serving the subject cluster, while accommodating demand-supply endogeneity and non-linear effects of frequency on ridership.

The stop-route level disaggregation of the model makes it easy to accommodate endogeneity between route-level service frequency and stop-route-level ridership. Specifically, endogeneity is addressed using the two-stage residual inclusion (2SRI) approach, where the residual from a route-level frequency model is used as an explanatory variable – along with the observed frequency and other variables – in the stop-route-level ridership model. In this context, it is demonstrated that passenger-kilometers served along a route – a variable commonly available with transit agencies – serves effectively as an instrumental variable (IV) in the route-level frequency model. The 2SRI approach allows the analyst to accommodate endogeneity (through the residual variable), while the non-linear effect of service frequency on ridership is incorporated through a piecewise-linear specification of the observed frequency.

The empirical model offers various insights and features of interest to bus transit agencies. First, transit agencies with access to only fare-stage cluster-level ridership data can use the proposed model to investigate the influence of disaggregate, stop-level catchment area variables on transit ridership. The model can potentially be used to disaggregate the estimated demand from the fare-stage-cluster level to the stop level. Second, it is important for bus transit agencies to account for the effects of inter-route relationships (both within the bus and metro/rail networks) in designing the transit networks and schedules. In this context, the proposed model can be used by transit agencies to quantify the ridership impacts of changes in network structure and assist in identifying route alignments that can enhance ridership. Third, the empirical model allows one to quantify the benefits of connectivity/accessibility via the transit network in enhancing ridership. Fourth, the empirical model results indicate endogeneity between service frequency and ridership, ignoring which would lead to overestimation of service frequency enhancements on ridership. In this context, the route-level passenger kilometers variable serves effectively as an instrument variable for addressing endogeneity between route-level service frequency and stop-route-level ridership. Fifth, while service

frequency is one of the strongest determinants of transit ridership, even after accounting for endogeneity, the largest benefits of service enhancement in Bengaluru appear to come from routes with a current frequency level between 1 to 3 buses per hour. Estimation results show that routes with higher frequency also benefit from increased frequency, albeit the benefits diminish as the current frequency levels increase. In this context, the proposed model can be used to quantify the demand elasticities with respect to service frequency while accounting for current frequency levels and other factors (such as catchment area population and employment, inter-route relationships, and connectivity).

In summary, the empirical model developed in this study can be used both as a descriptive tool to understand the factors that influence ridership, and as a predictive tool to forecast ridership in response to changes in various factors that can be controlled by transit agencies (*e.g.*, service frequency and network structure) and exogenous factors not in control of transit agencies (*e.g.*, the spatial structure of the population, employment, and land-use). The model, once developed, is easy to apply to answer a variety of *what-if* scenarios of interest to transit agencies. Further, the proposed conceptual and methodological framework and the empirical strategies presented in this paper (including the treatment of inter-route relationships and spatial disaggregation) are general enough to be applicable to model transit ridership in other cities within India and other countries.

Some limitations of this study open avenues for further research in this area. First, the influence of transit fares is not included because the fare structure did not change within one month for which the empirical data was available. How to incorporate the effect of fare structure in direct demand models of transit ridership is an important research question. Second, the empirical ridership data (electronic ticketing machine data) used in this research does not include a non-negligible population of bus-pass riders. In this context, obtaining stop-level ridership data of ticket sales and pass riders (through bus-stop and on-board surveys) will be useful. Such data can also help validate the proposed approach of disaggregating fare-stage cluster-level boardings into stop-level boardings. Third, this study does not consider the effect of variability in service characteristics (*e.g.*, travel time variability or headway variability due to uncertainty in network travel times and dwell times), which might significantly influence transit ridership in congested cities like Bengaluru. All these are fruitful avenues for near-future research. Equally important is to use more recent data to understand how transit ridership (and the influence of various factors) has changed post the COVID pandemic.

