The Benefits of Allowing Heteroscedastic Stochastic Distributions in Multiple Discrete-Continuous Choice Models

Sujan Sikder, Ph.D.
Department of Civil & Environmental Engineering
University of South Florida
4202 E. Fowler Ave., Tampa, FL 33620
Tel: (813) 507-3953; Fax: (813) 974-2957, Email: ssikder@mail.usf.edu

Abdul Rawoof Pinjari *(corresponding author)*
Assistant Professor
Department of Civil & Environmental Engineering
University of South Florida
4202 E. Fowler Ave., ENB 118, Tampa, FL 33620
Tel: (813) 974-9671; Fax: (813) 974-2957, Email: apinjari@usf.edu

September 2013

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ABSTRACT

This paper investigates the benefits of incorporating heteroscedastic stochastic distributions in random utility maximization-based multiple discrete-continuous (MDC) choice models. To this end, first, a Multiple Discrete-Continuous Heteroscedastic Extreme Value (MDCHEV) model is formulated to allow heteroscedastic extreme value stochastic distributions in MDC models. Next, an empirical analysis of individuals' daily time use choices is carried out using data from the National Household Travel Survey (NHTS) for three geographical regions in Florida. A variety of different MDC model structures are estimated: (a) the Multiple Discrete-Continuous Extreme Value (MDCEV) model with independent and identically distributed (IID) extreme value error structure, (b) the MDCHEV model, (c) the mixed-MDCEV model that allows heteroscedasticity by mixing a heteroscedastic distribution over an IID extreme value kernel, (d) the MDC generalized extreme value (MDCGEV) model that allows inter-alternative correlations using the multivariate extreme value error structure, (e) the mixed-MDCEV model that allows inter-alternative correlations using common mixing distributions across choice alternatives, and (f) the mixed-MDCEV model that allows both heteroscedasticity and inter-alternative correlations. Among all these model structures, the MDCHEV model provided the best fit to the current empirical data. Further, heteroscedasticity was prominent while no significant inter-alternative correlations were found. Specifically, the MDCHEV parameter estimates revealed the significant presence of heteroscedasticity in the random utility components of different activity type choice alternatives. On the other hand, the MDCEV model resulted in inferior model fit and systematic discrepancies between the observed and predicted distributions of time allocations, which can be traced to the thick right tail of the type-1 extreme value distribution. The MDCHEV model addressed these issues to a considerable extent by allowing tighter stochastic distributions for certain choice alternatives, thanks to its accommodation of heteroscedasticity among random utility components. Further, spatial transferability assessments using different transferability metrics also suggest that the MDCHEV model clearly outperformed the MDCEV model.

Keywords: discrete-continuous choice models, multiple discreteness, heteroscedasticity, distributional assumptions, time use behavior, spatial transferability

1 INTRODUCTION

1.1 Background

Numerous consumer choices are characterized by “multiple discreteness” where consumers can potentially choose multiple alternatives from a set of discrete alternatives available to them. Along with such discrete-choice decisions of which alternative(s) to choose, consumers typically make continuous-quantity decisions on how much of each chosen alternative to consume. Such multiple discrete-continuous (MDC) choices are being increasingly recognized and analyzed in a variety of scientific fields, including transportation, environmental economics, and marketing. Examples include: (1) individuals’ daily time-use choices, which involve decisions to engage in different types of activities in a day along with the allocation of available time to each activity, (2) households’ recreational destination choices and time allocation to the chosen destinations over a season, and (3) grocery shoppers’ brand choice and purchase quantity decisions.

A variety of approaches have been used in the literature to model MDC choices. Among these, a particularly attractive approach is based on the classical microeconomic consumer theory of constrained utility maximization. Specifically, consumers are assumed to optimize a direct utility function $U(\mathbf{x})$ over a set of non-negative consumption quantities $\mathbf{x} = (x_1,...,x_k,...,x_K)$ subject to a linear budget constraint, as:

$$\textit{Max } U(\mathbf{x}) \text{ such that } \mathbf{x}\mathbf{p} = y \text{ and } x_k \geq 0 \; \forall k = 1,2,...,K \tag{1}$$

In the above Equation, $U(\mathbf{x})$ is a quasi-concave, increasing and continuously differentiable utility function with respect to the consumption quantity vector $\mathbf{x}$, $\mathbf{p}$ is a vector of unit prices for all goods, and y is a budget for total expenditure. An increasingly popular approach for deriving the demand functions from the utility maximization problem in Equation (1), due to Hanemann (1978) and Wales and Woodland (1983), is based on the application of Karush-Kuhn-Tucker (KKT) conditions of optimality with respect to the consumption quantities. Since the utility function is assumed to be randomly distributed over the population, the KKT conditions are also randomly distributed and form the basis for deriving the probability expressions for consumption patterns.

Over the past decade, the above-discussed KKT approach has received significant attention for the analysis of MDC choices. A stream of research in environmental economics (Phaneuf et al., 2000; von Haefen et al., 2004; von Haefen and Phaneuf, 2005; Phaneuf and Smith, 2005; Vasquez-Lavin and Hanemann, 2009) advanced the approach to model individuals’ recreational demand choices for non-market valuation of environmental goods. Several studies in marketing research (Kim et al., 2002; Satomura et al., 2011) employed the approach to model situations when consumers purchase a variety of brands of a product (e.g., yogurt). In the transportation field, the multiple discrete-continuous extreme value (MDCEV) model formulated by Bhat (2005) and enlightened further by Bhat (2008) lead to an increased use of the KKT approach for analyzing a variety of choices, including individuals’ activity participation and time-use (Bhat 2005; Habib and Miller, 2008; Pinjari et al., 2009; Chikaraishi et al., 2010;

You et al., 2013), household vehicle ownership and usage (Ahn et al., 2008; Bhat et al., 2009; Jaggi et al., 2013), long-distance leisure destination choices (Van Nostrand et al., 2013), energy consumption choices (Pinjari and Bhat, 2011; Frontuto, 2011; Yu et al., 2012) and builders' land-development choices (Kaza et al., 2012; Farooq et al., 2013). Clever use of stochastic specifications has led to model formulations with closed-form likelihood expressions. Specifically, consider an additive utility form, as below:

$$U(x_1,...,x_K)=\sum_{k=1}^{K}U(x_k)=\sum_{k=1}^{K} f\left(u(x_k),\varepsilon_k\right) \qquad (2)$$

In the above equation, $U(x_k)$ is a random sub-utility function for good *k*, representing the utility derived from consuming x_k amount of good *k*, and expressed as a combination of a deterministic component $u(x_k)$ and a random component ε_k as: $U(x_k)= f\left(u(x_k),\varepsilon_k\right)$. Assuming that the random components (ε_k) enter the sub-utility functions $U(x_k)$ in a multiplicative fashion, as $U(x_k)=u(x_k)\times e^{\varepsilon_k}$, and are type-1 extreme value and independent and identically distributed (IID) across the choice alternatives leads to a very simple and elegant consumption probability expression (Bhat, 2005) making it easy for parameter estimation. In addition, computationally efficient procedures are now available for using these model systems for forecasting and policy evaluation (see von Haefen et al., 2004; and Pinjari and Bhat, 2011). Thanks to these advances, KKT-based MDC models are being increasingly used in empirical research and have begun to be used in operational travel forecasting models.

1.2 Gaps in Literature Relevant to this Paper

Recent research in this area has started to enhance the basic formulation in Equation (1) along three specific directions: (a) toward more flexible, non-additively separable functional forms for the utility specification so as to accommodate rich complementarity and substitution patterns in consumption (Vasquez-Lavin and Hanemann, 2009; Bhat et al., 2013a), (b) toward greater flexibility in the specification of the constraints faced by the consumer, such as multiple linear budget constraints as opposed to a single constraint (Satomura et al., 2011; Castro et al., 2012; Pinjari and Sivaraman, 2013), and (c) toward more flexible stochastic specifications for the random utility functions. The reader is referred to Pinjari et al. (2013) for a more detailed discussion of recent advances along the first two directions.

Within the context of stochastic specifications in KKT models, recent work has been geared toward relaxing the IID assumption of random utility components, in the following ways: (1) the specification of multivariate extreme value (MEV) distributions as opposed to IID extreme value distributions, which leads to the multiple discrete-continuous generalized extreme value (MDCGEV) structure (Pinjari and Bhat, 2010; Pinjari, 2011); (2) the specification of multivariate normal (MVN) distribution, which leads to the multiple discrete-continuous probit (MDCP) structure (Kim et al., 2002; Bhat et al., 2013b), and (3) the specification of additional error components mixed over an IID extreme value distributed kernel, which leads to the mixed-MDCEV structure (Bhat, 2005; Bhat and Sen, 2006; Spissu et al., 2009; Chikaraishi et al., 2010).

A number of the efforts to relax the IID assumption have been in the context of relaxing the independently distributed assumption, by allowing correlations between the random utility components of different choice alternatives (either by employing the MEV or MVN distributions or by employing the mixed-MDCEV structure). Such correlations help in accommodating flexible substitution patterns between the consumptions of different choice alternatives (Pinjari and Bhat, 2010). In addition, positive correlations between alternatives can capture the possibility that the consumptions of groups of alternatives can be complementary to each other in that an increase in the consumption of one alternative increases the consumption of other alternatives.

A handful of studies relax the identically distributed assumption by allowing for inter-alternative variations in the stochastic distributions (i.e., heteroscedastic distributions). Specifically, Bhat and Sen (2006) use the mixed-MDCEV structure to accommodate heteroscedasticity across choice alternatives. Spissu et al. (2009) also use the mixed-MDCEV structure, albeit with panel data, to accommodate variations in the influence of unobserved factors within and across individuals (i.e., inter-individual and intra-individual variations). In another notable study, Chikaraishi et al. (2010) use the mixed-MDCEV structure to accommodate variations in unobserved influences at multiple levels, including intra-individual variation, inter-individual variation, inter-household variation, and temporal and spatial variation. Both Spissu et al. (2009) and Chikaraishi et al. (2010) allow for the unobserved variations to be different across the different choice alternatives, thereby allowing for heteroscedasticity across choice alternatives. In all these studies, the primary reason for accommodating heteroscedasticity across choice alternatives is to recognize the differences in the variation of unobserved influences on the preferences for different choice alternatives. As often cited in the literature, doing so helps in improving the model fit to the data as well as accommodates the influence of heteroscedastic random variance on the elasticity effects of alternative attributes. For instance, the self-price elasticity estimate of a choice alternative is dampened by the variance in its random utility component (Bhat and Sen, 2006). However, what has been unknown (and unexplored) so far is the potential influence of heteroscedasticity across choice alternatives on the distributions of the consumptions implied by a KKT demand system such as the MDCEV model. It is this specific aspect that the current paper contributes to.

