A Time and Money Allocation Model of Household Vacation Travel Behavior: Formulation and Application of a Kuhn-Tucker Demand Model System

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Abstract: This study formulates and applies a joint model of annual, long-distance vacation destination and mode choices to simultaneously analyze the vacation destinations that a household visits over an entire year, along with the time and money allocations and the travel mode to each of the visited destinations. The proposed formulation enhances the Multiple Discrete-Continuous Extreme Value (MDCEV) model structure proposed by Bhat (2005, 2008) in several ways. First, an extended MDCEV framework is proposed to simultaneously consider the influence of both time and money budget constraints in household vacation travel decisions, as opposed to most previous MDCEV applications that consider only a single budget constraint. Second, the time- and money-constrained MDCEV framework of vacation destination choices is integrated with a multinomial logit (MNL) model of travel mode choice. The integrated framework recognizes that households make decisions on where to travel (i.e., vacation destinations) and how to travel (i.e., travel mode) in a joint fashion. Specifically, the framework recognizes that the vacation destinations are *imperfect substitutes* in that a household can potentially choose to visit multiple destinations over a year, while the travel mode alternatives to a destination are *perfect substitutes* in that only one mode of travel is chosen. Third, the proposed, time- and money-constrained MDCEV-MNL framework not only accommodates multiple budget constraints and a mix of imperfect and perfect substitutes in the choice set, but also recognizes the possibility of price variation across both imperfect and perfect substitutes. Finally, the paper highlights certain subtle, but important identification issues related to the specification of MDCEV models with multiple budget constraints. Simple normalizations are proposed that help with parameter identification as well as facilitate the derivation of closed form probability expressions.

The proposed framework is applied to the 1995 American Travel Survey (ATS), with the United States divided into 210 alternative long-distance vacation destinations. A variety of data sources, including the 1995 Consumer Expenditure Survey are used to synthesize information on destination attributes, and lodging costs and other costs of vacation at each of the 210 destinations. In addition to demonstrating the importance of the above-discussed methodological extensions, the empirical model provides insights into the determinants of households' vacation destination and mode choices and related time and money allocation behavior. The model can be incorporated into a larger national travel modeling framework for predicting the national-level origin-destination flows for long-distance vacation travel.

Keywords: *discrete-continuous choice, MDCEV, perfect substitutes, imperfect substitutes, time and money budgets, multiple constraints, long-distance travel, destination choice*

1. INTRODUCTION

1.1 Background

A significant portion of passenger travel miles in the United States (US) comes from long-distance travel, especially for leisure purposes such as vacation. Statistics from national travel surveys indicate that more than one half of all long-distance travel is for pleasure (BTS 2001). Further, the total amount of leisure travel in the nation has been increasing at a rate of at least two-fold every two decades (BTS 2001). The growing demand for long-distance leisure travel is expected to put significant pressure on the nation's transport network. At the same time, local congestion and inadequate multi-modal capacity is likely to hinder long-distance travel. In addition to the expected growth, long distance leisure travel garners particular attention due to its impact on the tourism industry. According to the U.S. Travel Answer Sheet, leisure travel resulted in a total direct spending of $526 billion supporting 14 million jobs in 2010, with the U.S. residents logging 1.5 billion trips in 2010 (U.S. Travel Association 2011). It is not surprising that the economy of several destinations thrives on tourism/leisure travel.

Due to the above discussed reasons, there is an increasing recognition of the need to better understand long-distance leisure travel patterns in the nation. From a transportation planning perspective, understanding the national long-distance travel flow patterns helps in assessing infrastructure needs and implementing appropriate policies. For instance, planning for a new modal network (e.g., high sped rail) across the nation requires an understanding of the long-distance travel flows in the nation along with the market share for different modes of travel between different origins and destinations. From a tourism industry standpoint, understanding the factors influencing where people travel for leisure can aid in: (a) taking measures to enhance the attractiveness of the destinations for increasing the tourism revenue, and (b) devising targeted promotional campaigns to specific traveler segments.

This paper contributes toward a better understanding of the long-distance leisure travel flows in the nation by formulating and applying a household-level vacation destination and mode choice modeling framework. The remainder of this section reviews the literature on long-distance leisure travel analysis and positions the current work vis-à-vis existing literature.

1.2 Literature Review

Long-distance leisure travel has been studied extensively in the tourism literature, and is steadily gaining importance in the transport planning/modeling literature. As reviewed in Van Nostrand et al. (2012), most work in the transport planning/modeling field on long-distance travel can be categorized into: (1) Statewide travel models in the US (Horowitz 2008), (2) National travel models in Europe[1], and (3) Inter-city travel demand analysis between specific city pairs[2]. An important end-goal of all these efforts is to estimate travel flows between different regions by different modes of travel. This information is used to inform various policy and investment initiatives.

A weak element of the statewide models in the US is how the inter-state trips are treated – as "external" or "through" trips and estimated using aggregate growth factor techniques.

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[1] These include the national model systems for Denmark (PETRA, Fosgerau, 2001), Sweden (SAMPERS; Beser and Algers, 2001), Holland (LMS, HCG 1990), Germany (VALIDATE; Vortsih and Wabmuth, 2007), UK, Switzerland, and other countries. Another example is the TRANS-TOOLS model, built for travel demand prediction in and between the European Union countries (see Rich et al., 2009).

[2] Koppelman and Sethi (2005), Bhat (1995), Yao and Morikawa (2005)

Thankfully though, given a majority (62%) of long distance trips are intrastate (Bureau of Transportation Statistics 2001), several state-wide models capture a majority of the long distance trips occurring within the state using better methods such as household/individual-level discrete choice models (Outwater et al., 2010; Hunt et al., 2011). However, it yet leaves about 38% of the remaining, inter-state trips to be estimated appropriately. This issue could be overcome by implementing a national level model to estimate travel across the different states in the nation (CS 2008). However, since the completion of inter-state highway system, there has been a long paucity of efforts aimed at estimating nationwide long-distance travel in the US. Recent efforts toward a national travel model system in the US (Ashaibor et al., 2007; Baik et al., 2008; Moeckel and Donnelly, 2010) are primarily based on the traditional 4-step modeling approach that is limited in its ability to answer to several policy questions.

In addition to the above identified issues, it is worth noting here that most long-distance travel literature and modeling practice in the transport planning arena treats leisure travel in a very limited fashion – generally as "visitor" trips estimated using aggregate methods. On the other hand, leisure travel is one of the most studied topics in tourism research, with a significant focus on understanding various behavioral aspects of leisure travel, including length of stay at destinations, monetary budget allocation, and the cognitive and attitudinal factors influencing destination choices. As categorized by LaMondia et al. (2009), some studies[3] attempt to estimate the "outbound" tourism demand from one origin (e.g., a country) to multiple destinations, while others[4] analyze the "inbound" tourism to a single destination. However, few studies analyze destination choices between multiple origins and multiple destinations toward developing tools for forecasting leisure travel flows in the nation.

A drawback of most work in both the travel demand and tourism literature is that the analysis is typically limited to smaller time frames such as a day or a few weeks. However, analysis of the 1995 American Travel Survey data indicates that on average, a household makes less than 4 vacation trips over a year. Given the infrequent nature of long-distance leisure travel, a smaller time-frame of analysis (e.g., a day) is likely to provide a distorted picture of leisure travel flows in the nation. Intuitively, vacations are planned over longer time frames, as opposed to daily travel decisions for which shorter time frames may suffice. In this context, Eugenio-Martin's (2003) theoretical framework for tourism demand analysis suggests one year as appropriate for vacation travel analysis (also see Morley 1992).

To be sure, a few studies do consider longer time-frames for analyzing leisure travel. Most of these studies follow a sequential approach by first predicting the frequency of vacation trips over a given time frame and then analyzing the destination choices and other decisions separately for each trip (see, for example, the annual leisure travel framework of van Middlekoop et al., 2004 and the holiday travel module of Rich et al., 2009). A second stream of studies attempts a simultaneous analysis of vacation/recreational travel choices over the entire time frame of interest (e.g., a year). LaMondia et al. (2008), Phaneuf and Smith (2005) and Van Nostrand et al. (2012) belong to this category. Among these studies, as discussed in the next section, the paper by Van Nostrand et al. is the most relevant to the current research.

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[3] Eymann and Ronning (1997), Gonzalez and Moral (1995), DeCrop and Snelders (2004), Lise and Tol (2001), Haliciolgu (2008)

[4] Greenidge (2001), Garin-Munoz and Amaral (2000), Chan et al. (2005)

1.3 Current Research

This paper contributes to the long-distance travel modeling literature by providing an analysis of American households' annual vacation destination and mode choices. More specifically, the paper formulates a household-level time and money allocation model to simultaneously analyze the different vacation destinations that a household visits over the time-frame of a year, along with the time (no. of days) and monetary allocations and the mode of travel to each of the visited destinations.

To model the destination choices, a Kuhn-Tucker demand model system called the Multiple Discrete-Continuous Extreme Value (MDCEV) framework (Bhat, 2005, Bhat 2008) is used. The model assumes that, over the time period of a year, households allocate a part of the total time (365 days) and money (annual income) available with them to one or more vacation destinations so as to maximize the utility derived from their choices. The multiple discrete modeling framework recognizes that households may visit a variety of destinations over the time frame of a year and allocate different amounts of time and money to each vacation destination, depending on the destination characteristics, travel costs, and household demographic characteristics.

The proposed time- and money-allocation formulation builds on our previous work (Van Nostrand et al., 2011) that involved the application of the MDCEV framework for modeling households' annual vacation destination choices, however only as a time allocation framework – not as a time and money allocation framework. In other words, the Van Nostrand et al. framework considers time as the only constraint governing (or, time as the only resource needed for) long-distance leisure travel. However, along with time, money is also essential for "consuming" leisure travel, and is an important constraint in governing related decisions. Households are typically faced with the problem of allocating limited amounts of time and money in making their leisure travel decisions such as whether to make vacation trip(s), which place(s) to visit over a certain time-frame, and how much time and money to expend on these trips. For example, a household may have the time to make another vacation trip but may not be able to afford the monetary costs of doing so. Similarly, a household may have the time to travel to an exotic, faraway destination but not enough money to do so. On the other hand, some households may simply not have the time for long vacations even if they are able to afford the expenses. In most cases, both time and money constraints are likely to influence the choices. Neglect of such constraints, when present, can lead to a confounding of the ignored constraints into the estimated preference structure. To address these issues, the current paper makes a significant methodological extension to the Van Nostrand et al. paper by simultaneously considering both time and money budget constraints within the MDCEV framework. This extended MDCEV framework explicitly recognizes that households make vacation travel decisions under both time and money constraints. To be sure, a handful of recent studies (Satomura et al., 2011; Castro et al., 2012; and Parizat and Sachar, 2010) do consider multiple budget constraints while modeling discrete-continuous choices. The next section reviews these studies and describes how our study is methodologically different from these studies.