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AUTHOR CONTRIBUTIONS

Study conception and design: Pinjari, A.R., Deepa, L., Srinivasan, K.K., Rambha, T.; Data curation: Deepa, L; Supervision of analysis: Pinjari, A.R.; Formal analysis, Validation, Investigation: Deepa, L., Pinjari, A.R., Nirmale, S., Srinivasan, K.K., Rambha, T.; Model estimation software: Nirmale, S., Deepa, L.; Writing-Original draft preparation: Deepa, L., Pinjari, A.R.; Writing-reviewing and editing: Rambha, T., Srinivasan, K.K., Nirmale, S.; All authors reviewed the results and approved the final version of the manuscript.

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APPENDIX A

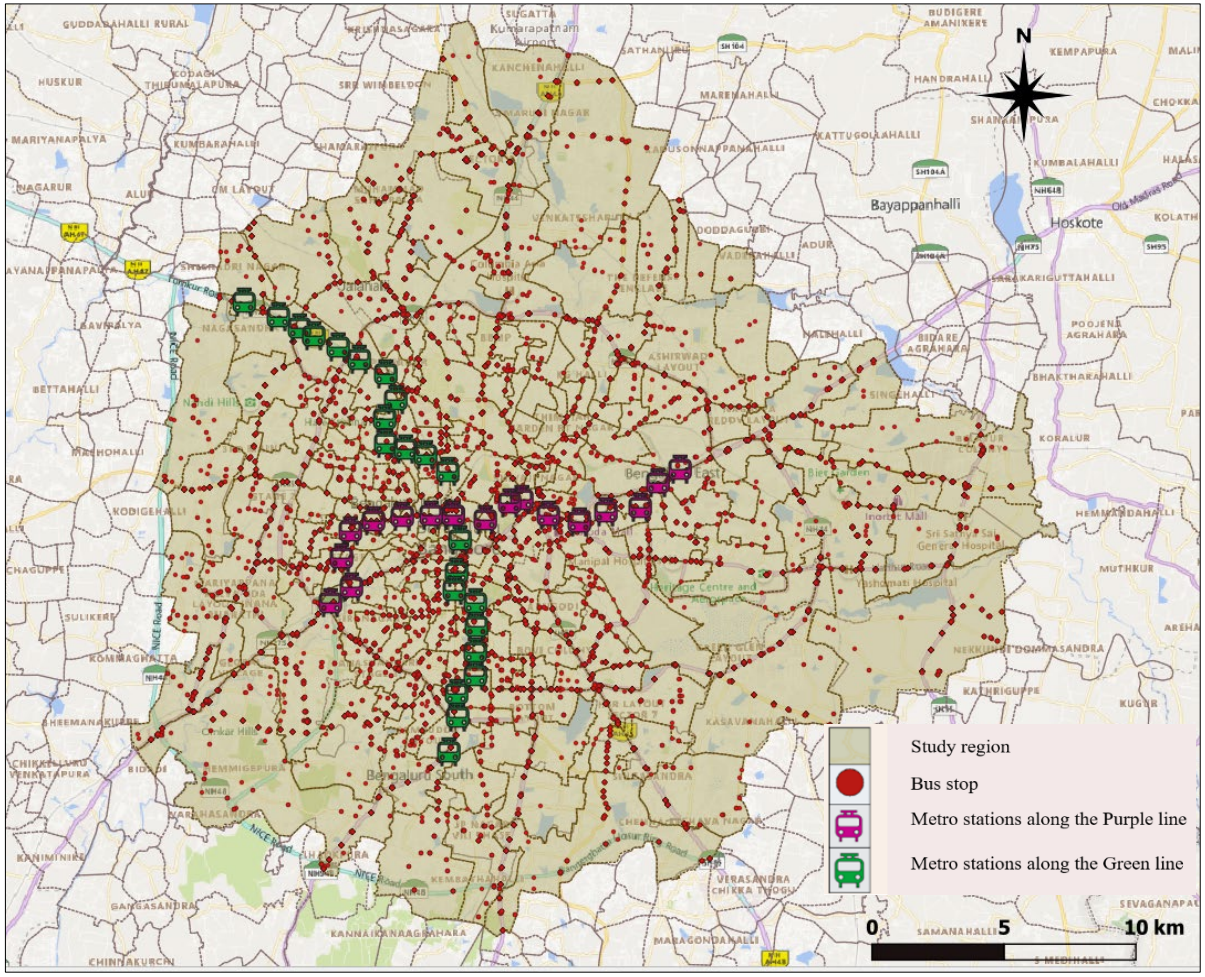

Fig. A.1 Study area: Bruhat Bengaluru Mahanagara Palike (BBMP) region of Bengaluru