Another line of recent research has been on evaluating the predictions obtained from KKT-based MDC models. A first step in this direction is the study by Jaggi et al. (2013) who analyzed the residuals between observed consumptions in the estimation data and predicted consumptions (on the same data) using the MDCEV model. In a recent study, Sikder and Pinjari (2013) compared the predictions from the MDCEV model with those observed in the estimation data. Their empirical analysis reveals both strengths and weakness of the MDCEV formulation in predicting the aggregate-level discrete choices and continuous consumption quantities in the context of individuals’ daily time-use decisions. Specifically, they reported that the MDCEV model performs very well in predicting the aggregate-level discrete choices observed in the estimation data (i.e., the market shares for each choice alternative). However, the continuous consumption quantities (i.e., daily time allocations) were reported to have been overestimated for certain alternatives and underestimated for other alternatives, when compared to the average consumptions in the estimation data. Alluding to the possibility that this problem could be attributed to the *fat right tail* of the extreme value distributions assumed in the MDCEV model, Sikder and Pinjari (2013) identify a research need to explore alternative distributional assumptions to overcome the issue. In the current paper, we explore the benefits of incorporating heteroscedasticity in the stochastic distributions across different choice alternatives in addressing the afore-mentioned prediction related issues of the MDCEV model.

Both the mixed-MDCEV and MDCP approaches can be used to relax the assumption of identically distributed errors. The advantage of both these approaches is that they are very general; the analyst can allow for correlated and non-identically (or heteroscedastically) distributed random utility terms simultaneously, while also allowing random coefficients on explanatory variables and recognizing correlations across observations. However, both the approaches have their own drawbacks. The likelihood function of the mixed-MDCEV formulation for allowing heteroscedasticity is a multidimensional integral of as many dimensions as the number of heteroscedastic choice alternatives. The dimensionality of the integral increases further when other features such as inter-alternative correlations and random coefficients are incorporated (along with heteroscedasticity) using the mixed-MDCEV approach. This integral cannot be evaluated analytically and necessitates the use of computationally intensive simulation techniques as the dimensionality of integration increases beyond a modest number. The typically used approach to estimating the mixed-MDCEV models is the maximum simulated likelihood (MSL) method, where the likelihood is simulated using pseudo-Monte Carlo or quasi-Monte Carlo simulation. The desirable asymptotic properties of the MSL estimator come with a computational cost, because the number of simulation draws ought to rise faster than the square root of the number of observations in the estimation sample. As widely noted in the literature (Train, 2009), the accuracy of such simulation techniques degrades quickly at high dimensions of integration unless a large number of simulation draws are used. Even if using a large number of draws is not impossible, it certainly increases the model estimation time thereby discouraging the analyst from exploring a variety of alternative empirical specifications. Another important issue that has not received due attention in the literature (but see Bhat, 2011) is the accuracy of the covariance matrix of the MSL estimator (not just the accuracy of the simulator itself), which is important for good statistical inference. As stated in Bhat (2011), simulating the log-likelihood function with even three to four decimal places of accuracy in the probabilities might not be sufficient for an accurate estimation of the covariance matrix of the MSL simulator. The implication is that the likelihood function needs to be simulated with a very high level of accuracy and precision (Bhat, 2011), which further increases the computational intensity. Finally, another drawback with the simulation methods is that parameter (un)identification issues arise quickly when the number of random coefficients to be estimated increases beyond a modest number. Suffice it to say that while the mixed-MDCEV approach is valuable for simultaneously allowing a variety of features such as inter-alternative correlations and heteroscedasticity, it is worth incorporating these features using techniques that help reduce the dimensionality of integration in the likelihood function. For example, the MDCGEV structure (Pinjari, 2011) can be used to allow inter-alternative correlations while retaining the closed-form of the likelihood expressions. Along similar lines, it would be useful to explore a simpler approach to incorporating heteroscedasticity in MDC models. One possibility of doing so, as discussed in the next section, is to use heteroscedastic extreme value (HEV) distributions in lieu of the homoscedastic extreme value distributions used in the MDCEV model.

As mentioned earlier, the MDCP approach can also be used to allow heteroscedasticity. The MDCP approach, similar to the mixed-MDCEV approach, leads to multi-dimensional integrals (in the likelihood function) that are not analytically tractable. An advantage of this approach is that the dimensionality of integration is always equal to one less than the number of choice alternatives, regardless of the number of random parameters and the presence of heteroscedasticity and inter-alternative correlations. This property is advantageous for situations with small to medium number of choice alternatives. The MDCP models are typically estimated using the GHK simulator to evaluate the integral appearing in the likelihood function (see Kim et al., 2002). However, in addition to being computationally intensive, the GHK simulator is not easy to implement, a reason why most analysts resort to the mixed-MDCEV approach. More recently, Bhat et al. (2013b) demonstrated the benefits of an analytic approximation method to evaluate the integrals in the MDCP structure. This development might make it simpler to use MDCP models in empirical research.

1.3 Current Research

In view of the above discussed gaps in the literature, the primary objective of this paper is to investigate the benefits of incorporating heteroscedastic stochastic distributions in KKT-based MDC choice models. To this end, we first formulate a Multiple Discrete-Continuous Heteroscedastic Extreme Value (MDCHEV) model that employs heteroscedastic extreme value (HEV) distributed random utility components in KKT-based MDC models. The HEV distribution was originally used by Bhat (1995) for modeling single discrete choice situations. One advantage of the proposed MDCHEV approach over the other two approaches (i.e., mixed-MDCEV and MDCP) is that the resulting likelihood function is a uni-dimensional integral that can be easily and accurately evaluated using quadrature methods; a reason why Bhat (1995) used it. Note, however, that the MDCHEV structure does not accommodate either inter-alternative correlations or random coefficients on explanatory variables. As discussed earlier, the mixed-MDCEV and the MDCP approaches are more general than the MDCHEV as they can simultaneously accommodate heteroscedasticity, inter-alternative correlations, and random coefficients. In practice, however, it can be very difficult to empirically identify a large number of random parameters needed to accommodate all these different features in the mixed-MDCEV model, especially with cross-sectional datasets. To overcome this issue, one can potentially formulate a mixed-MDCHEV model that superimposes a mixing distribution over a HEV kernel to accommodate inter-alternative correlations and random coefficients along with heteroscedasticity. Alternatively, one can conceive of a mixed-MDCHGEV model that is based on a heteroscedastic generalized extreme value (HGEV) kernel to allow both heteroscedasticity and inter-alternative correlations, and superimposes a mixing distribution over the HGEV kernel to allow random coefficients. However, these extensions as well as the MDCP approach are beyond the scope of this paper.[1]

In addition to the formulation of the MDCHEV model, an empirical analysis is presented in the context of modeling individuals' daily time-use choices using data from the 2009 National Household Travel Survey (NHTS) from Florida. The empirical analysis proceeds in two stages. In the first stage, a series of model estimations are carried out to select the best fitting MDC model structure for the current empirical data. Specifically, the following six different model structures are estimated: (1) The MDCEV model with IID extreme value stochastic distributions, (2) The mixed-MDCEV model to accommodate heteroscedasticity across choice alternatives, (3) The MDCHEV model, (4) The mixed MDCEV model to accommodate inter-alternative correlations, (5) The MDCGEV model to incorporate inter-alternative correlations using closed-form probability expressions, and (6) The mixed-MDCEV model to accommodate both inter-alternative correlations and heteroscedasticity. As will be revealed in the empirical results section, in the current empirical context, inter-alternative correlations were not significant but heteroscedasticity was prominent. Further, among all the above six model structures, the MDCHEV structure provided the best fit to the data, with goodness-of-fit measures far better than the mixed-MDCEV structure. In the second stage, we focus on assessing the benefits of incorporating heteroscedastic stochastic distributions in the context of modeling individuals' daily time-use choices. Considering the findings in the first stage of the empirical analysis, we compare the prediction performance of the MDCEV and MDCHEV model structures. In these comparisons, we first demonstrate that the distributions of the MDCEV-predicted continuous quantity decisions for certain choice alternatives can potentially have longer right tails than the observed distributions; implying overestimation of the continuous quantity predictions for those choice alternatives. We discuss how this problem is related to the *fat right tail* of the IID extreme value distributions assumed in the MDCEV model. We also demonstrate empirically that allowing for heteroscedasticity through the MDCHEV model helps in addressing this problem to a considerable extent. This is because the heteroscedastic model results in smaller variances (hence tighter distributions) for the random utility components of the choice alternatives for which the MDCEV model over-predicts the continuous quantity choices. Such tightly distributed random utility components, as will be demonstrated later in the paper, reduce the probability of unreasonably large continuous quantity predictions. In addition to comparing the in-sample prediction performance of the MDCEV and MDCHEV models, we also compare the out-of-sample prediction

[1] In this context, we identify the following potentially fruitful avenues for future research: (1) Exploration of the pros and cons of alternative approaches (e.g., mixed-MDCEV, MDCP, and mixed-MDCHGEV) to simultaneously accommodate inter-alternative correlations, heteroscedasticity, random coefficients, and (2) Assessment of the relative importance of each of these unobserved effects in different empirical contexts.

performance by assessing the spatial transferability of the models among different geographical regions in Florida.

The remainder of the paper is organized as follows. The next section presents the structure of the MDCHEV model and outlines the estimation procedure. Section 3 overviews the empirical data and geographical contexts considered for the empirical analysis. Section 4 presents the empirical results. Section 5 summarizes and draws conclusions from the paper.

2 THE MDCHEV MODEL

2.1 Model Formulation

Consider the following random utility function proposed by Bhat (2008) for modeling multiple discrete-continuous choice situations:

$$U(\mathbf{t}) = \psi_1 \ln t_1 + \sum_{k=2}^{K} \left\{ \psi_k \gamma_k \ln\left(\left(t_k / \gamma_k \right) + 1 \right) \right\} \tag{3}$$

In the above function, $U(\mathbf{t})$ is the total utility derived by an individual from his/her daily time-use. It is the sum of sub-utilities derived from allocating time (t_k) to each of the activity types *k* (*k =1, 2,…,K*). Individuals are assumed to make their activity participation and time-use decisions such that they maximize $U(\mathbf{t})$ subject to a linear budget constraint $\sum_k t_k = T$, where *T* is the total available time budget. Note that the subscript for the individual is suppressed for simplicity in notation.