In addition to modeling the destination choices, the time- and money-constrained MDCEV framework of vacation destination choices is integrated with a single discrete choice multinomial logit (MNL) model of travel mode choice. The integrated framework recognizes that households make decisions on where to travel (i.e., vacation destinations) and how to travel (i.e., mode of travel) in a joint fashion. An important methodological contribution of this framework is it recognizes that the choice alternatives comprise a combination of *imperfect* and *perfect* substitutes. Specifically, the destination choice alternatives are *imperfect* substitutes in that a household can potentially chose to visit multiple vacation destinations over a year, whereas the mode choice alternatives for each vacation destination are perfect substitutes in that a single mode[5] is chosen to travel to a destination. That is, the choice of one vacation destination does not preclude the choice of another destination at a different time in the year, whereas, for each destination visited, the choice of one mode precludes the choice of other modes of travel to that destination. The vast majority of choice modeling literature is focused on modeling consumer choice from a set of *perfectly substitutable* discrete choice alternatives, while there has been recent interest in modeling consumer choices of potentially multiple alternatives from a set of *imperfectly substitutable* choice alternatives. However, not much exists in the literature on modeling consumer choice(s) from a combination of perfect and imperfect substitutable choice alternatives. Only a few studies (Bhat et al., 2006; Bhat et al., 2009; Eluru et al., 2010) use joint multiple discrete-continuous (MDC) and single discrete choice modeling frameworks for such choice situations. These studies, however, do not consider multiple budget constraints. Further, the model formulations in these studies assume that the prices per unit consumption do not vary across the perfect substitutes. In the current empirical context, consideration of the differences in the travel prices by different modes of travel (which are perfect substitutes) is very important. Thus, to our knowledge, the current study is the first in the econometric literature to propose a discrete-continuous modeling framework for choice situations involving both imperfect and perfectly substitutable choice alternatives, multiple linear budget constraints, and price variation across all choice alternatives including perfect substitutes.

The proposed model formulation is applied to the 1995 American Travel Survey (ATS) data to estimate the empirical model parameters, with the United States divided into 210 alternative destinations. The ATS does not contain information on the travel times and travel costs to the destinations visited by the respondent households. Such data on travel level of service characteristics and data on the characteristics of alternative destinations were compiled from a variety of different data sources. Further, the ATS did not collect contact information on the monetary costs of vacation at any of the destinations. This information, necessary to analyze households' money-allocation to the different vacation destinations, was synthesized using the 1995 Consumer Expenditure Survey (CEX) data (obtained from ICPSR, 2011). In all, a rich database was compiled to build an empirical model of household-level annual vacation destination choices and mode choices. The empirical model can fit into a larger national leisure travel modeling framework to forecast the leisure travel flows in the nation and mode shares under alternative demographic and policy scenarios.

The remainder of the paper is organized as follows. The next section provides an overview of the methods used in the literature to incorporate both time and money constraints in choice modeling, and to accommodate perfect and imperfect substitutes in the choice alternatives. Section 3 formulates the proposed modeling methodology. Section 4 provides an overview of the data sources and the additional procedures used to synthesize information on the monetary costs of

[5] We use the term mode to refer to the primary mode of travel. It is possible that a particular trip to a destination involves travel by multiple modes of travel, which is not of concern in this paper. For example, one can travel by car to the airport, and then by air to the final destination. But the entire trip involves only one primary mode of travel (air mode in this example), which is the focus of this analysis. The primary modes of travel available to a household for traveling to a destination can be viewed as perfect substitutes. Although a household's travel mode choices may vary across the different destinations it visits over a year, the mode choice alternatives for each destination are perfect substitutes. As we illustrate later, the 1995 ATS data suggests that even if the household visited a destination multiple times over a year, a single primary mode of travel has been used every time the household traveled to that destination. Thus, mode choice alternatives to a destination are considered as perfect substitutes.

vacation at each of the 210 destinations in the U.S. Empirical model estimation results and discussion are provided in Section 5. Section 6 concludes the paper.

2 Time and Money Constraints in Consumer Choice Modeling

Literature on incorporating multiple constraints in consumer choice analysis dates back to Becker's (1965) time allocation theory. Per his formulation, households utilize time and goods purchased in the market (using money) to produce commodities, and consume those commodities to maximize the utility derived. Since both time and money are treated as resources used in producing commodities, both the constraints come into picture. He collapsed the two constraints into a single one, using "full-prices" that convert time into money based on a money-value of time and "full-budget" that a household could potentially earn by converting all the available time into work.

Literature in the recreation demand area has long recognized the importance of the roles played by both time and money in individuals' recreation choices (Cesario, 1976; Bockstael et al. 1987). However, until about a decade ago, most recreation demand studies considered only the money budget constraint explicitly while treating time as only a price (e.g., by converting travel time into an equivalent monetary cost), not as an explicit constraint. Larson and Shakih (2001) were the first in recreation demand literature to consider both time and money budget constraints explicitly. They follow Becker's (1965) full-price and full-budget approach to combine the time constraint with the money constraint into a single effective budget constraint. This same approach was used in a weekly travel demand analysis by Kockelman and Krishnamurthy (2001). Building on Larson and Shaikh (2001), Hanemann (2006) proposed a general framework to accommodate multiple linear constraints in utility maximization-based consumer choice problems, by assuming that different constraints can be collapsed into one single constraint. Also, see Carpio et al (2008) for a recent application of this approach.

Although the full-price and full-budget approach helps in simplifying the problem, it makes an implicit assumption that the different constraints are substitutable with each other. In several empirical situations, however, this assumption may not hold because different resources (e.g., time and money) cannot always be exchanged with one another. As discussed earlier, a household may have the time to travel to an exotic, faraway destination but money constraints may not allow them to do so, since the available time cannot necessarily be exchanged into money. Similarly, a household that can very well afford to take a vacation trip may not do so because of time constraints that cannot be relieved by converting money into extra leisure time. Thus, it is important to consider both the time and money constraints *in their own right* (Castro et al., 2012), as opposed to substituting one into another.

The above discussed studies, in addition to assuming free exchangeability of constraints, solve the consumer's direct utility maximization problem using its dual version (i.e., the cost minimization problem). Specifically, indirect utility functions and Roy's identity are applied to derive the demand functions based on the argument that the direct utility maximization approach is difficult and doesn't yield tractable demand functions. While the indirect utility method has been a standard approach in discrete-continuous choice modeling literature (due to Hanemann 1984), there is emerging recognition that the direct utility approach is more closely tied to behavioral theory and not difficult to work with. Further, the direct utility approach is more transparent in its assumptions, offers a clear interpretation of parameters for consumer preferences, and provides better insights into identification issues (Bunch, 2009; Bhat and Pinjari, 2010). Thus, in this study, we use the direct utility approach to formulate and solve the time and money constrained utility maximization problem. In this context, three studies of particular relevance are discussed below.

The first study is by Satomura et al. (2011) who propose a direct utility maximization framework for modeling multiple discrete-continuous (MDC) choices with any number of linear binding constraints. Specifically, they setup an incomplete demand system specification with constraint-specific Hicksian composite outside goods[6] (one outside good for each constraint). Their utility specification takes the form of an additively separable Linear Expenditures System (LES) with no stochasticity on the utility contributions of the outside goods. This assumption facilitates the derivation of closed-form likelihood expressions based on Kuhn-Tucker conditions of optimality to solve the direct utility maximization problem with multiple constraints. Their empirical application involves two constraints in the context of beverage purchases – one for money and the other for space.

The second study is by Castro et al. (2012) who propose general frameworks that can be used to model MDC choices as both complete and incomplete demand systems. In contrast to Satomura et al., Castro et al. consider stochasticity on the utility contributions of all goods including outside goods, which leads to likelihood expressions that are multivariate integrals (of as many dimensions as the number of linear constraints under consideration). Our formulation (described in the next section) is similar to that of Castro et al. in that stochasticity is considered for the outside good as well. Their empirical application is on modeling weekly time-allocation among different leisure activities considering time and money constraints. Another contribution of Castro et al. is that they discuss several identification considerations necessary for parameter estimation in the context of MDC models with multiple constraints. They also highlight the need for a "*deeper analysis of empirical identification and stability issues during estimation*". Thus, in the current paper, we probe deeper into the empirical identification issues for incomplete demand systems with constraint-specific Hicksian outside goods. As discussed in the next section, we highlight the need for additional normalizations (to what was proposed in Castro et al.) that not only help with parameter identification and stability during model estimation, but also help in achieving closed-form likelihood expressions which greatly facilitated the estimation of a large demand system with 210 inside goods in our empirical context.