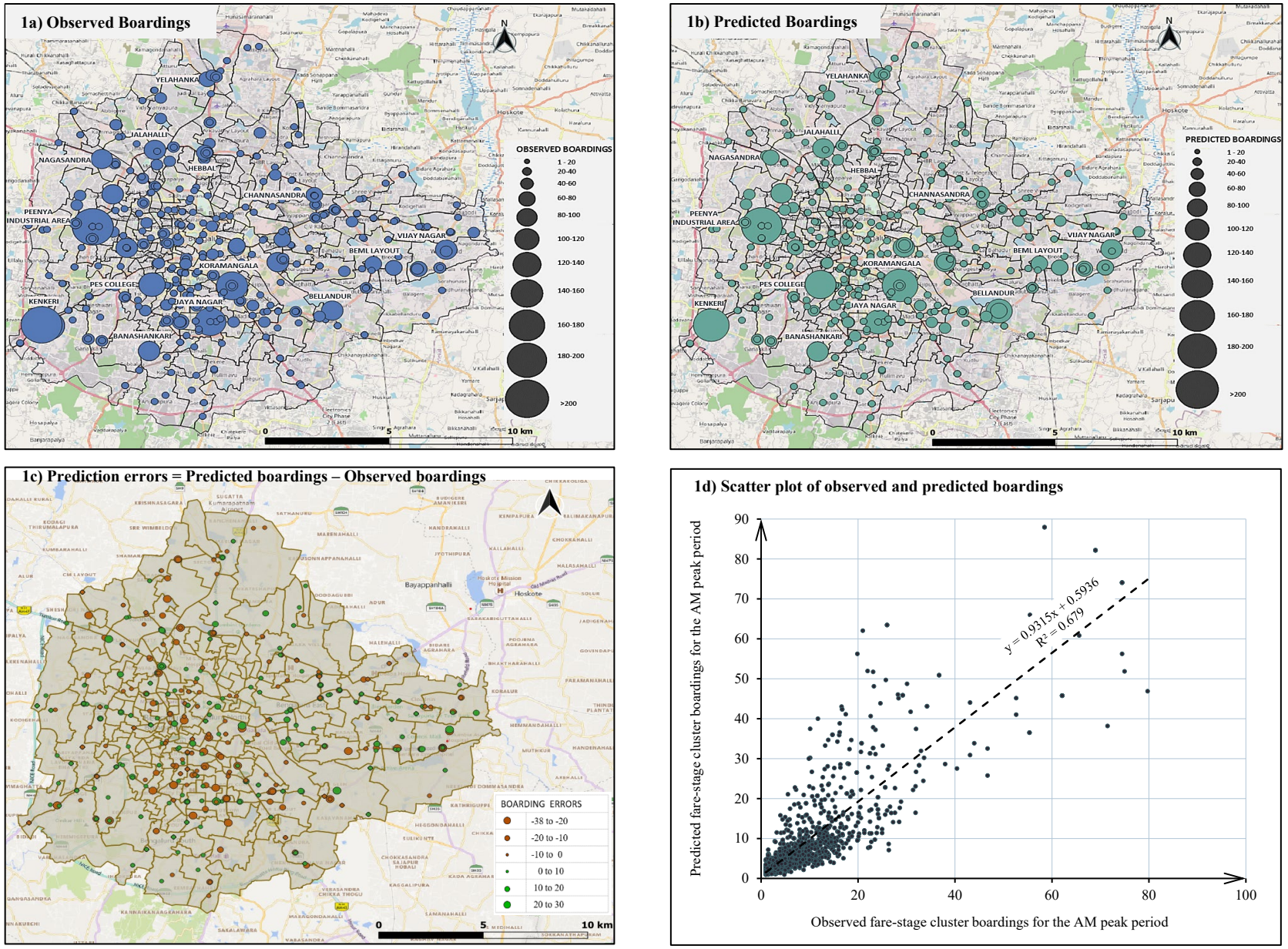

Fig. A.2 Validation results of the proposed fare-stage cluster boarding model

TABLE A.1 Summary of bus transit direct demand modeling studies in the literature

Author & Year	Study region	Modeling approach	Spatial unit of analysis	Temporal unit of analysis	Dependent variable	Demand-supply endogeneity	Non-linear effects of frequency	Inter-route relationships	Accessibility/ connectivity	Socio-demographics & Land use
Kyte *et al.* (1988)	Portland, (U.S.)	Time series analysis (Box & Jenkins approach)	System, sector, and route level	Monthly for system & Quarterly for sector, route (1971-1982)	Transit ridership	✓	✓	×	×	✓
Peng *et al.* (1997)	Portland, (U.S.)	Three stage lease squares regression (3SLS), 2SLS	Route-segment (by fare zones)	AM peak, PM peak, midday, evening, & night (inbound and outbound for 1989-1990)	Demand (boardings), Service supply, Demand on competing routes	✓	×	✓	×	✓
Kimpel *et al.* (2000)	Oregon, (U.S.)	Ordinary Least Squares Regression	Segments based on time-points	AM peak, PM peak, midday, evening, & night (4-29 Oct 1999)	Mean boardings, Mean scheduled headway, Headway delay variability	✓	×	✓	×	✓