Within the utility function in Equation (3), ψ_k, called the baseline marginal utility for alternative *k*, is the marginal utility of time allocation to activity *k* at the point of zero time allocation. ψ_k governs the discrete choice decisions in that an activity type with greater baseline marginal utility is more likely to be chosen than other activities. γ_k accommodates corner solutions (i.e., the possibility of not choosing an alternative). Both ψ_k and γ_k accommodate differential satiation effects (diminishing marginal utility with increasing consumption) for different activity types. Thus, both these parameters influence the time allocation decisions. Specifically, a greater value of either ψ_k or γ_k implies a larger allocation of time to the corresponding activity. Note that the 1st alternative, designated as in-home activity, does not have a γ_k parameter since all individuals in the data allocate some time to the in-home activity (i.e., there is no need of corner solutions for this activity). From now on, this alternative will be called the *outside good*, while all other activities (out-of-home activities) are called *inside goods*.[2]

[2] The outside good is a composite good that represents all goods other than the *K-1 inside* goods of interest to the analyst. The presence of the outside good helps in ensuring that the budget constraint is binding. Besides, the outside good helps in endogenously determining the total resource allocation for (or total consumption of) inside goods. It is not uncommon to treat the outside good as a numeraire with unit price, assuming that the prices and characteristics of the goods grouped into the outside category do not influence the choice and resource allocation among the inside goods (see Deaton and Muelbauer 1980). While the current empirical context is such that the

The influence of observed and unobserved individual characteristics and activity-travel environment (ATE) measures are accommodated into the utility function as $\psi_1 = \exp(\varepsilon_1)$; $\psi_k = \exp(\beta' z_k + \varepsilon_k)$; and $\gamma_k = \exp(\theta' w_k)$; where, z_k and w_k are observed socio-demographic and ATE measures influencing the choice of and time allocation to activity *k*, β and θ are corresponding parameter vectors, and ε_k (*k=1,2,…,K*) is the random error term capturing unobserved and unmeasured influences on the utility contribution of time allocation in activity type *k*. Note that ψ_1 does not include any observed explanatory variables as the coefficients of all explanatory variables for this alternative are normalized to zero for identification purposes.

To obtain the optimal time allocations ($t_1^*, t_2^*, ..., t_K^*$), one can form the Lagrangian and derive the Karush-Kuhn-Tucker (KKT) conditions of optimality (Bhat 2008). The Lagrangian function for the utility function and budget constraint considered in this study is

$$\mathbf{L} = \psi_1 \ln t_1 + \sum_{k=2}^{K} \left\{ \psi_k \gamma_k \ln\left(\left(t_k / \gamma_k \right) + 1 \right) \right\} - \lambda \left[\sum_{k=1}^{K} t_k - T \right] \tag{4}$$

where λ is the Lagrangian multiplier associated with the budget constraint. The KKT conditions of optimality are:

$$V_k + \varepsilon_k = V_1 + \varepsilon_1 \text{ if } t_k^* > 0, (k = 2,3,....,K)$$
$$V_k + \varepsilon_k < V_1 + \varepsilon_1 \text{ if } t_k^* = 0, (k = 2,3,....,K) \tag{5}$$
$$\text{where, } V_1 = \ln\left(t_1^*\right), \text{ and } V_k = \beta' z_k + \ln\left(\left(t_k^* / \gamma_k \right) + 1 \right), (k = 2,3......,K)$$

The above stochastic KKT conditions form the basis for the derivation of likelihood expressions. In the general case, if the joint probability density function of the ε_k terms is $g(\varepsilon_1, \varepsilon_2, ..., \varepsilon_k)$, and if M alternatives are chosen out of the available K alternatives, and if the consumptions of these M alternatives are $(t_1^*, t_2^*, t_3^*, ..., t_M^*)$, as given in Bhat (2008), the joint probability expression for this consumption pattern is as follows:

$$P(t_1^*, t_2^*, t_3^*, ..., t_M^* \, 0, 0, 0..., 0) =$$
$$|J| \int_{\varepsilon_1=-\infty}^{+\infty} \int_{\varepsilon_{M+1}=-\infty}^{V_1 - V_{M+1} + \varepsilon_1} .. \int_{\varepsilon_{k-1}=-\infty}^{V_1 - V_{k-1} + \varepsilon_1} \int_{\varepsilon_k=-\infty}^{V_1 - V_k + \varepsilon_1} g(\varepsilon_1, V_1 - V_2 + \varepsilon_1, V_1 - V_3 + \varepsilon_1, ..., V_1 - V_M + \varepsilon_1, \varepsilon_{M+1}, \varepsilon_{M+2}, ..., \varepsilon_{K-1}, \varepsilon_K)$$
$$d\varepsilon_k d\varepsilon_{k-1} ... d\varepsilon_{M+2} d\varepsilon_{M+1} d\varepsilon_1 \tag{6}$$

outside good is an essential good (where all individuals consume some amount of it), it is not always necessary for the outside good to be specified as an essential good.

where $|J|$ is the determinant of a Jacobian whose elements are given by (see Bhat, 2005)

$$J_{ih} = \frac{\partial\left[V_1 - V_{i+1} + \varepsilon_1\right]}{\partial t^*_{h+1}} = \frac{\partial\left[V_1 - V_{i+1}\right]}{\partial t^*_{h+1}}; i, h = 1, 2, ..., M-1 \tag{7}$$

For the MDCHEV model, we assume that the random components in the baseline marginal utilities of different choice alternatives are independent but heteroscedastically extreme value (HEV) distributed. Specifically, the random error term ε_k of each alternative k $(k = 1, 2, 3, ..., K)$ is assumed to have a type-1 extreme value distribution with a location parameter equal to zero and a scale parameter equal to σ_k. With the HEV distribution, the probability expression in Equation (6) becomes:

$$P(t_1^*, t_2^*, ..., t_M^*, 0, ..., 0) = |J| \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{\left(\prod_{j=2}^{M} \frac{1}{\sigma_j} g\left[\frac{V_1 - V_j + \varepsilon_1}{\sigma_j}\right]\right)\right\} \times \left\{\prod_{s=M+1}^{K} G\left[\frac{V_1 - V_s + \varepsilon_1}{\sigma_s}\right]\right\} \times g\left(\frac{\varepsilon_1}{\sigma_1}\right) d\left(\frac{\varepsilon_1}{\sigma_1}\right) \tag{8}$$

where g(.) and G(.) are the probability density function and cumulative distribution function, respectively, of the standard type I extreme value distribution; Specifically, $g(w) = e^{-w} e^{-e^{-w}}$ and $G(w) = e^{-e^{-w}}$. If the scale parameters σ_k across all alternatives are assumed to be equal, then the above expression simplifies to the closed-from MDCEV model derived by Bhat (2005).

2.2 Model Estimation

The parameters of the MDCHEV model can be estimated using the familiar maximum likelihood procedure. However, there is no analytical form for the integral appearing in the probability expression of Equation (8), which enters the likelihood function. In this paper, we employ the Laguerre Gaussian Quadrature (Press et al., 1986) to compute the integral. To do so, define $w = \frac{\varepsilon_1}{\sigma_1}$ and $u = e^{-w}$. These two equations can be combined to write $\varepsilon_1 = -\sigma_1 \ln u$. Further, $\frac{du}{dw} = \frac{d}{dw}(e^{-w}) = -e^{-w}$ or $dw = -e^{w} du$. Now, the term $g\left(\frac{\varepsilon_1}{\sigma_1}\right) d\left(\frac{\varepsilon_1}{\sigma_1}\right)$ in Equation (8) can be written as $g(w)dw$, which can further be expanded as $e^{-w} e^{-e^{-w}} dw$. Substituting $-e^{w} du$ for dw and u for e^{-w}, one can write $g\left(\frac{\varepsilon_1}{\sigma_1}\right) d\left(\frac{\varepsilon_1}{\sigma_1}\right) = -e^{-u} du$. Finally, substituting $-\sigma_1 \ln u$ for ε_1 and $-e^{-u} du$ for $g\left(\frac{\varepsilon_1}{\sigma_1}\right) d\left(\frac{\varepsilon_1}{\sigma_1}\right)$, the probability expression in Equation (8) can be re-written as follows:

$$P(t_1^*, t_2^*, t_3^*, ..., t_M^* 0, 0, ..., 0) = |J| \int_{u=0}^{\infty} f(u) e^{-u} du \quad (9)$$

where $f(u) = \left\{ \left(\prod_{j=2}^{M} \frac{1}{\sigma_j} g\left[\frac{V_1 - V_j - \sigma_1 \ln u}{\sigma_j} \right] \right) \right\} \times \left\{ \prod_{s=M+1}^{K} G\left[\frac{V_1 - V_s - \sigma_1 \ln u}{\sigma_s} \right] \right\}$

According to the Laguerre Gaussian Quadrature technique, the integral of the form in Equation (9) can be approximated as a summation of terms over a certain number (*I*) of support points as follows:

$$\int_{u=0}^{\infty} f(u) e^{-u} du \approx \sum_{i=1}^{I} w_i f(u_i) \quad (10)$$

where, i is the support point at which the function $f(u_i)$ is evaluated (support points are the roots of the Laguerre polynomial of order *I*) and w_i is the weight or probability mass associated with support point i (see Press et al., 1986). Since the integral being evaluated is uni-dimensional, the quadrature method is computationally efficient and accurate. In this paper, preliminary tests suggested that increasing the number of support points (*I*) beyond 15 did not increase the accuracy of the integral or influence the final model results (log-likelihood function value and parameter estimates). Therefore, all estimations of MDCHEV models were performed using 15 support points to evaluate the integral in the likelihood function. The likelihood function was coded in the maximum likelihood estimation module of the GAUSS matrix programming language.

Note that, since there is no variation in the prices of unit consumption of different activity alternatives in the current empirical context, for identification purposes, at least one of the scale parameters need to be fixed to an arbitrary value (Bhat, 2008). It is convenient to fix the scale parameter of the essential outside good (in-home activity) to 1. Therefore, the interpretation of all other scale parameters would be in reference to that of the outside good. Specifically, a σ_k value less (greater) than 1 implies that the unobserved variation in utility derived from time investment in activity type *k* is smaller (larger) than that in the in-home activity.