The third study, by Parizat and Sachar (2010), is a significant departure from the two studies discussed above. Specifically, both Satomura et al and Castro et al work within the realm

[6] An incomplete demand system is described here by contrasting it with a complete demand system. A complete demand system models the consumption of all goods from all possible categories of consumption. Thus, setting up a complete demand system requires data on the prices and consumptions of each (and every) good that can potentially be consumed with the available resources (e.g., money). However, in many empirical analyses, the analyst might be interested in studying the consumption patterns of only one category of goods (e.g., vacation destination choices over a year). Further, data may not be available on the consumptions and prices of each (and every) good in other categories (e.g., education, housing, etc.). In such situations, the analyst can setup an incomplete demand system, where the consumptions of only those goods in the category of interest to the analyst are modeled in a detailed fashion, while the expenditures for other categories of consumption are modeled in an aggregate fashion. There are two ways to do so: (1) A two-stage budgeting approach, where the first stage involves expenditure allocations to each broad category of consumption and the second stage involves detailed modeling of the consumption of goods in the specific category of interest, (2) A Hicksian composite outside good approach, where the expenditure allocation to all goods other than those of interest are pooled into one or few categories called the "outside" goods and the goods in the specific category of interest are finely categorized and analyzed as "inside" goods. A common practice is to have a single Hickisan composite outside good that aggregates all consumption "outside" the purview of interest to the analyst. Usually, such an outside good is "essential" in that all consumers will have consumed at least some of it.

of linear prices with no initial (or fixed) costs of consuming goods. The resources (e.g., money) are assumed to be expended in proportion to the amount of consumption. Thus, assuming a constant price per unit of consumption suffices in their framework. However, Parizat and Sachar consider a more general framework where prices can be non-linear due to fixed costs. Specifically, they consider the case of daily time allocation among different leisure activities where participation in any activity is associated with fixed costs. The fixed costs include travel times, travel costs, and the cost of activity participation which is independent of the amount of time spent in the activity (e.g., eat out activity involves some expenditure for purchasing the meal). Due to such fixed costs, prices become non-linear and destroy the continuous and twice differentiable properties of the utility functions. Therefore, the consumer's constrained utility maximization problem cannot be solved by Kuhn-Tucker (KT) conditions alone. Thus, instead of the KT conditions, Parizat and Sachar employ numerical search methods for locating optimal consumptions while considering the non-linear time and money constraints. While their problem setup is behaviorally appealing due to the accommodation of non-linear prices, the numerical approach to solving for the optimal utility is rather cumbersome. In our empirical context of vacation destination choices with 210 destination choice alternatives, the numerical search approach becomes simply impractical. Even with 12 choice alternatives (i.e., 12 different leisure activity types) in the empirical application, Parizat and Sachar had to make several simplifying assumptions (such as same setup costs for every individual) to keep the numerical search from blowing up computationally. Further, such empirical assumptions may, in fact, offset the benefits of considering non-linear pricing and multiple constraints.[7] Besides, a purely numerical approach (ignoring KT conditions) to locating the optimal point loses sight of the insights one can obtain from the KT conditions. Thus, in this paper, we assume that travel costs can be amortized into a constant travel-price per unit of consumption (i.e., travel price per unit time allocation to the destination). That is, the travel costs are treated as *variable* with the amount of consumption with a constant price per unit consumption. In reality, however, the travel costs are *fixed* and do not vary linearly with the amount of time allocation at the destination. As discussed earlier, incorporating such fixed costs leads to non-linear and non-smooth budget constraints making it extremely difficult to solve the consumer's utility maximization problem. Resolving this issue is beyond the scope of this paper, but a very important avenue for future research.

Finally, as discussed earlier, our formulation accommodates the possibility that some of the choice alternatives may be perfect substitutes while others are imperfect substitutes and that prices (of unit consumption) can vary across both imperfect and perfect substitutes, a feature that none of the above three studies considers.

3 MODEL FORMULATION

3.1 Choice Alternatives

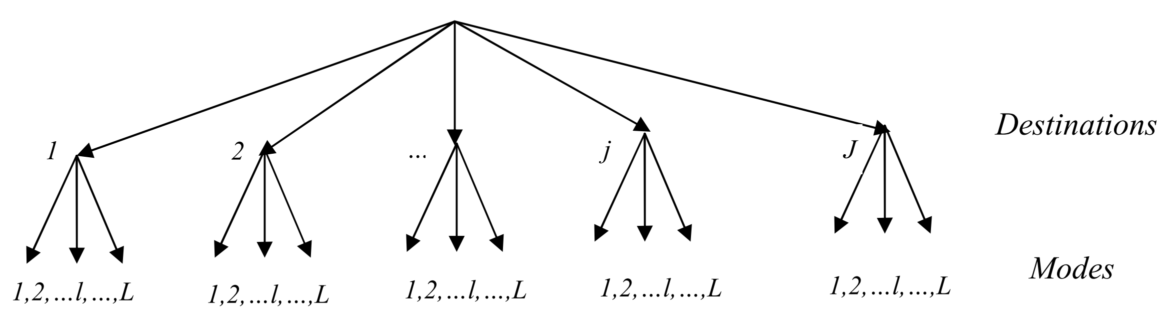
Let j be the index to represent the vacation destination alternatives available to the household, l be the index to represent the travel mode alternatives, and jl be the index to represent a vacation destination and travel mode combination. Let the U.S be divided into J number of destination alternatives ($j = 1,2,3,\ldots J$) which can be reached by any of the L number of travel modes ($l =$

[7] In the current empirical context (of vacation choices), travel times and travel costs to a destination can vary significantly from one individual to another, depending on the residential location of the individuals as well as the mode chosen for travel. Thus, we cannot make the same assumptions as in Parizat and Sachar (2010) that the fixed costs are same for all individuals.

1,2,…,L). Let $\mathbf{t} = (t_1, t_2, .., t_j, .., t_J)$ be the vector of vacation time allocations by the household to each of the destination alternatives *j* (*j = 1,2,3,…,J*). Considering that one can travel to a destination by any of the *L* available modes, one can expand each element t_j of $\mathbf{t}$ as a sub-vector $(t_{j1}, t_{j2}, ..., t_{jL})$ representing the vacation time allocation to destination *j* reached by each of the available *L* travel modes. Thus, one can expand $\mathbf{t}$ as
$\mathbf{t} = ((t_{11}, t_{12}, .., t_{1L}), (t_{21}, .., t_{2L}), ..., (t_{j1}, ..t_{jl}, .., t_{jL}), .., (t_{J1}, .., t_{JL}))$, a vector of vacation time allocations by the household to each of the destination alternatives *j* reached by each of the travel modes *l*.

Over the time frame of a year, a household may choose to visit none, one, or more destinations (although not necessarily all destinations, due to time and money constraints). Thus, one can expect the data to exhibit *imperfect substitution* (hence, *multiple discreteness*) among destination choice alternatives. For the chosen destinations, however, households are observed to travel by a single mode of travel regardless of the number of times they visited the destination. Thus, if a destination *j* is visited, the entire time t_j allocated for the destination would be allocated to only one element in the time-allocation sub-vector $(t_{j1}, ..t_{jl}, .., t_{jL})$ for that destination while all other elements would be zero, exhibiting *perfect substitution* (hence, *single discreteness*) among mode choice alternatives. If the destination is not visited, then all elements of the corresponding time-allocation sub-vector would be zero. Let l_j be the index to denote the chosen mode of travel to a chosen destination *j*. Then, if a household visits the first *M* of the *J* destinations over a year, the time allocations to these chosen destinations by the corresponding chosen modes of travel can be represented as $(t_{1l_1}, t_{2l_2}, .., t_{jl_j}, .., t_{Ml_M})$, where t_{jl_j} is the time allocated to a chosen destination *j* by the corresponding chosen travel mode l_j. Alternatively, $\mathbf{t}$ can be expressed with each of the first *M* sub-vectors having only one positive element (for the corresponding chosen destination-mode combination) and all other zeros as: $\left((0,..,t_{1l_1},..,0),(0,..,t_{2l_2},..,0),..,(0,..,t_{jl_j},..,0),..,(0,..,t_{Ml_M},..,0),(0,..,0,..,0),..,(0,..,0,..,0)\right)$.

Pictorially, we represent the destination and mode choice alternatives available to a household over the time frame of a year as follows, where the upper level tree represents the vacation destination choice alternatives with multiple discreteness and the lower level trees represent the mode choice alternatives available to each destination with single discreteness:



3.2 Model Formulation for Choice Situations with Imperfect Substitutes Only

In this section, we outline the time- and money-constrained model formulation for choice situations with alternatives that are imperfect substitutes. To suit the current empirical context, formulation is laid out for households' choice of vacation destinations over a year. Perfect substitutes are not considered temporarily. The subsequent section builds the formulation to consider both imperfect and perfect substitutes (i.e., both destination choices and mode choices).

3.2.1 Household's Constrained Utility Maximization Problem Formulation

To model a household's vacation destination choices over an annum, consider the following utility function (the subscript for the household is suppressed for simplicity):

$$U(\mathbf{t}, t_0, e_0) = \sum_{j=1}^{J} \psi_j \gamma_j \ln\left((t_j / \gamma_j) + 1\right) + (\psi_0 / \alpha) t_0^{\alpha} + (\phi_0 / \rho) e_0^{\rho} \qquad (1)$$

In the above utility function, the first term represents the utility accrued due to vacation. Specifically, the term $\psi_j \gamma_j \ln\left((t_j / \gamma_j) + 1\right)$ is a sub-utility function representing the utility accrued due to spending t_j amount of time to a vacation destination *j*. Further, utility is assumed to be additively separable in that the total utility from vacation over the time frame of a year is the sum of the utility accrued from the time spent at all the vacation destinations *j* (= *1, 2, …, J*) over the year.

The second and third terms in Equation (1), $(\psi_0 / \alpha) t_0^{\alpha}$ and $(\phi_0 / \rho) e_0^{\rho}$ complete the utility function to form an incomplete demand system with the time- and money-specific Hicksian composite outside goods, respectively. Specifically, t_0 is the Hicksian composite outside good for time representing all the non-vacation time in a year (i.e., 365 days – annual number of days spent on vacation) and e_0 is the Hicksian composite outside good for money (i.e., income – annual expenditure on vacation). The presence of these terms recognizes that neither all the time available to a household (i.e., an entire year) nor all the money (i.e., annual income) is spent completely on vacation. Thus, both the outside goods are assumed to be "essential" with some positive consumption by all households.

ψ_j is the baseline marginal utility parameter (i.e., marginal utility at zero time allocation) for the vacation destination *j*. γ_j is a translation parameter that allows the possibility of a corner solution (i.e., zero consumption) for vacation destination *j*, and accommodates differential satiation effects (i.e., diminishing marginal utility with increasing time allocation) across different destinations. The terms ψ_0 and ϕ_0 represent the baseline marginal utility parameters, while α and ρ are the satiation parameters for the two Hicksian composite outside goods t_0 and e_0, respectively.

Households are assumed to allocate the annual time (*T*) and income (*E*) available to them to maximize the utility in Equation (1) subject to the following two constraints:

$$\sum_{j=1}^{J} q_j t_j + t_0 = T \qquad (2)$$

$$\sum_{j=1}^{J} p_j t_j + e_0 = E \qquad (3)$$

These equations represent the time and money constraints, respectively, with q_j and p_j representing the time-prices and money-prices, respectively, of spending unit time at a vacation destination *j*. As can be observed from the two constraints, the Hicksian composite outside good for time (t_0) is assumed to have unit time-price and zero money-price (i.e., it doesn't appear in the time constraint), while the money-specific outside good (e_0) has unit money-price and zero time-price (i.e., it doesn't appear in the money constraint).