Pendyala and Ubaka (2000)	Volusia, (U.S.)	3SLS	Route-segment (by time points)	Daily (1998)	Daily ridership, seat supply, Ridership on competing & complemntary routes	✓	×	✓	×	✓
Kikuchi and Miljkovic (2001)	Delaware, (U.S.)	Fuzzy inference and Regression	Stop level	Daily (1997, 1998, 1999)	Daily ridership	×	×	×	✓	✓
Thompson and Brown (2006)	82 MSAs in U.S.	Ordinary Least Squares Regression	Metropolitan Statistical Area	(1990 and 2000)	Passenger miles per capita	×	×	×	✓	✓
Chu *et al.* (2007)	Jacksonville, (U.S.)	Poisson regression	Stop level	AM peak, PM peak, off peak, & night period for weekdays & Saturday, Sunday	Direct and Transfer boarding	×	×	✓	✓	✓
Kimpel *et al.* (2007)	Oregon, (U.S.)	Ordinary Least Squares Regression	Stop level	Morning peak hour (7:30–8:30 AM)	Average passenger boardings	×	×	×	✓	✓
Estupinan and Rodrıguez (2008)	Bogota, (Columbia)	2SLS	Stop level	Annual (2005)	Transit ridership and Transit supply	✓	×	✓	✓	✓