3 DATA

The primary data source used for the empirical analysis is the 2009 National Household Travel Survey (NHTS) for the state of Florida. The survey collected detailed information on all out-of-home travel undertaken by the respondents in a day, including the purpose, mode of travel, start and end time, and the dwell time (i.e., time spent) at the destination of all trips made in the day. This information was used to define eight out-of-home (OH) activity categories: (1) Shopping, (2) Other maintenance (buy services), (3) Social/Recreational (visit friends/relatives, go out/hang out, visit historical sites, museums and parks), (4) Active recreation (working out in gym, exercise, and playing sports), (5) Medical, (6) Eat out (such as meal, coffee, and ice cream) (7) Pickup/drop-off, and (8) Other activities. For each individual, the daily time-allocation to each of these activity categories was derived by aggregating the dwell time of each trip made for that activity purpose. The time spent in in-home (IH) activities was computed as total time in a day (24 hours) minus the time allocated to the above out-of-home activities, sleep, and travel. Based on the information from the 2010 American Time Use Survey (ATUS) for Florida, an average amount of 8.7 hours was assumed for sleep. For each individual in the data, the time spent in in-home activities and in all out-of-home activities together forms the available time budget (*T*) for subsequent analysis.

The demographic segment of focus in this study is unemployed adults (age >18 years) with survey information on weekdays. Further, the current empirical analysis focused on the following three geographical regions in Florida: (1) Southeast Florida (SEF), (2) Central Florida (CF), and (3) Tampa Bay (TB). It is worth noting here that this was the same dataset used in a previous study by Sikder and Pinjari (2013).[3] Therefore, only the patterns of relevance to this paper are quickly summarized here. The observed participation rates in different out-of-home activities are: 49.2% for shopping, 30.4% for other maintenance, 29.2% for social/recreational, 20.5% for active recreation, 23.3% for medical, 25.3% for eat out, 15.6% for pick-up/drop-off and 6.0% for other activities. Note that all the percentages add up to more than 100 because several individuals participated in multiple activities over a day. The average daily time allocations to each of the activities are (averaged among those who participated in the activities): 55 minutes for shopping, 50 minutes for other maintenance, 124 minutes for social/recreational, 48 minutes for active recreation, 60 minutes for medical, 49 minutes for eat out, 15 minutes for pick-up/drop-off, and 21 minutes for other activities. Overall, the activity participation rates and time allocation patterns were found to be reasonable for the most part. For example, among all the out-of-home activities considered in this study, the highest activity participation rate was observed for shopping followed by other maintenance, social/recreational and so on. Further, time allocation to social/recreational activities was observed to be larger than that to other activities while that to pickup/drop-off activities was smaller. However, it is worth noting one anomaly that was observed in the context of daily time allocation to active recreational activities. According to the data, a large proportion (more than 30%) of those who participated in active recreation appear to have done so for only 2 minutes or less in a day. Given the activities considered in this category (e.g., exercising, working out in gym, or playing sports), there is a high chance that such unreasonably small activity durations for a large proportion of the sample is a result of measurement error; presumably due to misreporting by the respondents or errors in coding of the data[4]. Such measurement errors can potentially have bearing on the estimated variance of the random error term for the active recreation activity.

[3] Sikder and Pinjari (2013) used this dataset to assess the spatial transferability of a time-use model with an MDCEV structure. On the other hand, the current study uses the dataset to assess the extent to which the MDCHEV helps resolve the prediction-related issues associated with the MDCEV model.

[4] To be sure, we considered the possibility of activities of very short duration such as walking around the house. Such a trip would begin and end at the same location. But the NHTS collected information on only those trips that were made to a different address. Also, the auto travel mode was used to arrive at many of these activities, suggesting that these activities are not likely to be short strolls.

4 EMPIRICAL RESULTS

4.1 Model Structure

Six different model structures were estimated on the above-described activity generation and time-use data from each of the three geographic regions considered in this study: (1) The MDCEV model with IID extreme value stochastic distributions, (2) The mixed-MDCEV model to accommodate heteroscedasticity, (3) The MDCHEV model[5], (4) The mixed MDCEV model to accommodate inter-alternative correlations, (5) The MDCGEV model, and (6) The mixed-MDCEV model to accommodate both inter-alternative correlations and heteroscedasticity. The mixed-MDCEV models were estimated using 200 quasi-Monte Carlo random draws (specifically, Halton draws) to adequately cover the space of the mixing distributions. Normal distribution and triangular distribution were explored for the mixing distributions. Normal distribution provided a better data fit in all cases. The goodness-of-fit measures for all the six model structures on the data from the South East Florida (SEF) region is presented in Table 1. These include log-likelihood values as well as the Bayesian Information Criterion (BIC) measures on the estimation data. One can make several observations from these results and the parameters estimates[6] from all the six models, as discussed next.

First, the MDCHEV model (model #3 in the table) provides the best fit to the data both in terms of log-likelihood and the Bayesian Information Criterion (BIC). Further, between the mixed-MDCEV and the MDCHEV approaches to incorporate heteroscedasticity, the latter approach provides a far better fit to the data. This suggests that the MDCHEV approach performs better than the mixed-MDCEV approach for capturing heteroscedasticity in the current empirical context. Of course, this doesn't necessarily imply that the MDCHEV approach would always be better than the mixed-MDCEV approach for introducing heteroscedastic stochastic distributions in MDC Models. However, the MDCHEV is a convenient approach since it helps reduce the dimensionality of integration in the log-likelihood function. In the above mentioned model estimations, the mixed-MDCEV models took 2 to 3 hours to estimate, even after coding the analytical gradients of the simulated log-likelihood function and providing starting values for the parameters (obtained from the MDCEV models). On the other hand, the MDCHEV models took 15 to 30 minutes to estimate on the same machine, without having to code the gradients of the log-likelihood function.

Second, the mixed-MDCEV model for inter-alternative correlations (model #4 in the table) does not provide significant improvement in model fit over the MDCEV model (model # 1 in the table). A variety of different specifications of inter-alternative correlations were explored, but none turned out to be

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[5] In addition, the MDCEV model was estimated using the MDCHEV likelihood expression in Equation (10) and different number of support points (e.g., 5, 10, 15 and 20) but fixing all scale parameters to 1. While not reported in the tables, for 15 support points, the resulting parameter estimates, standard errors, and log-likelihood values were all very close to those from the MDCEV model estimated using Bhat's closed-form likelihood expression. This, on one hand, demonstrates the accuracy of the Laguerre Gaussian Quadrature technique used for estimating the MDCHEV model, and on the other hand, indicates the number of reasonable support points required to estimate the MDCHEV model with the current empirical data.

[6] For brevity, detailed estimation results for all the models estimated in the study are not reported in the paper, but are available from the authors.

statistically significant (i.e., the standard deviations of the random coefficients of the common error components were not significantly different from zero). The MDCGEV model (model #5 in table) yielded a marginally better log-likelihood over the MDCEV model. This is due to correlations between the random utility components of social/recreational and eat out activities (detailed estimation results are available from the authors). However, the corresponding nesting parameter was not statistically different from 1 at a 95% confidence level, suggesting weak correlation between the two random utility components. These results suggest that inter-alternative correlations are relatively less prominent compared to heteroscedasticity in the current empirical context. This same finding is echoed by the mixed-MDCEV model that captures both heteroscedasticity and inter-alternative correlations (model #6), which does not show significant improvement in log-likelihood over the mixed-MDCEV model that captures only heteroscedasticity (model #2).

4.2 Model Estimation Results

Table 2 presents the parameter estimates from the MDCEV and MDCHEV models of activity generation and time-use for each of the three geographic regions considered in this study. The parameter estimates of the mixed-MDCEV models are neither reported not discussed as the model had an inferior model fit compared to the MDCHEV model. Similarly the parameter estimates of the MDCGEV models are not reported as inter-alternative correlations are not a focus of this paper (besides, as reported earlier, no significant correlations were found).

4.2.1 Scale Parameters

The scale parameter estimates are reported first in the table. As discussed earlier, the MDCEV model restricts all the scale parameters for all activities as equal to 1. On the other hand, the MDCHEV model allows the scale parameters to be different across different activities while normalizing the scale of in-home activity to 1. In the current empirical context, the MDCHEV estimates of scale for all out-of-home activities except active recreation and "other" activities are significantly smaller than 1, while that for active recreation is greater than 1 and that for "other" activity is not different from (therefore fixed to) 1.[7] Similar patterns can be observed from the parameter estimates for all three geographical regions. Plausible reasons for these patterns in the scale parameter estimates are discussed next.

As discussed in many references on choice modelling (e.g., Ben-Akiva and Lerman, 1985, Koppelman and Bhat, 2006), the random error terms ε_k represent a sum of errors (made by the analyst) in characterizing the consumers' utility functions. Commonly cited sources of errors include omitted

[7] Further, based on statistical tests, the scale parameters of several pairs of alternatives have been constrained to be equal for the sake of parsimony in model specification. Specifically, shopping and medical activities share a common scale parameter, other maintenance and pickup/drop-off activities share a common scale parameter, and social/recreational and eat-out activities share a common scale parameter, while active-recreation has its own unique scale parameter. The scale parameters of the in-home and other activities have been fixed to 1. Therefore, only 4 unique scale parameters were estimated for each geographical region.

alternative attributes and decision-maker characteristics, measurement errors in the explanatory variables included in the utility functions, and errors in the functional form of the utility function. In the current context, we attribute the specific patterns observed in the scale parameter estimates to the following three major sources of unobserved variation. First, recall from Section 3 that each activity category (i.e., choice alternatives) used in the model specification is an aggregation of many finely categorized activity types. The influence of explanatory variables included in the utility function of an aggregate activity category can potentially vary by each disaggregate activity type in that category. Such variation resulting from aggregation of choice alternatives is unobservable and manifests in the form of additional variance of random error terms (Daly, 1982). Among the nine activity categories considered in the current empirical context, the in-home activity is an aggregation of a wider variety of finer activities when compared to out-of-home activities. Recall that the in-home activity category combines all activities other than out-of-home activities into a *composite outside good*. This is one reason why the stochastic component of in-home activity has greater variance compared to most out-of-home activity categories. Second, note from Table 2 that the utility specifications for all activities except the in-home and "other" activity categories include explanatory variables. While the in-home activity category was treated as a reference alternative in the specification for identification purposes, no explanatory variable turned out to be significant in the utility function for the "other" activity category; presumably due to the arbitrary nature of the "other" activity category. Besides, similar to the in-home activity category, the "other" activity category combines all out-of-home activities other than those of interest into a single composite category. Thus, the final empirical specification of the deterministic utility components views in-home and "other" activities as similar (except the alternative-specific constant for "other" activity). This is perhaps a reason why the scale parameter for the "other" activity is not different from the in-home activity. Third, in the context of discrete-continuous choice modelling, measurement errors in the continuous dependent variables can potentially be significant. This is unlike traditional discrete choice models, where there might not be significant errors in dependent variables (because it is easier to elicit information on the discrete choice decisions made by the consumers than to measure the continuous quantity decisions). In the current empirical application, recall from Section 3 that time allocation to the active recreational activity might be associated with substantial measurement errors leading to greater unobservable variation. This may be a reason why the estimated scale parameter for the active recreational activity is greater than 1.