3.2.1 KT Conditions for Constrained Utility Maximization

To solve the above described utility maximization problem, one can form a Lagrangian function as:

$$L = U(\mathbf{t}, t_0, e_0) + \lambda\left(\sum_{j=1}^{J} q_j t_j + t_0 - T\right) + \mu\left(\sum_{j=1}^{J} p_j t_j + e_0 - E\right) \tag{4}$$

where λ and μ are the Lagrangian multipliers for the time and budget constraints, respectively. Subsequently, one can employ the following Kuhn-Tucker (KT) first-order conditions of optimality:

$$\left.\begin{aligned} &\frac{\partial L}{\partial t_0^*} = 0 \;\text{ since }\; t_0^* > 0 \\ &\frac{\partial L}{\partial e_0^*} = 0 \;\text{ since }\; e_0^* > 0 \\ &\frac{\partial L}{\partial t_j^*} = 0 \;\text{ if }\; t_j^* > 0 \\ &\frac{\partial L}{\partial t_j^*} < 0 \;\text{ if }\; t_j^* = 0 \end{aligned}\right\} \tag{5}$$

The optimal time allocations and expenditures satisfy the KT conditions above and the time and money constraints in Equations (2) and (3). The first two conditions above result in the following Lagrangian multipliers: $\lambda = \psi_0 t_0^{*\alpha-1}$ and $\mu = \phi_0 e_0^{*\rho-1}$, representing the marginal utility of time and money, respectively. Similarly, the KT conditions for the inside goods j (= *1, 2,…, J*) can be expressed as below:

$$\begin{aligned} &\psi_j\left((t_j^*/\gamma_j)+1\right)^{-1} - \lambda q_j - \mu p_j = 0 \;\text{ if }\; t_j^* > 0;\; j = 1,2,...,J \\ &\psi_j\left((t_j^*/\gamma_j)+1\right)^{-1} - \lambda q_j - \mu p_j < 0 \;\text{ if }\; t_j^* = 0;\; j = 1,2,...,J \end{aligned} \tag{6}$$

or, after algebraic arrangements,

$$\begin{aligned} &\frac{\psi_j\left((t_j^*/\gamma_j)+1\right)^{-1}}{\lambda q_j + \mu p_j} = 1 \;\text{ if }\; t_j^* > 0\;;\; j = 1,2,...,J \\ &\frac{\psi_j\left((t_j^*/\gamma_j)+1\right)^{-1}}{\lambda q_j + \mu p_j} < 1 \;\text{ if }\; t_j^* = 0;\; j = 1,2,...,J \end{aligned} \tag{7}$$

or, after taking logarithms and substituting $\psi_0 t_0^{\alpha-1}$ and $\phi_0 e_0^{\rho-1}$ for the Lagrangian multipliers λ and μ, respectively,

$$\ln\psi_j - \ln\left((t_j^*/\gamma_j)+1\right) - \ln\left(\psi_0 t_0^{*\alpha-1} q_j + \phi_0 e_0^{*\rho-1} p_j\right) = 0 \text{ if } t_j^* > 0;\; j = 1,2,...,J$$
$$\ln\psi_j - \ln\left((t_j^*/\gamma_j)+1\right) - \ln\left(\psi_0 t_0^{*\alpha-1} q_j + \phi_0 e_0^{*\rho-1} p_j\right) < 0 \text{ if } t_j^* = 0;\; j = 1,2,...,J \quad (8)$$

Note that the above KT conditions are based on the gradients of the Lagrangian function with respect to optimal consumptions $(t_0^*, e_0^*, t_1^*, t_2^*, \dots, t_J^*)$, not expenditures. That is, the consumer's utility maximization problem is solved for optimal consumptions, not optimal expenditures. While the KT conditions from solving for optimal expenditures can be rearranged algebraically to obtain the same KT conditions as above, the problems arises when deriving the probability expressions for optimal expenditures. Specifically, the analyst will end up with invalid probability density functions when (s)he derives the probability expressions for optimal expenditures. Since the consumer derives utility from (and is assumed to optimize the total utility of) consumption, it is important to solve for optimal consumptions and derive the probability expressions for optimal consumptions, not for optimal expenditures. This is true even in the context of the MDCEV model derived by Bhat (2008) with a single budget constraint.

3.2.2 Econometric Structure

To complete the model specification, define the baseline marginal utility for inside goods, ψ_j as a function of observed and unobserved household characteristics and destination (*j*) characteristics as:

$$\psi_j = \exp(\mathbf{\Delta}'\mathbf{z}_j + \varepsilon_j) \quad (9)$$

where,

$\mathbf{z}_j$ is a vector of destination characteristics influencing the destination choices, and their interactions with household characteristics, and $\mathbf{\Delta}$ is a corresponding vector of parameters;

ε_j is a destination-specific random term to accommodate the unobserved factors influencing the choice of destination *j*.

Similarly, define ψ_0 as $\exp(\mathbf{v}'\mathbf{v_0} + \varepsilon_0)$ and ϕ_0 as $\exp(\mathbf{w}'\mathbf{w_0} + \xi_0)$, where, $\mathbf{v_0}$ is a vector of household characteristics influencing annual time allocation for vacation; $\mathbf{w_0}$ is a vector of household characteristics influencing annual expenditure allocation for vacation ($\mathbf{v}'$ and $\mathbf{w}'$ are corresponding parameter vectors); and ε_0 and ξ_0 are random terms to accommodate unobserved factors influencing the total annual vacation time allocation and total annual expenditure allocation respectively.

Now, the KT conditions in Equation (8) can be written as:

$$\varepsilon_j = -\mathbf{\Delta}'\mathbf{z}_j + \ln\left((t_j^*/\gamma_j)+1\right) + \ln\left(\psi_0 t_0^{*\alpha-1} q_j + \phi_0 e_0^{*\rho-1} p_j\right) \text{ if } t_j^* > 0;\; j = 1,2,...,J$$
$$\varepsilon_j < -\mathbf{\Delta}'\mathbf{z}_j + \ln\left((t_j^*/\gamma_j)+1\right) + \ln\left(\psi_0 t_0^{*\alpha-1} q_j + \phi_0 e_0^{*\rho-1} p_j\right) \text{ if } t_j^* = 0;\; j = 1,2,...,J \quad (10)$$

Without loss of generality, say that the household chooses to visit the first *M* of the available vacation destinations *j* (= *1,2,…, M*) over a year. Assume that the random terms ε_j (*j* = *1,2,…,J*), and ε_0 and ξ_0 are independent and identical type-1 extreme value distributed with a scale parameter σ. Then, conditional upon the error terms ε_0 and ξ_0, the conditional probability expression that the household allocates T-t_o amount of time for vacation (or t_0^* amount of time for the time-specific outside good), E- e_0 amount of money for vacation (or e_0^* amount for the money-specific outside good), and $(t_1^*, .., t_M^*, 0, \ldots, 0)$ amounts of time to each of the J vacation destinations is:

$$P\left\{\left(t_0^*, e_0^*, t_1^*, t_2^* ..., t_M^*, 0, ..., 0\right) \mid \varepsilon_0, \xi_0\right\}$$
$$= \left\{\prod_{j=1}^{M} \frac{1}{\sigma} g\left(\frac{W_j \mid \varepsilon_0, \xi_0}{\sigma}\right) \left|J / \varepsilon_0, \xi_0\right|\right\} \times \left\{\prod_{j=M+1}^{J} G\left(\frac{W_j \mid \varepsilon_0, \xi_0}{\sigma}\right)\right\} \quad (11)$$

In the above expression, the first term corresponds to all chosen destination alternatives, while the second term corresponds to all non-chosen destination alternatives. Further, the terms in the expression are as below:

$W_j \mid \varepsilon_0, \xi_0 = -\mathbf{\Delta}'\mathbf{z}_j + \ln\left((t_j^* / \gamma_j) + 1\right) + \ln\left(\psi_0 t_0^{*\alpha-1} q_j + \phi_0 e_0^{*\rho-1} p_j\right),$

g and G are standard Gumbel probability density and cumulative density functions, and

$\left|J / \varepsilon_0, \xi_0\right|$ is the determinant of the Jacobian matrix for transformation of variables from the random error terms to consumption (i.e., time allocation) variables for the chosen destination-mode alternatives. There is no compact form for the Jacobian, but the ih^{th} element of the Jacobian matrix can be computed as follows:

$$J_{ih} = \partial W_i / \partial t_h^* \quad (i = 1, 2, ..., M)$$
$$= \frac{\psi_0 t_0^{*\alpha-2}(1-\alpha) q_i q_h + \phi_0 e_0^{*\rho-2}(1-\rho) p_i p_h}{\left[\psi_0 t_0^{*\alpha-1} q_h + \phi_0 e_0^{*\rho-1} p_h\right]} + \frac{Z_{ih}}{(t_i^* + \gamma_i)} \quad (12)$$

where,

i(= 1,2,...,M) and *h(=1,2,...,M)* are the row and column indices, respectively for each chosen vacation destination alternative;

$Z_{ih} = 1$ if $i = h$, and 0 otherwise;

q_i, q_h are the time-prices for the destinations *i* and *h*, respectively; and

p_i, p_h are the money-prices for the destinations *i* and *h*, respectively.

The unconditional probability for the expression in Equation (11) is given by integrating it over the distributions of the random terms ε_0 and ξ_0, as:

$$P\left\{\left(t_0^*, e_0^*, t_1^*, t_2^*, ..., t_M^*, 0, ..., 0\right)\right\} =$$
$$\int_{\varepsilon_0=-\infty}^{\infty} \int_{\xi_0=-\infty}^{\infty} \prod_{j=1}^{M} \frac{1}{\sigma} g\left(\frac{W_j \mid \varepsilon_0, \xi_0}{\sigma}\right) \times \left|J / \varepsilon_0, \xi_0\right| \times \prod_{j=M+1}^{J} G\left(\frac{W_j \mid \varepsilon_0, \xi_0}{\sigma}\right) \times \frac{1}{\sigma^2} g\left(\frac{\varepsilon_0}{\sigma}\right) g\left(\frac{\xi_0}{\sigma}\right) d\varepsilon_0 d\xi_0$$

(13)

The above double integral does not have a closed form, but can be evaluated either using either simulation or quadrature methods.