Taylor *et al.* (2008)	265 Urbanized areas in U.S.	2SLS	Urbanized Area	Annual	Aggregate and per-capita transit ridership, total and per-capita vehicle service hours	✓	×	×	×	✓
Cervero *et al.* (2010)	Los Angeles, (U.S.)	Ordinary Least Squares and Hierarchical Linear regressions	Stop/ Station level	Daily average	Average daily boardings	×	×	✓	✓	✓
Pulugurtha and Agurla (2012)	North Carolina, (U.S.)	Negative binomial using spatial proximity and spatial weight method	Stop level	Daily average	Average daily boardings	×	×	×	✓	✓
Dill *et al.* (2013)	Oregon, (U.S.)	Ordinary Least Squares Regression	Stop level	Weekday average	Log transformed ridership	×	×	×	✓	✓
Alam *et al.* (2015)	273 MSAs in U.S.	Ordinary Least Squares Regression	Metropolitan Statistical Area	Annual	Passenger Miles per Capita	×	×	✓	✓	✓
Kerkman *et al.* (2015)	Arnhem–Nijmegen, (Netherlands)	Cross-sectional and fixed-effects regression Model	Stop level	Total weekday (March 2012 – March 2013	Log transformed ridership	✓	×	✓	✓	✓
Chakour and Eluru (2016)	Montreal, (Canada)	Ordered probit model using Composite Marginal Likelihood	Stop level	Time of day (AM Peak, PM Peak, Off-peak day, Off-peak Night)	Boarding, Alighting (low, medium, high)	×	×	×	✓	✓
Mucci and Erhardt (2018)	San Francisco, (U.S.)	Ordinary Least Squares Regression	Route-stop level	Daily Average	Log transformed average bus/ rail ridership	×	×	✓	✓	✓
Rahman *et al.* (2019) Rahman *et al.* (2021)	Orlando, (U.S.)	Simultaneous equations system using grouped response approach. Spatial panel model	Stop level	Average Weekday (May 2013 – Dec 2016)	Boarding, Alighting, Headway	✓	✓	×	✓	✓
Berrebi *et al.* (2021)	Portland, Miami, Minneapolis, Atlanta, (U.S.)	Poisson cross-section and fixed-effects model	Stop-cluster (by route and direction)	Weekday (2012-2018)	Boardings	✓	✓	×	✓	✓
Current study	Bengaluru, (India)	Non-linear regression (2SRI)	Stop-route level	Average weekday AM Peak (8:00–11:00 AM)	Log-transformed boardings	✓	✓	✓	✓	✓

TABLE A.2 Descriptive statistics of the variables used in the analysis

Continuous variables	**Mean**	**Std. Dev.**
ln (Fare-stage cluster level boardings in AM peak period, 8–11 AM)	2.35	1.41
Service frequency of subject route (buses/hour)	2.44	2.89
Difference of observed and predicted frequency obtained from first-stage regression	0.06	1.15
Service frequency of fully competing stops (max value across stops in cluster)	12.32	35.35
Service frequency of partly competing stops (max value across stops in cluster)	7.05	8.24
Service frequency of fully complementary stops (sum across stops in cluster)	87.21	184.86
Service frequency of partly complementary stops (max value across stops in cluster)	164.53	315.08
ln (No. of upstream overlaps subject route makes with interacting routes)	5.82	1.68
ln (No. downstream stops directly accessible on subject route from fare-stage cluster)	2.96	0.82
ln (No. of downstream stops accessible by making one transfer)	7.10	1.97
Number of downstream stops along the subject route from where one can transfer to a metro line to reach destinations different from those on the subject route	3.43	0.46
ln (Population in stop-specific catchment/Voronoi polygon)	9.69	0.96
ln (Employment in stop-specific catchment/Voronoi polygon)	8.69	1.49
ln (Commercial floor area in stop specific Voronoi polygon)	11.66	1.48
ln (Industrial floor area in stop specific Voronoi polygon)	9.43	3.06
ln (Public services floor area in stop specific Voronoi polygon)	10.52	1.52
Dummy variables	**Count**	**Percentage**
Dummy variable for stage stop (first stop) in the fare-stage cluster	2988	30.47
Presence of fully competing metro stops in fare-stage cluster	486	16.45
Presence of partly competing metro stops in fare-stage cluster	1081	36.58
Presence of fully complementary metro stops in fare-stage cluster	801	27.11
Presence of partly complementary metro stops in fare-stage cluster	310	10.49
None of the stops in fare-stage cluster interact with the metro lines	277	9.37
Bus service type is Ordinary	2589	87.61
Any bus stop in the cluster is a bus station, bus depot, or TTMC	267	9.04