In summary, the MDCHEV model estimates reveal the presence of substantial heteroscedasticity in the random utility components of choice alternatives and point to different sources of unobservable variation.

4.2.2 Baseline Utility and Satiation Parameters

All the parameter estimates in baseline utility and satiation functions have intuitive interpretations and identical signs in both the MDCEV and MDCHEV models for all three regions. The substantive interpretations are not a focus of this paper. Therefore only the influence of incorporating heteroscedasticity on parameter estimates is discussed. Specifically, for all out-of-home activities, except active recreation, the magnitude of baseline utility parameter estimates in the MDCHEV model is slightly smaller than that in the MDCEV model. For active recreation, however, the baseline utility parameter estimates from the MDCHEV model are of greater magnitude than those from MDCEV. This pattern can be attributed to the differences in scale parameters between the MDCEV and MDCHEV models. Specifically, the baseline parameter estimates in the MDCEV model are confounded with the unknown scale parameters (which are simply assumed to be equal to 1). But the MDCHEV model helps in disentangling the baseline parameter estimates from the scale difference between the out-of-home and in-home activities. As a result, all activities with smaller (greater) scale parameters in the MDCHEV model than those in the MDCEV model have smaller (larger) magnitudes for baseline parameter estimates in the former model.

In the context of satiation functions, the parameter estimates of MDCHEV model are greater (in magnitude) for all out-of-home activities that have a tighter distribution of the random utility component (i.e., smaller scale parameter) than that in the MDCEV model. For active recreation activity, the satiation function parameter estimates of the MDCHEV model are smaller in magnitude than those from the MDCEV model.

Since the true parameter values are unknown, it is difficult to assert which model provides better/less-biased parameter estimates. However, note from the log-likelihood measures for all three geographical regions (last two rows of the table) that the MDCHEV model yields a significantly better fit to the estimation data than the MDCEV model. For example, the likelihood ratio test statistic between the two models for the South East Florida region is 385.12, which is larger than the chi-squared statistic with four degrees of freedom at any reasonable level of significance. This suggests that ignoring heteroscedasticity (i.e., estimating an MDCEV model) can potentially lead to biased parameter estimates in both baseline marginal utility and satiation functions and inferior model-fit.

4.3 In-Sample Prediction Performance

All prediction exercises in this paper were performed using the forecasting algorithm proposed by Pinjari and Bhat (2011). In this subsection, we first provide a brief discussion of this forecasting algorithm and then compare the in-sample prediction performance of the MDCEV and MDCHEV models.

Given the observed characteristics of an individual (e.g., z_k), the available time budget, the estimated parameters, and the simulated error draws (ε_k), the forecasting algorithm first identifies the number of chosen alternatives, and then computes the optimal time allocation to each of the chosen alternatives. In the first step, the price-normalized baseline utility values (ψ_k / p_k) are computed for all choice alternatives. Next, the alternatives are sorted in the descending order of their price-normalized baseline utility values, with the outside good in the first place in this sorted arrangement. Subsequently, the number of chosen alternatives is determined. This begins with an assumption that only the first alternative in the above sorted arrangement is chosen. Based on this assumption, an estimate of the Lagrange multiplier (λ) of the utility maximization problem is computed. To check if the next alternative (with the next highest price-normalized baseline utility) is also chosen, the Lagrange multiplier estimated in the previous step is compared with the price-normalized baseline utility of the alternative. If the estimated Lagrange multiplier is greater than the price-normalized baseline utility of the next alternative, the optimal time allocations to the previously assumed chosen alternatives are calculated and the algorithm stops. If not, the next alternative is also added to the set of chosen alternatives, and a new estimate of the Lagrange multiplier is computed and compared with the next highest price-normalized baseline utility. This procedure is repeated until the exact number of chosen alternatives is determined. Once the number of chosen alternative is determined, the optimal time allocations are computed using price-normalized baseline utility and satiation parameters of the chosen alternatives and the available time budget.

In this paper, for all prediction exercises, the above-described forecasting procedure was conducted for 100 sets of quasi-Monte Carlo random draws (specifically Halton draws) for each individual in the data to adequately cover the distributions of the random error terms. The only difference between the MDCEV and the MDCHEV forecasting procedures is that the simulated error draws come from the IID extreme value distribution for the MDCEV model while they come from the heteroscedastic extreme value (HEV) distribution for the MDCHEV model.

Using the above-described forecasting procedure, first the predicted shares of individuals participating in each activity type (i.e., the discrete choice component) were computed. Table 3 presents these aggregate shares for both MDCEV and MDCHEV models for all three geographical regions. The predicted aggregate shares for each activity were computed as the proportion of instances the activity was predicted with a positive time allocation across all 100 sets of random draws for all individuals. For each prediction result presented, the corresponding observed values in the estimation sample are presented in the parentheses. As can be observed from the table, both the MDCEV and MDCHEV models perform well in predicting the aggregate shares of participation in each activity type.

To evaluate the model predictions of time allocations to each activity (i.e., the continuous choice component), we compared the distributions of the predicted time allocations (for only those predicted with positive time allocation) with the distributions of observed time allocations (again, for only those observed with positive time allocation). Such distributions are presented in the form of box-plots in Figure 1 (for South East Florida region only). There are 9 sub-figures in Figure 1, one for each activity type. In each sub-figure, the distributions of predicted activity durations from both MDCEV and MDCHEV models are presented as box-plots along with the distributions of observed activity durations. Several interesting observations can be made from these box-plots. First, in the context of in-home activities, the predicted distributions from both the MDCEV and MDCHEV models show larger left tails than the observed distribution. However, the discrepancy between predicted and observed distributions is much greater for the MDCEV model than for the MDCHEV model. This suggests a greater chance of under-prediction of in-home activity durations by the MDCEV model. Second, for all out-of-home activities other than active recreation, the distributions of activity durations predicted with the MDCEV model show a significant chance of over-prediction. For active-recreation, the MDCEV model shows under-prediction of activity durations when compared to the observed data. Third, the MDCHEV model rectifies all these issues to a considerable extent. As can be observed, the predicted distributions of the MDCHEV model are closer to the observed distributions than those of the MDCEV model for almost all activities.

The differences in the distributional assumptions between the MDCEV and MDCHEV models explain the above differences in performance between the two models. The MDCEV model assumes unit scale parameter for all activity categories. For all activities for which the "true" scale parameter is smaller than the assumed value, the MDCEV model shows significant over-prediction of activity durations. These include all out-of-home activities other than active recreation. This is due to the the asymmetry and the *fat right tail* of the standard Gumbel distribution used in its structure. For instance, the probability of drawing any less than -2 from a standard Gumbel distribution is very low (0.06%), while that of drawing greater than 2 is high (12.65%). Since the Gumbel terms enter the model in an exponentiated multiplicative fashion (i.e., $\psi_k = \exp(\beta' z_k) \times \exp(\varepsilon_k)$), there is a non-negligible chance that the ψ_k values become quite large and therefore lead to unrealistically large time allocations for several out-of-home activities. Whenever an out-of-home activity hogs up a large amount of available time budget, it leaves a very small amount of time for the in-home activity (hence the under-prediction of time allocation for the in-home activity). Therefore, employing a larger value (than what it is) for the scale parameter of an activity implies a fatter right tail for the random utility component, which in turn implies a fatter right tail (than what it should be) for the distribution of the predicted consumptions/durations. Similarly, a smaller value of the scale parameter assumed in the MDCEV model for active recreation (than what is revealed in the MDCHEV model) leads to under-estimation of the time allocated to active recreational activities.[8]

The MDCHEV model overcomes the above-discussed problems by allowing the scale parameters to be different from each other. Recall that the MDCHEV scale parameter estimates are smaller than 1 for all out-of-home activities except active recreation and "other" categories. This implies tighter distributions of the ψ_k values and therefore a smaller chance of over-prediction of time allocation for those activities. For active recreation, the estimated scale parameter in the MDCHEV model is greater than 1. This implies a more spread-out distribution of the corresponding ψ_k value than that in the MDCEV model, and hence a smaller chance of under-estimation.

In summary, the in-sample prediction exercises suggest that both the MDCEV and MDCHEV models perform similarly in predicting the aggregate discrete-choice shares for each activity type. However, the MDCHEV model performs far better than the MDCEV model in predicting the time allocation to different activities. Note, however, that the MDCHEV-predicted durations are still not very close to the observed durations. In this context, exploring the influence of alternative distributional assumptions to

[8] The under-estimation is with respect to the observed values, assuming that the observed values are free of errors.

extreme value distributions – including right-truncated extreme value distributions, multivariate normal distributions, and multivariate skew-normal distributions – on the prediction properties of MDC models is a useful avenue for further research.

4.4 Transferability Assessments

This section examines the influence of incorporating heteroscedasticity on out-of-sample prediction by comparing the transferability of MDCEV and MDCHEV models among different geographical regions in Florida. Specifically, both the models estimated for each of the three geographical regions (SEF, CF, and TB) were transferred to the other two regions. Two different types of transferability metrics were used to assess model transferability: (1) log-likelihood based measures, and (2) measures of aggregate-level predictive ability. The results obtained from these metrics are discussed next.

In all transferability assessments, the geographical context from which a model is transferred is called the "*estimation context*" and the geography to which a model is transferred is called the "*application context*". For the application context, a model estimated using data from the same geography is called the "*locally estimated model*" and a model transferred from a different geography is called the "*transferred model*".

4.4.1 Log-Likelihood Based Measures of Transferability

Table 4 presents the log-likelihood values of the transferred and locally estimated MDCEV and MDCHEV models for each of the 12 model transfers conducted in this study. One can observe that, for model transfers between any two regions (i.e., in any row of the table), the predictive log-likelihood of the transferred MDCHEV model (column 5) is better than that of the transferred MDCEV model (column 3), suggesting that an MDCHEV model transfers better than an MDCEV model. What is more interesting is that the log-likelihood of all transferred MDCHEV models (column 5) are better than that of the corresponding locally estimated MDCEV models (column 4). This highlights the importance of incorporating heteroscedasticity in MDC models.