The reader will note that the above probability expression is similar to what Castro et al. (2012) proposed for the case of MDCEV models with multiple linear constraints with constraint-specific Hicksian composite outside goods in that it is a double integral. In a general case with *R* number of linear constraints, the above approach leads to a probability expression of *R*-dimensional integral (Castro et al., 2012). In the following discussion, we highlight certain identification issues specific to empirical contexts with constraint-specific Hicksian composite essential outside goods (i.e., outside goods t_0 and e_0, each of which are specific to the time and money constraints, respectively) that warrant a different formulation leading to a different model structure.

3.2.3 Identification Issues

First, as discussed in Castro et al. (2012), it is difficult to estimate alternative-specific parameters of explanatory variables separately on the baseline utility parameters for inside goods (i.e., ψ_j) as well as those for outside goods (i.e., ψ_0 and ϕ_0). Thus, for each explanatory variable, the corresponding coefficients in the baseline utility parameters for at least as many goods as the number of budget constraints need to be normalized (to zero). In the current empirical context, the baseline utility parameters of the two outside goods are the natural candidates for such normalization. This is because, once the consumptions of the inside goods are known, the consumptions of the constraint-specific outside goods are automatically implied from the corresponding budget constraints. In other words, only the inside good consumptions are the decision variables of any household's utility maximization problem. The "leftovers" for the constraint-specific outside goods (i.e., t_0 and e_0) can be obtained from the budget constraint identities in Equations (2) and (3).

Second, for the same reasons just discussed, it is difficult to identify the baseline utility parameters ψ_0 and ϕ_0 separately. Thus, another important normalization is to set the baseline utility parameters of all the constraint-specific outside goods as equal. In the current empirical context, $\psi_0 = \phi_0$. While the former normalization has been discussed in Castro et al. (2012) and is applicable regardless of the presence/absence and the nature of the outside goods, the latter normalization is equally important in situations with constraint-specific Hicksian composite outside goods.[8] Neglecting this normalization can potentially lead to severe estimation problems. To better explain this, Figure 1 below illustrates the identification issues arising in absence of such normalization for the following utility expression with one inside good t_{in} and two constraint-specific outside goods t_0 and e_0:

$$U(t_{in}, t_0, e_0) = \psi_{in}\gamma_{in} \ln\left((t_{in} / \gamma_{in}) + 1\right) + (\psi_0 / \alpha)t_0^{\alpha} + (\phi_0 / \rho)e_0^{\rho} \quad (14)$$

subject to the constraints: $t_{in} + t_0 = T$ and $p_{in}t_{in} + e_0 = E$, where *T* is the time budget set to 365, *E* is the money budget set to 100, p_{in} is the price for the inside good set to 0.2, ψ_{in} is the baseline utility parameter for the inside good set to 5, γ_{in} is the translation parameter for the inside good set to 0.64. The figure shows the profiles for the total utility $U\left(t_{in}, t_0, e_0\right)$ as a function of the consumption of

[8] These two normalizations are overlapping but not equivalent to one another. For example, the former normalization ensures that the deterministic components of ψ_0 and ϕ_0 are equal but the random terms ε_0 and ξ_0 are still different, which leads to the double integral for the likelihood function as in Castro et al. (2012). Similarly, the latter normalization does not ensure that the coefficients of explanatory variables on the baseline utilities of the outside goods have to be normalized to zero.

inside good (t_{in})[9] for four different cases with distinct sets of values for the baseline utility parameters of the outside goods, ψ_0 and ϕ_0. The α and ρ parameters are both fixed to zero in the first three cases, while they take the values of 0.1 and 0.4, respectively in the fourth case. All other parameters (T, E, p_{in}, ψ_{in}, γ_{in}) are the same in all four cases. As can be observed from the first three cases, the utility curves for all cases follow each other closely, reflecting that different sets of baseline utility parameters for the outside goods can result in a similar utility profile. In addition, the optimal consumption values for the inside good (i.e., the t_{in} values where the utility curves peak) are very close to each other, if not exactly the same, in all three cases. Note that $\psi_0 = \phi_0$ in case 3, suggesting that, keeping all else same, a set of different ψ_0 and ϕ_0 parameters can be replaced with a single value while retaining a similar utility profile and optimal consumptions. These results point to the difficulty of identifying ψ_0 and ϕ_0 separately and suggest the need for setting $\psi_0 = \phi_0$.[10]

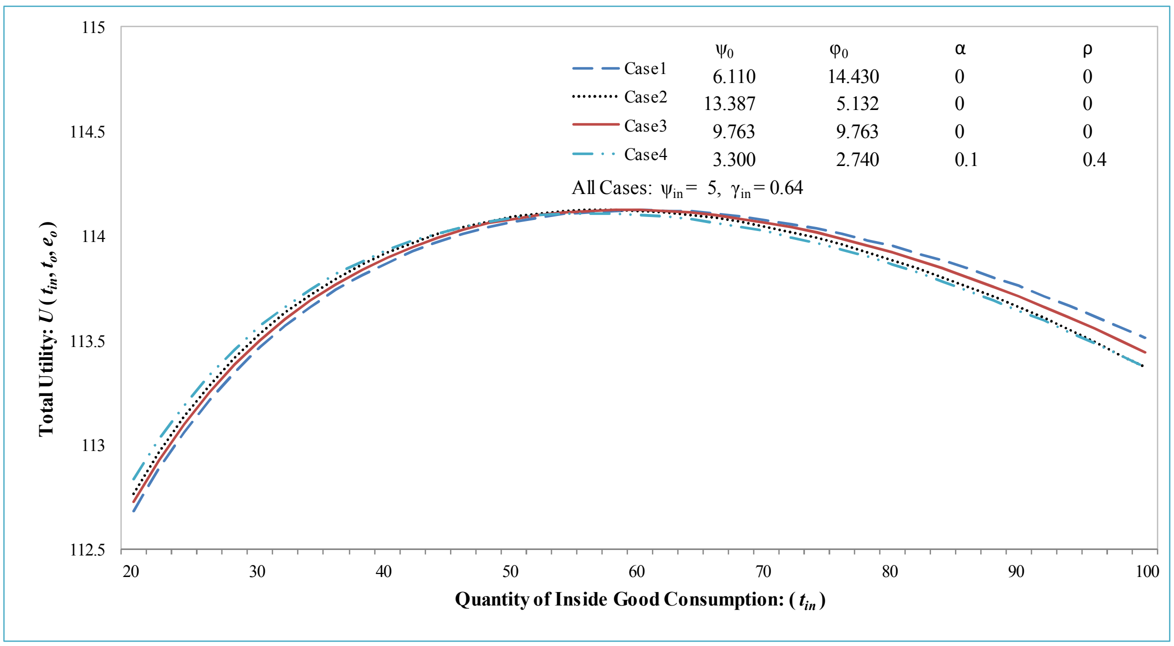


Figure 1. Total Utility as a Function of Inside Good Consumption for different values of ψ_0, ϕ_0, α, and ρ (keeping all else same) [11]

[9] For a given consumption t_{in} of the inside good, the outside good consumptions are implied automatically from the budget constraints.

[10] Additional exercises with different sets of parameters in the above-discussed 3-good utility function suggested the same identification issues. Further experiments with 4-good utility functions (with two-dimensional utility surfaces as a function of the consumptions of the two inside goods) also suggested that multiple sets of ψ_0 and ϕ_0 values can lead to similar utility profiles and optimal consumptions.

[11] The figure presents a "zoom-in" of the utility curves for inside good consumption values ranging from 20 to 100. Utility profiles over the entire consumption range for the inside good (i.e., 0 to 365) make it very difficult to distinguish the four curves from one another.

Third, the satiation parameters for the outside goods, α and ρ are also difficult to estimate, because the quantities of the Hickisan composite outside goods t_0 and e_0 are essentially determined based on the consumption quantities of the inside goods and the time and money budget constraint identities. Thus, the analyst may have to impose normalizations on the satiation parameters of the constraint-specific outside goods, such as $\alpha = 0$ and $\rho = 0$, for parameter identification and stability in estimation. This can be observed in Figure 1 from the utility profile in case 4 (with $\alpha = 0.1$ and $\rho = 0.4$), which is empirically indistinguishable from the other utility profiles (with α and ρ set to zero). Further, the utility profile with the most parsimonious specification is in case 3, with $\psi_0 = \phi_0$, $\alpha = 0$ and $\rho = 0$. This suggests that, keeping all else same, a utility profile with different values for ψ_0 and ϕ_0 and non-zero values for α and ρ can be replaced with another utility profile with a single value for the ψ_0 and ϕ_0 parameters and zero for α and ρ.

Finally, as discussed in Castro et al. (2012) the scale parameter of the random error terms σ can always be identified and estimated (at least in theory) as long as price variation exists at least either in time-prices or in money-prices.