To quantify how much better is the transferability of an MDCHEV model over that of an MDCEV model, we computed Transferability Index (TI) values as suggested in Koppelman and Wilmot (1982). TI measures the degree to which the log-likelihood of a transferred model exceeds that of a reference model relative to a locally estimated model in the application context.

$$TI_j(\beta_i) = \frac{L_j(\beta_i) - L_j(\beta_{reference,j})}{L_j(\beta_j) - L_j(\beta_{reference,j})} \quad (11)$$

where, $L_j(\beta_i)$ = log-likelihood of the transferred model applied to the application context data, $L_j(\beta_j)$ = log-likelihood of the locally estimated model, and $L_j(\beta_{reference,j})$ is the log-likelihood of a locally estimated reference model (e.g., a constants only model). In this paper, the constants only specification of the MDCEV structure is taken as the reference model. The closer the value of TI is to 1, the closer is the transferred model's performance to a locally estimated model (in terms of the information captured in the application context relative to the reference model). The TI values for all transfers conducted in the study are presented in Table 5. The diagonal elements in the table that have a TI value of 1 (in bold) are not of interest, because they are not for model transfers from one region to another. It can be observed from the non-diagonal elements in the table that incorporating heteroscedasticity leads to a considerable improvement in the TI value. For example, for models transferred from South East Florida and Central Florida, allowing for heteroscedasticity resulted in an improvement of the TI value from 0.53 to 0.77 (or 53% to 77%). Similar improvements in TI values can be observed for all other transfers conducted in the study.

4.4.2 Aggregate-level Predictive Accuracy

To assess the aggregate-level predictive accuracy of the transferred models, two types of Root Mean Square Error (RMSE) metrics were used in this study: (1) RMSE for the discrete (activity participation)

choice component, and (2) RMSE for the continuous (time allocation) component.

$$RMSE = \left(\frac{\sum_k P_k \times REM_k^2}{\sum_k P_k} \right)^{1/2} \quad (12)$$

where, P_k and O_k are the aggregate predicted and observed shares for activity type *k*, respectively (or durations averaged over all individuals who participated in activity type *k*), and $REM_k = \{(P_k - O_k)/O_k\}$ is the percentage error in the prediction of alternative *k*.

Table 6 reports the RMSEs for all transfers conducted in the study. As expected, in any row of the table, the aggregate errors of the locally estimated models (in bold) are lower than those of transferred models of the same model structure. For any transfer, the RMSEs for the discrete components of the two model structures (MDCEV and MDCHEV) are very similar. However, considerable differences can be observed in the RMSEs for the continuous components of the two model structures. Specifically, the RMSEs for the continuous component of the MDCHEV models (both transferred and locally estimated models) are considerably smaller than the corresponding values for the MDCEV models. A closer examination suggests that the RMSEs for the continuous component of even transferred MDCHEV models are smaller than those of locally estimated MDCEV models, suggesting that the transferred MDCHEV models are providing better prediction performance than locally estimated MDCEV models. Recall that predictive log-likelihood values of the transferred MDCHEV models were better than the log-likelihood values of locally estimated MDCEV models. These results reiterate the benefit of incorporating heteroscedasticity in MDC models.

4.4.3 Response to Changes in Explanatory Variables

To compare the transferability of MDCEV and MDCHEV models based on their responses to changes in explanatory variables, we simulated the influence of a scenario where the age of individuals older than 29 years was increased by 10 years (to reflect aging of the population). Each estimated model was applied to its own estimation sample as well as the other two geographical context datasets for both base and policy scenarios. To measure the resulting changes in the time-use patterns, a policy response measure was computed. To do so, first, for each set of error term draws for each individual, the overall change in activity participation and time-use patterns was measured as below (see Jaggi et al., 2013):

$$T_c = \frac{1}{T}\left(\sum_{k=1}^{K}\frac{|\hat{t}_k^{\,p} - \hat{t}_k^{\,b}|}{2}\right) \quad (13)$$

where, $\hat{t}_k^{\,p}$ is the predicted duration for alternative k in the policy case, and $\hat{t}_k^{\,b}$ = predicted duration for alternative k in the base case. Next, the above metric was averaged over all sets of error term draws for all individuals.

The *policy response* measure was computed for 50 sets of bootstrapped values drawn from the sampling distributions implied by the parameter estimates and their covariance matrix. Table 7 presents the policy response measures for all transferred and locally estimated models in the form of average policy response values (averaged over all bootstrapped estimates). The corresponding standard errors are provided in the parentheses next to each average policy response measure. Since the true policy response is unknown, the policy response obtained from the model with the best data fit (i.e., the locally estimated MDCHEV model) in each region is taken as the reference for that region. The corresponding cells in the table are shaded in gray. The transferability performance of transferred MDCEV and MDCHEV models are assessed by comparing their policy response measures to that from the corresponding reference model (i.e., the policy response measure from the locally estimated MDCHEV model).

It can be observed that, for each of the three regions, the policy response measures of transferred MDCHEV models are better than (i.e., closer to the policy response implied by the locally estimated MDCHEV model) those of the transferred MDCEV models. Further, except for transfers to and from the Tampa bay region, the policy response of a transferred MDCHEV model appears to be better even than that of a locally estimated MDCEV model. These results suggest that improvement in model structure (i.e., incorporation of heteroscedasticity) has not only resulted in a better data-fit but also a better ability to predict responses to changes in explanatory variables.

In summary, all the transferability assessments conducted in this study suggest that the proposed methodological extension (of incorporating heteroscedasticity) helps in enhancing the spatial transferability (hence the predictive ability) of time-use models.

5 SUMMARY AND CONCLUSIONS

This paper investigates the benefits of incorporating heteroscedastic stochastic distributions in random utility maximization-based multiple discrete continuous (MDC) choice models. To this end, the paper formulates a Multiple Discrete-Continuous Heteroscedastic Extreme Value (MDCHEV) model that employs heteroscedastic extreme value (HEV) distributed random utility components in MDC models. Heteroscedasticity is accommodated by allowing the scale parameters of the random utility components to be different across the different choice alternatives. Therefore, the MDCHEV model collapses to the MDCEV model when all the scale parameters are constrained to be equal. The likelihood of the MDCHEV model is a uni-dimensional integral that can be easily and accurately evaluated using quadrature techniques.

In addition to formulating the MDCHEV model, the paper investigates the benefits of incorporating heteroscedasticity for analyzing individuals' daily activity participation and time allocation choices, using data from the 2009 National Household Travel Survey (NHTS) data for three major urban regions in Florida – South East Florida, Central Florida, and Tampa Bay. The empirical analysis proceeds in two stages. In the first stage, a series of model estimations are carried out to select the best fitting MDC model structure for the current empirical data. Specifically, the following six different model structures are estimated: (1) The MDCEV model with IID extreme value stochastic distributions, (2) The mixed-MDCEV model to accommodate heteroscedasticity across choice alternatives, (3) The MDCHEV model, (4) The mixed MDCEV model to accommodate inter-alternative correlations, (5) The MDCGEV model to incorporate inter-alternative correlations while retaining closed-form probability expressions, and (6) The mixed-MDCEV model to accommodate both inter-alternative correlations and heteroscedasticity. The results from all these model estimations resulted in the following findings in the current empirical context. First, among all the above six model structures, the MDCHEV structure provided the best fit to the data. Second, no significant correlations were found between the random utility components of different activity type alternatives but heteroscedasticity was prominent. Given these results, the second stage analysis focuses on comparing the MDCHEV and the MDCEV models, in terms of their empirical parameter estimates, in-sample prediction performance, and transferability to different geographical regions. For spatial transferability assessments, the models estimated for each of the three regions were transferred to the other two regions.

The parameter estimates of the MDCHEV model reveal the presence of substantial differences in the scale parameters (i.e., heteroscedasticity) of the random utility components across different activity type choice alternatives. Plausible reasons for heteroscedasticity include aggregation of choice alternatives into broader activity categories and measurement errors in the continuous dependent variables. These findings suggest that data collection efforts and model specifications for discrete-continuous choice models need to be cognizant of potential aggregation and measurement errors.

Neglecting heteroscedasticity (when present) in MDC models can have several ramifications. As revealed from the current empirical application, ignoring heteroscedasticity can potentially lead to biased parameter estimation and inferior statistical fit to the estimation sample. Furthermore, the predicted distributions of the continuous quantity decisions (time allocations, in the current empirical context) can be distorted when compared to the distributions observed in the estimation sample. Specifically, the MDCEV-predicted distributions of continuous quantities exhibit thicker right tails (i.e., greater chance of over-prediction) for some alternatives and thinner right tails (i.e., greater chance of under-prediction) for other alternatives when compared to the distributions observed in the estimation sample. In the current empirical context, the time allocations for many out-of-home activities were over-estimated and those for in-home and active recreation activities were under-estimated. The MDCHEV model overcomes these issues to a considerable extent by allowing the scale parameters to be different from each other. Specifically, the MDCHEV model results in tighter (wider) distributions of random utility components for the alternatives for which the MDCEV over-predicts (under-predicts) the time allocations and therefore reduces the chances of over-prediction (under-prediction).

Spatial transferability assessments using a variety of different assessment metrics suggest better predictive ability for MDCHEV models transferred from other regions than MDCEV models transferred from those same regions. More interestingly, in most cases, the transferred MDCHEV models appear to perform not only better than transferred MDCEV models but also better than locally estimated MDCEV models. These results reiterate the importance of incorporating heteroscedasticty in MDC choice models.

The findings in this paper not only demonstrate the benefits of employing HEV distributions over IID extreme value distributions in MDC models, but also raise a more general issue of the importance of distributional assumptions in MDC Models. In this context, exploration of the influence of alternative distributional assumptions – such as multivariate heteroscedastic extreme value and multivariate normal distributions – on the prediction performance of MDC models is a potentially fruitful avenue for further research. Equally important is the need for investigating the relative importance of inter-alternative correlations vis-à-vis heteroscedasticity in error terms (across alternatives as well as across individuals). Further studies using both simulated data and empirical data to compare the performance of a variety of different model structures, including MDCEV, MDCHEV, MDCGEV, mixed-MDCEV, and MDCP models are warranted to address these issues.

ACKNOWLEDGEMENTS

The second author's efforts on this paper are dedicated to Mr. Siva Subramanyam Jonnavithula, a transportation engineer, who passed away in July 2013. The authors would like to thank three anonymous reviewers for constructive comments on an earlier manuscript.

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Table 1: Goodness of Fit of Different MDC Model Structures on the Empirical Time-Use Data Used in the Current Study

	Model with IID Stochastic Distribution	Models with Heteroscedastic Stochastic Distributions		Models with Correlated Stochastic Distributions		Model with Heteroscedastic and Correlated Stochastic Distributions
	MDCEV (Model #1)	Mixed-MDCEV (Model #2)	MDCHEV (Model #3)	Mixed-MDCEV (Model #4)	MDCGEV (Model #5)	Mixed-MDCEV (Model #6)
Log-likelihood at convergence	-29397.2	-29345.0	-29204.4	-29396.7	-29392.1	-29344.5
Number of Parameters	51	53	55	52	52	53
BIC = -2*LL+Ln(N)*K	59184.2	59095.1	58829.2	59191.9	59181.7	59094.1

Note: LL is the log-likelihood value at convergence, N is the number of observations, and K is the number of parameters.