3.2.4 Revised Model Structure for the Imperfect Substitutes Case

Considering the above-discussed identification issues and corresponding normalizations, we propose the following revised utility form:

$$U(\mathbf{t}, t_0, e_0) = \sum_{j=1}^{J} \psi_j \gamma_j \ln\left((t_j / \gamma_j) + 1\right) + (\varphi_0 / \alpha) t_0^{\alpha} + (\varphi_0 / \rho) e_0^{\rho}, \tag{15}$$

Note that the baseline utility parameter φ_0 in the revised utility form is same for both the outside goods, unlike the different parameters ψ_0 and ϕ_0 in Equation (1). Further, to recognize that the coefficients of explanatory variables in the baseline utility parameters cannot be identified separately for the outside goods and all inside goods, we express φ_0 as only a function of a random error term as: $\varphi_0 = \exp(\varsigma_0)$. That is, the coefficients of explanatory variables in the outside good baseline utility components are normalized to zero. Note that the role of the random error term ς_0 is not to directly capture the influence of unobserved factors on t_0 and e_0, but to induce correlations among the utility contributions of all inside goods due to common unobserved factors affecting the choice of inside goods. Similar to how the influence of observed variables cannot be estimated on the outside goods (i.e., it is difficult to identify the coefficients) but captured indirectly via their influence on the consumption of the inside goods and the budget constraints, the role of unobserved factors is also captured via the same mechanism.[12]

The proposed utility form in Equation (15) retains the satiation parameters for the outside goods α and ρ, despite the discussions in the previous section that they cannot be estimated. The

[12] This is a reason why several model specifications in the environmental economics literature (as well as in Satomura et al., 2011) specify no stochasticity on the utility contribution of outside good(s). While this normalization in not theoretically inappropriate, it precludes the possibility of an easy way to induce correlation between the utility contributions of the inside goods. Such correlation is useful for inducing greater competition among the inside goods. For example, it is likely that increase in price of an inside good will induce a greater shift in the consumption to another inside good than to the outside good(s). Thus, we prefer the specification with stochasticity on the outside good(s) to that without stochasticity.

subsequent derivations also include the terms α and ρ. But one can simply normalize these parameters to zero for identification. The utility contributions of the outside good terms then simplify to $\varphi_0 \ln t_0$ and $\varphi_0 \ln e_0$, instead of $(\varphi_0 / \alpha)t_0^{\alpha}$ and $(\varphi_0 / \alpha)t_0^{\alpha}$. Similarly, all the subsequent derivations can be simplified by setting both α and ρ to zero.

With the reformulated utility form in Equation (15), the KT conditions from Equation (10) can be expressed as:

$$\begin{aligned} \varepsilon_j &= V_j + \varsigma_0 \ \text{ if } \ t_j^* > 0 \\ \varepsilon_j &< V_j + \varsigma_0 \ \text{ if } \ t_j^* = 0 \end{aligned} \tag{16}$$

where, $V_j = -\boldsymbol{\Delta}'\mathbf{z}_j + \ln\left((t_j^* / \gamma_j) + 1\right) + \ln\left(t_0^{*\alpha-1} q_j + e_0^{*\rho-1} p_j\right)$.

The above KT conditions result in the following conditional probability given the error term ς_0:

$$P\left\{\left(t_0^*, e_0^*, t_1^*, t_2^*, ..., t_M^*, 0, ..., 0\right) | \varsigma_0\right\} = \left\{\prod_{j=1}^{M} \frac{1}{\sigma} g\left(\frac{V_j + \varsigma_0}{\sigma}\right) |J|\right\} \times \left\{\prod_{j=M+1}^{J} G\left(\frac{V_i + \varsigma_0}{\sigma}\right)\right\} \tag{17}$$

where,

$|J|$ is the determinant of the Jacobian matrix whose ih^{th} element can be computed as:

$$\begin{aligned} J_{ih} &= \partial(V_i + \varsigma_0) / \partial t_h^* \quad (i = 1, 2, ..., M) \\ &= \frac{t_0^{*\alpha-2}(1-\alpha) q_i q_h + e_0^{*\rho-2}(1-\rho) p_i p_h}{\left[t_0^{*\alpha-1} q_h + e_0^{*\rho-1} p_h\right]} + \frac{Z_{ih}}{(t_i^* + \gamma_i)} \end{aligned} \tag{18}$$

and other terms are as described earlier in the context of Equation (12). Note that the above Jacobian element is slightly different (and simpler) than that in Equation (12) as the outside good-specific baseline utility parameters are normalized to be equal and thus drop out of the above expression.

The unconditional probability for the expression in Equation (17) can be obtained by integrating it over the distribution of ς_0, which leads to a closed-form probability expression similar to the MDCEV probability expression Bhat (2005) derived for the case with a single linear constraint:[13]

$$P\left(t_0^*, e_0^*, t_1^*, t_2^*, ..., t_M^*, 0, ..., 0\right) = \frac{1}{\sigma^M} |J| \frac{\left\{\prod_{j=1}^{M} e^{V_j/\sigma}\right\} M!}{\left\{1 + \sum_{i=1}^{J} e^{V_i/\sigma}\right\}^{M+1}} \tag{19}$$

As can be observed from the above expression, imposing a normalization that the baseline utility parameters of both the outside goods ψ_0 and ϕ_0 are equal (to φ_0) helps not only in parameter identification but also in obtaining closed-form probability expressions that are much easier to

[13] On the other hand, if the error term on the outside goods ς_0 is assumed to collapse on zero (i.e., no error term exists) and if α_0 , ρ_0, and $\gamma_j \forall j = 1, 2, ..., K$ are equal to zero, Equation (17) results in Satomura et al.'s (2011) model.

evaluate (compared to integrals with no closed form as in Equation 12). It is worth noting here that the closed form of the probability expression remains regardless of the number of linear budget constraints with constraint-specific Hicksian composite outside goods. Of course, these advantages are applicable only for cases with constraint-specific Hicksian composite outside goods. This is because the normalization that the baseline utility parameters of the outside goods ought to be equal is applicable only for the case with constraint-specific outside goods. Whether the same normalization is necessary (or at least innocuous) for situations with outside goods that are not constraint-specific outside goods and situations without outside goods is a question beyond the scope of this paper. See Castro et al. (2012) for model structures without imposing this normalization for situations without constraint-specific outside goods and situations without outside goods.

It is worth noting here that the above formulation simplifies to the standard MDCEV model with a single linear budget constraint (as in Bhat, 2008) when the outside good quantity corresponding to one of the constraints is infinity. That is, when an infinite amount of one of the sources (e.g., time) is available, then the corresponding budget constraint becomes unnecessary and the entire model formulation, including the KT conditions and the probability expression along with its Jacobian, collapses to the MDCEV model with a single budget constraint.

3.3 Model Structure for Choice Situations with Imperfect and Perfect Substitutes

The above-discussed model formulations are for MDC choice situations with choice alternatives that are imperfect substitutes. However, in many situations, choice alternatives can comprise a combination of imperfect substitutes and perfect substitutes. For example, in the current empirical context, the vacation destination choices are imperfect substitutes while the travel mode choice alternatives for each destination are perfect substitutes. Thus, in this section, we extend the above time- and money-constrained MDC model formulations for annual vacation destination choices to include perfect substitutes (i.e., mode choices for each of the chosen destinations) in a simultaneous fashion. To be sure, Bhat et al. (2006) derived a joint model formulation that considers both imperfect substitutes (i.e., multiple discrete-continuous choices) and perfect substitutes (i.e., single discrete choices) with a single linear budget constraint. However, their formulation can be used only for situations when the price variation across the choice alternatives is limited to imperfect substitutes, but not when prices vary across the perfect substitutes. In the current empirical context, the price variation across perfect substitutes cannot be ignored because travel costs vary significantly between the different modes of travel to a destination.

To jointly model a household's vacation destination and mode choices over an annum, consider the following utility function:

$$U(\mathbf{t}, t_0, e_0) = \sum_{j=1}^{J} \sum_{l=1}^{L} \psi_{jl} \gamma_j \ln\left((t_{jl} / \gamma_j) + 1 \right) + (\varphi_0 / \alpha) t_0^{\alpha} + (\varphi_0 / \rho) e_0^{\rho} \qquad (20)$$

The first term in the above utility function represents the utility accrued due to vacation. Specifically, $\psi_{jl} \gamma_j \ln\left((t_{jl} / \gamma_j) + 1 \right)$ is the sub-utility function for destination-mode combination *jl* representing the utility accrued from allocating t_{jl} amount of time for a vacation destination *j* reached via travel mode *l*. ψ_{jl} is the corresponding baseline marginal utility parameter, expressed as: $\psi_{jl} = \exp(\mathbf{\Delta}' \mathbf{z}_j + \mathbf{\beta}' \mathbf{x}_{jl} + \varepsilon_{jl})$, where $\mathbf{z}_j$ is a vector of destination characteristics and their interactions with household characteristics influencing destination choices, and $\mathbf{\Delta}$ is a corresponding vector of parameters; $\mathbf{x}_{jl}$ is a vector of mode-specific characteristics and their interactions with household characteristics influencing mode choice for the destination j, and $\boldsymbol{\beta}$ is a corresponding vector of parameters; ε_{jl} is a destination-mode specific random error term to accommodate the unobserved factors influencing the choice of destination j and mode l. γ_j is a destination-specific parameter to allow corner solutions and to accommodate differential satiation effects across vacation destinations. γ_j can be expressed as a function of destination characteristics as: $\gamma_j = \exp(\nabla'\mathbf{y}_j)$.[14]

The second and third terms in Equation (20), $(\varphi_0/\alpha)t_0^{\alpha}$ and $(\varphi_0/\rho)e_0^{\rho}$ complete the utility function to form an incomplete demand system with the time- and money-specific Hicksian composite outside goods, t_0 and e_0, respectively. Note that, based on the discussion in Section 3.2.3, for identification purposes, the baseline utility parameter is specified as the same (φ_0) for both the constraint-specific outside goods. Further, since the coefficients on the explanatory variables cannot be identified together for inside goods and outside goods, φ_0 is defined as a function of only a random error term as: $\exp(\varsigma_0)$ (i.e., for any explanatory variable in the baseline utility parameters, the outside good-specific coefficients are normalized to zero). α and ρ are the satiation parameters for the two Hicksian composite outside goods.

Specifying the joint cumulative distribution F of the random error terms $\left(\varsigma_0,(\varepsilon_{11},..,\varepsilon_{1l},..,\varepsilon_{1L}),...,(\varepsilon_{j1},..,\varepsilon_{jl},..,\varepsilon_{jL}),,...,(\varepsilon_{J1},..,\varepsilon_{Jl},..,\varepsilon_{JL}),\right)$ completes the utility specification. In this paper, we assume that the random error terms have a nested extreme value distributed error term structure with the following joint cumulative distribution:

$$F\left(\varsigma_0,(\varepsilon_{11},..,\varepsilon_{1l},..,\varepsilon_{1L}),..,(\varepsilon_{j1},..,\varepsilon_{jl},..,\varepsilon_{jL}),..,(\varepsilon_{J1},..,\varepsilon_{Jl},..,\varepsilon_{JL})\right) = \left[\exp\left\{-\exp\left(\frac{-\varsigma_0}{\sigma}\right)\right\}\right] \times \prod_{j=1}^{J} \exp\left[-\left\{\sum_{l=1}^{L} \exp\left(\frac{-\varepsilon_{jl}}{\sigma\theta}\right)\right\}^{\theta}\right] \quad (21)$$

In the above cumulative distribution function, the error terms of all modal alternatives for a specific destination j, $(\varepsilon_{j1},..,\varepsilon_{jl},..,\varepsilon_{jL})$ are grouped into a nest, with a (dis)similarity parameter θ introduced to capture correlations among the random utility contributions of all destination-mode combination alternatives jl sharing a destination j. σ is a scale parameter that can be estimated due to the variation in prices (either in money-prices, or time-prices, or both) across the choice alternatives. Note that the error term on the outside goods, ς_0 is in its own nest with no other alternative in it.