Table 2: Model Estimation Results

	South East Florida (SEF)		Central Florida (CF)		Tampa Bay (TB)	
	MDCEV Coef. (t-stat)	MDCHEV Coef. (t-stat)	MDCEV Coef. (t-stat)	MDCHEV Coef. (t-stat)	MDCEV Coef. (t-stat)	MDCHEV Coef. (t-stat)
Scale Parameters (t-stats against 1)						
In-home Activity	1.00(fixed)	1.00(fixed)	1.00(fixed)	1.00(fixed)	1.00(fixed)	1.00(fixed)
Shopping	1.00(fixed)	0.73(11.11)	1.00(fixed)	0.68(12.08)	1.00(fixed)	0.68(11.06)
Other Maintenance	1.00(fixed)	0.52(22.06)	1.00(fixed)	0.42(27.84)	1.00(fixed)	0.47(25.72)
Social/Recreational	1.00(fixed)	0.60(16.96)	1.00(fixed)	0.58(16.16)	1.00(fixed)	0.55(17.29)
Active Recreation	1.00(fixed)	1.14(1.77)	1.00(fixed)	1.18(1.87)	1.00(fixed)	1.40(3.46)
Medical	1.00(fixed)	0.73(11.11)	1.00(fixed)	0.68(12.08)	1.00(fixed)	0.68(11.06)
Eat out	1.00(fixed)	0.60(16.96)	1.00(fixed)	0.58(16.16)	1.00(fixed)	0.55(17.29)
Pick-up/ Drop-off	1.00(fixed)	0.52(22.06)	1.00(fixed)	0.42(27.84)	1.00(fixed)	0.47(25.72)
Other Activities	1.00(fixed)	1.00(fixed)	1.00(fixed)	1.00(fixed)	1.00(fixed)	1.00(fixed)
Baseline Utility Parameters						
Constants						
Shopping	-7.45(-74.79)	-7.26(-90.07)	-7.55(-53.30)	-7.30(-65.61)	-6.69(-89.77)	-6.69(-122.18)
Other Maintenance	-8.90(-49.05)	-7.98(-71.42)	-8.54(-53.07)	-7.74(-68.50)	-7.41(-83.86)	-7.02(-131.87)
Social/Recreational	-8.48(-77.19)	-7.94(-92.90)	-8.68(-53.06)	-7.98(-64.67)	-8.18(-34.51)	-7.52(-53.68)
Active Recreation	-8.99(-67.10)	-8.98(-48.37)	-9.33(-44.29)	-9.33(-33.58)	-8.69(-30.04)	-9.63(-19.88)
Medical	-8.75(-75.57)	-8.25(-85.18)	-8.78(-45.52)	-8.13(-55.34)	-7.99(-45.27)	-7.62(-60.85)
Eat out	-9.48(-51.48)	-8.56(-65.60)	-9.65(-29.19)	-8.50(-39.64)	-8.07(-31.25)	-7.50(-49.88)
Pick-up/ Drop-off	-8.46(-56.68)	-7.91(-77.51)	-9.85(-18.27)	-8.26(-33.45)	-8.99(-26.47)	-7.94(-39.83)
Other Activities	-10.20(-84.54)	-9.97(-87.96)	-10.26(-61.8)	-9.52(-31.74)	-9.04(-84.49)	-9.04(-84.81)
Gender (Male is base)						
Female - Shopping	0.06(0.79)	0.02(0.40)	0.13(1.56)	0.10(1.70)	0.16(1.82)	0.11(1.81)
Female - Active Recreation	-0.20(-1.97)	-0.26(-2.23)	-	-	-	-
Female - Pick-up/ Drop-off	-	-	-	-	0.27(1.71)	0.13(1.48)
Age (30 – 54 years is base)						
18-29 years - Social/Recreational	0.75(3.57)	0.50(3.82)	-	-	-	-
55-64 years - Medical	-	-	0.15(0.77)	0.07(0.54)	0.39(1.80)	0.28(1.87)
55-64 years - Eat out	-	-	0.39(2.01)	0.20(1.76)	-	-
55-64 years - Pick-up/Drop-off	-0.48(-2.64)	-0.24(-2.34)	-0.38(-1.65)	-0.23(-2.20)	-	-
65-74 years - Medical	0.28(2.33)	0.21(2.47)	0.16(0.93)	0.08(0.64)	0.30(1.47)	0.21(1.51)
65-74 years - Eat out	-	-	0.43(2.44)	0.21(1.97)	-	-
65-74 years - Pick-up/Drop-off	-0.62(-3.78)	-0.29(-3.12)	-0.43(-1.96)	-0.26(-2.61)	-	-
$\geq$ 75 years - Social/Recreational	-	-	-	-	-0.31(-2.52)	-0.18(-2.49)
$\geq$ 75 years - Active Recreation	-	-	-	-	-0.16(-1.21)	-0.21(-1.12)
$\geq$ 75 years - Medical	0.24(2.11)	0.20(2.39)	0.20(1.14)	0.11(0.94)	0.36(1.79)	0.26(1.92)
$\geq$ 75 years - Eat out	-	-	0.39(2.16)	0.19(1.77)	-	-
$\geq$ 75 years - Pick-up/ Drop-off	-1.00(-5.99)	-0.49(-5.07)	-0.65(-2.82)	-0.34(-3.32)	-0.59(-3.21)	-0.33(-3.14)
White race - Eat out	0.27(1.73)	0.17(1.81)	0.44(1.72)	0.24(1.63)	0.28(1.08)	0.15(1.00)
Driver (Non-driver is base)						
Driver - Other Maintenance	0.44(2.36)	0.14(1.41)	-	-	-	-
Driver - Social/Recreational	-	-	-	-	0.68(2.97)	0.32(2.40)
Driver - Active Recreation	-	-	-	-	0.60(2.29)	0.90(2.41)
Driver - Pick-up/ Drop-off	-	-	1.06(2.07)	0.37(1.66)	0.72(2.26)	0.33(1.81)
Education (High Sch./low base)						
College - Other Maintenance	0.35(3.10)	0.17(2.76)	-	-	0.33(2.64)	0.15(2.33)
Bac. /High - Other Maintenance	0.50(4.76)	0.27(4.52)	0.22(1.94)	0.07(1.32)	0.32(2.52)	0.13(1.95)
Bac./High - Active Recreation	0.20(1.81)	0.24(1.91)	0.39(2.96)	0.49(3.09)	0.21(1.51)	0.31(1.67)
Born in US						
Social/Recreational	0.17(1.77)	0.10(1.74)	-	-	-	-
Eat out	0.49(4.18)	0.30(4.19)	0.14(0.66)	0.08(0.66)	-	-

Table 2: Model Estimation Results (continued…)

	South East Florida (SEF)		Central Florida (CF)		Tampa Bay (TB)	
	MDCEV Coef. (t-stat)	MDCHEV Par. (t-stat)	MDCEV Coef. (t-stat)	MDCHEV Par. (t-stat)	MDCEV Coef. (t-stat)	MDCHEV Coef. (t-stat)
Number of Children						
0-5 years - Shopping	-	-	-0.50(-2.55)	-0.33(-2.42)	-0.14(-0.85)	-0.12(-1.05)
0-5 years - Other Maintenance	-0.29(-1.68)	-0.17(-1.64)	-0.26(-1.38)	-0.10(-1.15)	-	-
0-5 years - Pick-up/Drop-off	0.26(1.81)	0.16(1.44)	0.58(3.90)	0.30(3.86)	0.23(1.30)	0.11(1.07)
6-18 years - Pick-up/Drop-off	0.48(5.09)	0.28(4.88)	0.46(2.95)	0.20(2.65)	0.58(3.95)	0.34(3.81)
Income (<25K is base)						
25 -55 K - Other Maintenance	-	-	0.34(2.43)	0.15(2.28)	-	-
25 -55 K - Social/Recreational	-	-	0.29(2.11)	0.16(1.89)	-	-
25 -55 K - Active Recreation	-	-	0.39(2.34)	0.44(2.29)	-	-
25 -55 K - Eat out	0.29(2.08)	0.17(1.94)	0.31(2.10)	0.16(1.86)	-	-
55 - 75k - Other Maintenance	-	-	0.28(1.62)	0.12(1.47)	-	-
55 - 75k - Social/Recreational	-	-	0.27(1.61)	0.13(1.31)	-	-
55 - 75k - Active Recreation	0.28(2.03)	0.33(2.10)	0.43(2.16)	0.49(2.11)	0.19(1.02)	0.20(0.76)
55 - 75k - Eat out	0.30(1.86)	0.17(1.75)	0.33(1.91)	0.17(1.62)	0.31(1.87)	0.20(2.05)
>75 K - Other Maintenance	-	-	0.37(2.26)	0.15(1.89)	-	-
>75 K - Social/Recreational	-	-	0.38(2.48)	0.18(1.96)	-	-
>75 K - Active Recreation	0.55(4.59)	0.63(4.41)	0.51(2.69)	0.54(2.45)	0.67(4.36)	0.88(4.06)
>75 K - Eat out	0.82(5.98)	0.46(5.42)	0.47(2.85)	0.23(2.25)	0.51(3.63)	0.29(3.53)
No. of Workers						
Shopping	-0.14(-2.26)	-0.09(-2.12)	-0.10(-1.20)	-0.06 (-1.05)	-	-
Pick-up/ Drop-off	-	-	0.14(1.21)	0.09(1.52)	0.38(3.24)	0.21(3.07)
# Recreation sites in a mile from HH. Social/Recreational	0.005(3.55)	0.003(3.81)	0.07(2.02)	0.04(1.99)	0.004(2.42)	0.002(2.13)
# Intersections in 0.25 miles from HH. Active Recreation	-	-	0.006(1.25)	0.005(1.02)	0.01(1.59)	0.01(1.74)
No. of Cul-de-sacs in 0.25 miles from HH. Active Recreation	0.009(0.92)	0.01(1.08)	-	-	-	-
Day of the Week						
Monday - Eat out	-0.28(-2.04)	-0.16(-1.87)	-0.16(-1.10)	-0.11(-1.21)	-	-
Friday - Social/Recreational	-	-	0.22(1.80)	0.11(1.54)	0.19(1.38)	0.13(1.66)
Friday - Eat out	-	-	0.30(2.29)	0.16(2.06)	0.18(1.23)	0.12(1.42)
Satiation Parameters						
Constants						
Shopping	2.82(33.98)	3.25(38.31)	3.04(46.97)	3.55(48.76)	3.01(44.73)	3.51(45.49)
Other Maintenance	3.17(46.65)	3.96(54.83)	2.94(37.04)	3.89(50.04)	2.72(21.76)	3.66(29.62)
Social/ Recreational	4.31(49.46)	4.99(52.25)	4.19(49.04)	4.90(50.80)	4.44(46.92)	5.21(49.13)
Active Recreation	1.64(8.56)	1.46(6.60)	1.57(9.02)	1.37(6.54)	2.04(16.81)	1.48 (8.07)
Medical	3.38(42.48)	3.86(43.66)	3.11(32.69)	3.70(36.05)	3.19(31.79)	3.76(34.10)
Eat out	3.02(34.73)	3.71(40.20)	3.15(36.18)	3.86(41.14)	3.05(28.65)	3.80(34.81)
Pick-up/Drop-off	1.44(15.93)	2.32(23.59)	1.41(12.98)	2.49(22.49)	1.59(13.58)	2.37(19.41)
Other Activities	2.41(16.38)	2.41(16.41)	1.97(11.77)	2.20(10.04)	2.09(12.84)	2.09(12.88)
Gender (Male is Base)						
Female - Shopping	0.34(3.13)	0.34(3.54)	-	-	0.34(2.12)	0.26(2.06)
Female - Active Recreation	-0.25(-1.30)	-0.22(-1.14)	-	-	-	-
Age. 35-45 years - Social/Recreational	-0.32(-1.77)	-0.37(-2.25)	-	-	-	-
Education (< college is base)						
Some College - Active Recreation	0.36(1.49)	0.34(1.38)	0.31(1.10)	0.27(0.94)	-	-
Bachelor/ Higher - Active Recreation	0.94(4.28)	0.86(3.81)	0.76(2.93)	0.65(2.45)	-	-
Day of the Week						
Friday - Social/Recreational	0.31(1.83)	0.34(2.19)	-	-	-	-
Friday - Eat out	0.26(1.41)	0.27(1.66)	-	-	0.39(1.66)	0.40(1.99)
Log-likelihood at constants	-29681.3	-29454.6	-20518.7	-20297.6	-18390.8	-18234.3
Log-likelihood at convergence	-29397.2	-29204.4	-20386.6	-20180.0	-18302.1	-18148.3