Households are assumed to allocate the annual time (T) and income (E) available to them to maximize the utility in Equation (20) subject to the following, time and money constraints:

$$\sum_{j=1}^{K}\sum_{l=1}^{L} q_{jl}t_{jl} + t_0 = T \quad (22)$$

[14] γ_j is not specified to vary by mode because there is no reason for the satiation effects to differ by mode of travel. Besides, constraining the γ_j parameter to be the same across the perfect substitutes facilitates the model formulation.

$$\sum_{j=1}^{K}\sum_{l=1}^{L} p_{jl}t_{jl} + e_0 = E \tag{23}$$

In the above constraints, q_{jl} and p_{jl} represent the time-prices and money-prices, respectively, of spending unit time at a vacation destination *j* traveled by mode *l*. These prices comprise two components – (a) destination prices that do not depend on the mode of travel (e.g., lodging prices, dining prices, entertainment prices, etc.) and (b) travel prices that depend on the mode of travel, with the latter leading to the difference between the prices by different modes of travel (*l*) for a same destination *j*.[15] As explained earlier, the Hicksian composite outside goods have unit prices for (i.e., appear with unit prices in) their own constraint and zero prices for (i.e., it do not appear in) the other constraint.

To solve the above constrained utility maximization problem, one can form a Largrangian function as below:

$$L = U\left(\mathbf{t}, t_0, e_0\right) + \lambda\left(T - \sum_{j=1}^{K}\sum_{l=1}^{L} q_{jl}t_{jl} - t_0\right) + \mu\left(E - \sum_{j=1}^{K}\sum_{l=1}^{L} p_{jl}t_{jl} - e_0\right) \tag{24}$$

As described in Section 3.2.1, applying the KT optimality conditions for the outside goods results in the following Lagrangian multipliers: $\lambda = \psi_0 t_0^{*\alpha-1}$ and $\mu = \phi_0 e_0^{*\rho-1}$, representing the marginal utility of time and money, respectively. Further, the following KT conditions can be derived for the inside goods:

$$\begin{aligned} &V_{jl} - \varsigma_0 + \varepsilon_{jl} = 0 \quad if \;\; t_{jl}^* > 0, \; j = 1,..,J; l = 1,..,L \\ &V_{jl} - \varsigma_0 + \varepsilon_{jl} < 0 \quad if \;\; t_{jl}^* = 0, \; j = 1,..,J; l = 1,..,L \end{aligned} \tag{25}$$

where, $V_{jl} = \boldsymbol{\Delta}'\mathbf{z}_j + \boldsymbol{\beta}'\mathbf{x}_{jl} - \ln\left(\left(t_{jl}^* / \gamma_j\right) + 1\right) - \ln\left(t_0^{*\alpha-1} q_{jl} + e_0^{*\rho-1} p_{jl}\right)$.

Since the mode choice alternatives for any chosen destination are perfect substitutes, let the chosen mode of travel for a destination *j* be denoted as l_j. Then the entire time allocated to the destination t_j^* would be allocated to the destination-mode alternative l_j while no time would be allocated to other modal alternatives for that destination. Thus, the above KT conditions can be rewritten as:

$$\begin{aligned} &\boldsymbol{\Delta}'\mathbf{z}_j + \boldsymbol{\beta}'\mathbf{x}_{jl} - \ln\left(\left(t_j^* / \gamma_j\right) + 1\right) - \ln\left(t_0^{*\alpha-1} q_{jl} + e_0^{*\rho-1} p_{jl}\right) - \varsigma_0 + \varepsilon_{jl} = 0 \quad if \;\; t_{jl}^* > 0; \; j = 1,..,J; l = 1,..,L \\ &\boldsymbol{\Delta}'\mathbf{z}_j + \boldsymbol{\beta}'\mathbf{x}_{jl} - \ln\left(t_0^{*\alpha-1} q_{jl} + e_0^{*\rho-1} p_{jl}\right) - \varsigma_0 + \varepsilon_{jl} < 0 \quad if \;\; t_{jl}^* = 0; \; j = 1,..,J; l = 1,..,L \end{aligned} \tag{26}$$

[15] As discussed earlier, it is assumed here that the travel costs for each available mode of travel to a destination can be amortized into a travel-price per unit of consumption (i.e., travel price per unit time allocation to the destination). In reality, the travel costs are *fixed* and do not vary linearly with the amount of time allocation at the destination. But incorporating such fixed costs leads to non-linear and non-smooth budget constraints making it extremely difficult to solve the consumer's utility maximization problem. Resolving this issue is beyond the scope of this paper, but a very important avenue for future research.

Note that unlike the KT conditions in (25), the left hand side of the first of the above KT conditions has the destination-level time allocation variable t_j^* instead of the destination-mode level variable t_{jl}^*. This is because the entire time allocated to the destination t_j^* will be allocated to the chosen destination-mode alternative *jl*. On the other hand, the second condition doesn't have such a term since $t_{jl}^* = 0$ for all non-chosen alternatives. Now, one can expand the above KT conditions for each travel mode *l* (*=1,2,…,L*) available to a destination *j* as below.

$$\begin{bmatrix} \boldsymbol{\Delta}'\mathbf{z}_j + \boldsymbol{\beta}'\mathbf{x}_{j1} - \ln\left(\left(t_j^* / \gamma_j\right)+1\right) - \ln\left(t_0^{*\alpha-1} q_{j1} + e_0^{*\rho-1} p_{j1}\right) - \varsigma_0 + \varepsilon_{j1} = 0 \;\; if \;\; t_{j1}^* > 0 \;\; j=1,2,...,J \\ \boldsymbol{\Delta}'\mathbf{z}_j + \boldsymbol{\beta}'\mathbf{x}_{j2} - \ln\left(\left(t_j^* / \gamma_j\right)+1\right) - \ln\left(t_0^{*\alpha-1} q_{j2} + e_0^{*\rho-1} p_{j2}\right) - \varsigma_0 + \varepsilon_{j2} = 0 \;\; if \;\; t_{j2}^* > 0 \;\; j=1,2,...,J \\ \vdots \\ \boldsymbol{\Delta}'\mathbf{z}_j + \boldsymbol{\beta}'\mathbf{x}_{jL} - \ln\left(\left(t_j^* / \gamma_j\right)+1\right) - \ln\left(t_0^{*\alpha-1} q_{jL} + e_0^{*\rho-1} p_{jL}\right) - \varsigma_0 + \varepsilon_{jL} = 0 \;\; if \;\; t_{jL}^* > 0 \;\; j=1,2,...,J \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{\Delta}'\mathbf{z}_j + \boldsymbol{\beta}'\mathbf{x}_{j1} - \ln\left(t_0^{*\alpha-1} q_{j1} + e_0^{*\rho-1} p_{j1}\right) - \varsigma_0 + \varepsilon_{j1} < 0 \;\; if \;\; t_{j1}^* = 0 \;\; j=1,2,...,J \\ \boldsymbol{\Delta}'\mathbf{z}_j + \boldsymbol{\beta}'\mathbf{x}_{j2} - \ln\left(t_0^{*\alpha-1} q_{j2} + e_0^{*\rho-1} p_{j2}\right) - \varsigma_0 + \varepsilon_{j2} < 0 \;\; if \;\; t_{j2}^* = 0 \;\; j=1,2,...,J \\ \vdots \\ \boldsymbol{\Delta}'\mathbf{z}_j + \boldsymbol{\beta}'\mathbf{x}_{jL} - \ln\left(t_0^{*\alpha-1} q_{jL} + e_0^{*\rho-1} p_{jL}\right) - \varsigma_0 + \varepsilon_{jL} < 0 \;\; if \;\; t_{jL}^* = 0 \;\; j=1,2,...,J \end{bmatrix} \quad (27)$$

As can be observed, the KT conditions are arranged into two sets – the equality conditions in the first set and the inequality conditions in the second set. If a destination *j* is chosen, only one of the equality conditions will hold for that destination (because mode choice alternatives are perfect substitutes), while *L-1* of the inequality conditions will hold. Specifically, the mode choice alternative with the maximum value of the left hand side of the first set of conditions will be chosen. On the other hand, if the destination *j* is not chosen, then none of the equality conditions but all of the inequality conditions will hold. As a result, for each destination *j*, the above KT conditions can be *reduced* to a new set of KT conditions as below:

$$\mathbf{\Delta}'\mathbf{z}_j - \ln\left(\left(t_j^* / \gamma_j\right) + 1\right) + \underset{l=1,2,\ldots,L}{Max}\begin{bmatrix} \boldsymbol{\beta}'\mathbf{x}_{j1} - \ln\left(t_0^{*^{\alpha-1}} q_{j1} + e_0^{*^{\rho-1}} p_{j1}\right) + \varepsilon_{j1} \\ \boldsymbol{\beta}'\mathbf{x}_{j2} - \ln\left(t_0^{*^{\alpha-1}} q_{j2} + e_0^{*^{\rho-1}} p_{j2}\right) + \varepsilon_{j2} \\ \vdots \\ \boldsymbol{\beta}'\mathbf{x}_{jL} - \ln\left(t_0^{*^{\alpha-1}} q_{jL} + e_0^{*^{\rho-1}} p_{jL}\right) + \varepsilon_{jL} \end{bmatrix} - \varsigma_0 = 0 \quad if \;\; t_j^* > 0; \; j = 1,2,\ldots,J$$

$$\mathbf{\Delta}'\mathbf{z}_j - \ln\left(\left(t_j^* / \gamma_j\right) + 1\right) + \underset{l=1,2,\ldots,L}{Max}\begin{bmatrix} \boldsymbol{\beta}'\mathbf{x}_{j1} - \ln\left(t_0^{*^{\alpha-1}} q_{j1} + e_0^{*^{\rho-1}} p_{j1}\right) + \varepsilon_{j1} \\ \boldsymbol{\beta}'\mathbf{x}_{j2} - \ln\left(t_0^{*^{\alpha-1}} q_{j2} + e_0^{*^{\rho-1}} p_{j2}\right) + \varepsilon_{j2} \\ \vdots \\ \boldsymbol{\beta}'\mathbf{x}_{jL} - \ln\left(t_0^{*^{\alpha-1}} q_{jL} + e_0^{*^{\rho-1}} p_{jL}\right) + \varepsilon_{jL} \end{bmatrix} - \varsigma_0 < 0 \quad if \;\; t_j^* = 0; \; j = 1,2,\ldots,J$$

(28)

Note that the above KT conditions are for the imperfect substitutes (i.e., destination choice alternatives). Hence, the time allocation variables in the above equations do not have a subscript *l* for the perfect substitutes (i.e., mode choice alternatives). The *Max [...]* terms in these conditions represent the information from all the mode choice alternatives available for a destination.