Table 3: Predicted and Observed Activity Participation (% participation) Rates

		In-home	Shopping	Other Maintenance	Social/ Recreational	Active Recreation	Medical	Eat Out	Pick Up/ Drop Off	Other Activities
SEF	% Participation(MDCEV)	100.0 (100.0)	49.2 (51.0)	29.9 (30.6)	29.0 (30.5)	19.1 (20.6)	23.1 (24.8)	22.8 (24.3)	16.0 (17.0)	5.3 (5.7)
	% Participation(MDCHEV)	100.0 (100.0)	47.6 (51.0)	29.3 (30.6)	29.0 (30.5)	19.8 (20.6)	22.9 (24.8)	22.2 (24.3)	16.0 (17.0)	5.5 (5.7)
CF	% Participation(MDCEV)	100.0 (100.0)	49.3 (49.9)	30.9 (30.4)	29.1 (30.0)	20.4 (21.9)	23.0 (24.3)	26.2 (27.2)	15.5 (16.2)	5.3 (5.7)
	% Participation(MDCHEV)	100.0 (100.0)	47.1 (49.9)	30.3 (30.4)	28.5 (30.0)	21.0 (21.9)	22.8 (24.3)	25.3 (27.2)	15.3 (16.2)	5.5 (5.7)
TB	% Participation(MDCEV)	100.0 (100.0)	47.9 (48.5)	31.9 (31.6)	26.3 (27.1)	19.6 (21.2)	22.4 (23.4)	23.6 (24.4)	14.4 (15.5)	6.6 (7.0)
	% Participation(MDCHEV)	100.0 (100.0)	45.9 (48.5)	31.1 (31.6)	25.8 (27.1)	20.3 (21.2)	21.9 (23.4)	22.9 (24.4)	14.3 (15.5)	6.8 (7.0)

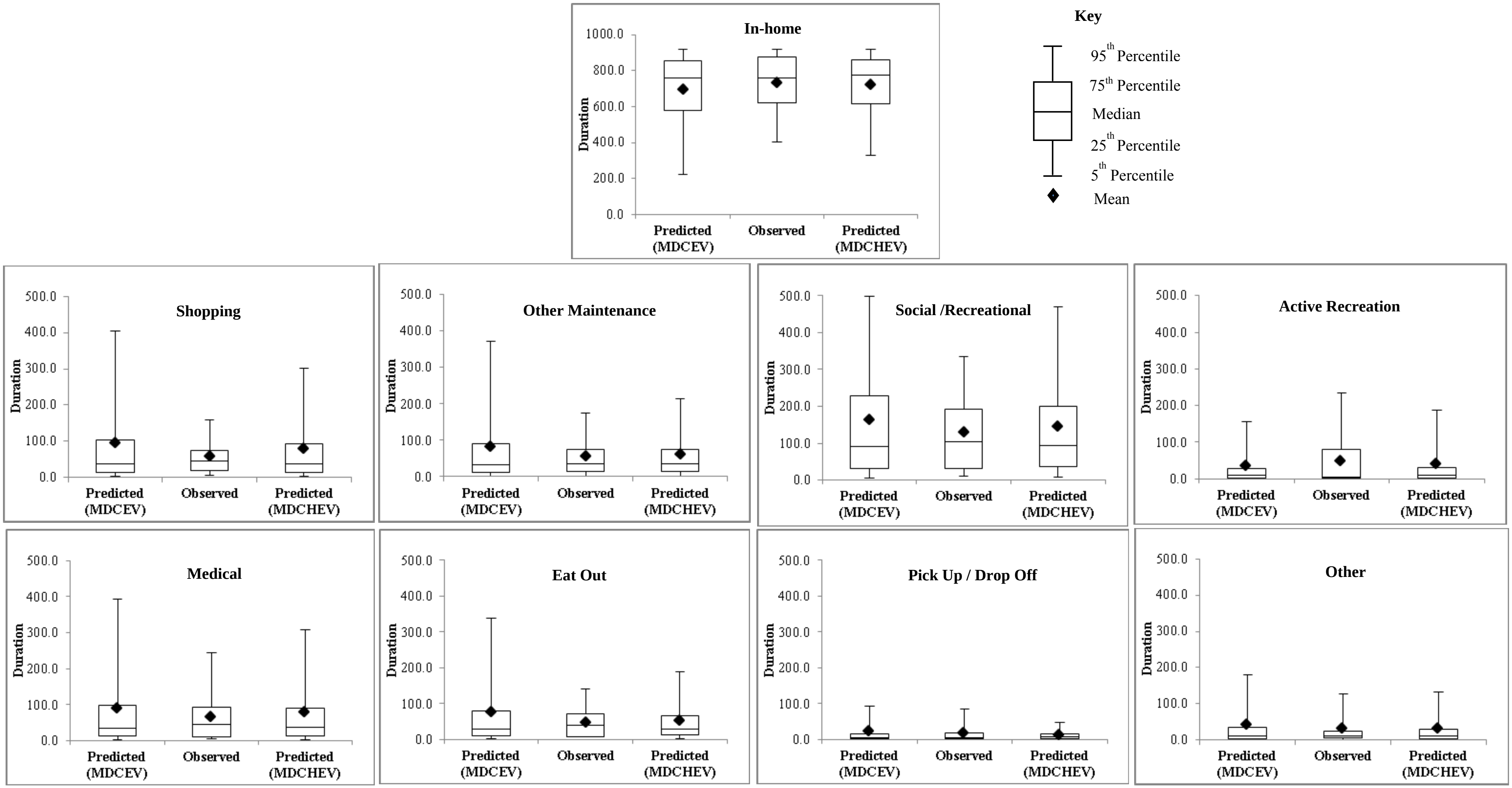

Figure 1: Observed and Predicted Distributions of Activity durations (for the Southeast Florida Region)

Table 4: Transferability Assessment Results: Log-likelihood of Transferred and Locally Estimated Models

Transferred Form	Transferred To	Log-likelihood Values			
		Transferred MDCEV	**Local MDCEV**	**Transferred MDCHEV**	**Local MDCHEV**
SEF	CF	-20448.60	-20386.63	-20257.74	-20180.03
	TB	-18367.31	-18302.08	-18223.37	-18148.27
CF	SEF	-29513.25	-29397.16	-29348.51	-29204.44
	TB	-18349.97	-18302.08	-18217.24	-18148.27
TB	SEF	-29598.86	-29397.16	-29393.82	-29204.44
	CF	-20481.40	-20386.63	-20274.07	-20180.03

Table 5: Transferability Assessment Results: Transfer Index (TI)

Transferred To / **Transferred From**	SEF		CF		TB	
	MDCEV	MDCHEV	MDCEV	MDCHEV	MDCEV	MDCHEV
SEF	**1.00**	**1.00**	0.53	0.77	0.26	0.69
CF	0.59	0.70	**1.00**	**1.00**	0.46	0.72
TB	0.29	0.60	0.28	0.72	**1.00**	**1.00**

Table 6: Transferability Assessment Results: Root Mean Square Error (RMSE)

	Transferred To / Transferred From	SEF		CF		TB	
		MDCEV	MDCHEV	MDCEV	MDCHEV	MDCEV	MDCHEV
Discrete Component	SEF	**0.03**	**0.05**	0.04	0.04	0.07	0.06
	CF	0.04	0.08	**0.04**	**0.04**	0.04	0.06
	TB	0.05	0.08	0.06	0.09	**0.03**	**0.04**
Continuous Component[1]	SEF	**0.11**	**0.07**	0.31	0.16	0.31	0.16
	CF	0.16	0.07	**0.16**	**0.07**	0.18	0.10
	TB	0.17	0.08	0.16	0.10	**0.17**	**0.08**

Table 7: Transferability Assessment Results: Policy Response Measures

Transferred To / Transferred From	SEF		CF		TB	
	MDCEV	MDCHEV	MDCEV	MDCHEV	MDCEV	MDCHEV
SEF	2.76 (0.71)	**2.30 (0.57)**	3.19 (0.84)	2.68 (0.68)	2.39 (0.80)	2.59 (0.65)
CF	2.92 (0.72)	1.96 (0.49)	3.40 (0.85)	**2.30 (0.59)**	2.17 (0.78)	2.13 (0.54)
TB	5.42 (1.43)	4.31 (1.10)	6.01 (1.61)	4.80 (1.24)	5.46 (1.44)	**4.33 (1.11)**