Recall from Equation (21) that the random error terms for all modal alternatives to a particular destination *j* $(\varepsilon_{j1},..\varepsilon_{jl},..,\varepsilon_{jL})$ follow a nested extreme value distribution as: $F(\varepsilon_{j1},\varepsilon_{j2},...,\varepsilon_{jL}) = \exp\left[-\left\{\sum_{l=1}^{L}\exp\left(-\varepsilon_{jl}/\sigma\theta\right)\right\}^{\theta}\right]$. Invoking the property of Gumbel distribution (i.e., type-1 extreme value distribution) that the maximum of different Gumbel distributed random variables is another Gumbel distributed random variable, the above KT conditions can be re-written as:

$$H_j + \eta_j - \varsigma_0 = 0 \quad if \;\; t_j^* > 0; \; j = 1,2,\ldots,J$$
$$H_j + \eta_j - \varsigma_0 < 0 \quad if \;\; t_j^* = 0; \; j = 1,2,\ldots,J \qquad (29)$$

where,

$$H_j = \mathbf{\Delta}'\mathbf{z}_j - \ln\left(\left(t_j^* / \gamma_j\right) + 1\right) + \theta \ln \sum_{l=1}^{L} \exp\left(H_{jl} / \sigma\theta\right)$$

$$H_{jl} = \boldsymbol{\beta}'\mathbf{x}_{jl} - \ln\left(t_0^{*^{\alpha-1}} q_{jl} + e_0^{*^{\rho-1}} p_{jl}\right), \text{ and}$$

η_j is a type-1 extreme value distributed random term with scale parameter σ.

In the above KT conditions, the term $\theta \ln \sum_{l=1 \text{ to } L} \exp\left(H_{jl}/\sigma\theta\right)$ is analogous to the log-sum term in the nested logit model and carries the information from the perfect substitutes (i.e., mode choice alternatives) to the imperfect substitutes (i.e., destination choice alternatives). Further, using the above KT conditions, following the derivation of the MDCEV model in Bhat (2005), the probability that the household allocates t_o amount of time for the time-specific outside good, e_0

amount of money for the money-specific outside good, and chooses the first M of the J destinations and allocates $(t_1^*, t_2^*, \ldots, t_M^*)$ amounts of time to each of the chosen destinations may be written as:

$$P\left(t_0^*, e_0^*, t_1^*, t_2^*, \ldots, t_M^*, 0, \ldots, 0\right) = \frac{1}{\sigma^M} \, | \, Jac \, | \, \frac{\left\{ \prod_{j=1}^{M} e^{H_j / \sigma} \right\} M!}{\left\{ 1 + \sum_{i=1}^{J} e^{H_i / \sigma} \right\}^{M+1}} \tag{30}$$

In the above expression, all elements except $|Jac|$ have been defined earlier. $|Jac|$ is the determinant of the Jacobian matrix which will be defined later (but very soon).

The conditional probability that a mode l_j is chosen given that the destination j is chosen (i.e., $t_j^* > 0$) is given by:

$$P(l_j \mid t_j^* > 0) = P\left(H_{jl_j} + \varepsilon_{jl_j} > H_{jl} + \varepsilon_{jl} \quad \forall l \neq l_j\right) \tag{31}$$

The above expression leads to a multinomial logit (MNL) type of probability given by:

$$P(l_j \mid t_j^* > 0) = \frac{\exp\left(H_{jl_j} / \sigma\theta\right)}{\sum_{l=1}^{L} \exp\left(H_{jl} / \sigma\theta\right)} \tag{32}$$

Now, the joint probability for the complete consumption pattern involving both imperfect substitutes (destination choices) and perfect substitutes (mode choices) is presented. The joint probability of the entire consumption pattern of a household is nothing but the probability that the household allocates t_o amount of time for the time-specific outside good, e_0 amount of money for the money-specific outside good, chooses the first M of the J destinations and allocates $(t_1^*, t_2^*, \ldots, t_M^*)$ amounts of time to each of the chosen destinations, and chooses the l_j^{th} mode of travel to each of the j^{th} chosen destination. The probability may be written as a product of marginal and conditional probabilities in the form of a joint MDCEV-MNL model as below:

$$\begin{aligned} & P\left(t_0^*, e_0^*, (0, .., t_{1l_1}^*, .., 0), \ldots, (0, .., t_{jl_j}^*, .., 0), \ldots, (0, .., t_{Ml_M}^*, .., 0), (0, .., 0, .., 0), \ldots, (0, .., 0, .., 0)\right) = \\ & P\left(t_0^*, e_0^*, t_1^*, t_2^*, \ldots, t_M^*, 0, \ldots, 0\right) \times \prod_{j=1}^{M} P(l_j \mid t_j^* > 0) = \\ & \frac{1}{\sigma^M} \, | \, Jac \, | \, \frac{\left\{ \prod_{j=1}^{M} \exp\left(H_j / \sigma\right) \right\} M!}{\left\{ 1 + \sum_{i=1}^{J} \exp\left(H_i / \sigma\right) \right\}^{M+1}} \times \left\{ \prod_{j=1}^{M} \frac{\exp\left(H_{jl_j} / \sigma\theta\right)}{\sum_{l=1}^{L} \exp\left(H_{jl} / \sigma\theta\right)} \right\} \end{aligned} \tag{33}$$

where,

$$H_j = \mathbf{\Delta}'\mathbf{z}_j - \ln\left(\left(t_j^* / \gamma_j\right) + 1\right) + \theta \ln \sum_{l=1}^{L} \exp\left(H_{jl} / \sigma\theta\right)$$

$$H_{jl} = \boldsymbol{\beta}'\mathbf{x}_{jl} - \ln\left(t_0^{*\alpha-1} q_{jl} + e_0^{*\rho-1} p_{jl}\right), \text{ and}$$

$$H_{jl_j} = \boldsymbol{\beta}'\mathbf{x}_{jl_j} - \ln\left(t_0^{*\alpha-1} q_{jl_j} + e_0^{*\rho-1} p_{jl_j}\right).$$

The term $|Jac|$ in the above expression is the determinant of the Jacobian matrix obtained from applying change of variables calculus between the vector of stochastic terms $(\varepsilon_{1l_1}, \varepsilon_{2l_2}, ..., \varepsilon_{jl_j}, ..., \varepsilon_{Ml_M})$ for all chosen destination-mode combination alternatives and the corresponding vector of time allocation variables $(t_{1l_1}, t_{2l_2}, ..., t_{jl_j}, ..., t_{Ml_M})$. As discussed earlier, the determinant of the Jacobian does not have a compact form but the *ih^th^* element of the matrix can be computed as below:

$$Jac_{ih} = \partial(V_{il_i} + \varsigma_0) / \partial t_{hl_h} \quad (i, h = 1, 2, ..., M)$$
$$= \frac{t_0^{*\alpha-2}(1-\alpha) q_{il_i} q_{hl_h} + e_0^{*\rho-2}(1-\rho) p_{il_i} p_{hl_h}}{\left[t_0^{*\alpha-1} q_{hl_h} + e_0^{*\rho-1} p_{hl_h}\right]} + \frac{Z_{ih}}{(t_i^* + \gamma_i)} \qquad (34)$$

The terms V_{il_i} used in defining the Jacobian element J_{ih} are defined as in the context of the stochastic KT conditions in Equation (25). Specifically, $V_{il_i} = \mathbf{\Delta}'\mathbf{z}_i + \boldsymbol{\beta}'\mathbf{x}_{il_i} - \ln\left(\left(t_{il_i}^* / \gamma_i\right) + 1\right) - \ln\left(t_0^{*\alpha-1} q_{il_i} + e_0^{*\rho-1} p_{il_i}\right)$, with the subscript il_i representing the combination of a chosen destination i and the chosen mode of travel l_i to that destination. The subscript hl_h is defined in similar fashion.

The joint probability expression in Equation (33) can be used to form likelihoods and estimate the utility function parameters simultaneously for imperfect substitutes and perfect substitutes. The closed form of the likelihoods facilitates an easy estimation using the familiar maximum likelihood estimation method even in situations with large numbers of choice alternatives (e.g., 210 destination alternatives and 2 mode choice alternatives for each destination in the current empirical context). Note that even if θ = 1 (which implies absence of correlations among the random utility components of the destination-mode alternatives), the H_{jl} terms (which are a part of the H_j terms) cause jointness between the marginal and conditional probabilities in the above expression. Further, it is important to note that the probability expression does <u>not</u> collapse to an MDCEV probability expression even if θ = 1 and γ_j is the same across the different destination-mode alternatives sharing the same destination. This is because the MDCEV probability expression, despite the above restrictions, does not recognize that the destination-mode alternatives sharing the same destination are perfect substitutes. Neglecting perfect substitution between alternatives (if present) can lead to either estimation problems or poor model fit (even if the parameters can be estimated). This is true even in contexts with a single linear budget constraint and the absence of price variation across choice alternatives.

The above formulation simplifies to a joint MDCEV-MNL model with a single linear budget constraint when the outside good quantity corresponding to one of the constraints is infinity.

Consider, for example, that the household has unlimited amount of time available. Then the KT conditions in Equation (25) could be rewritten as:

$$V'_{jl} - \varsigma_0 + \varepsilon_{jl} = 0 \quad if \;\; t^*_{jl} > 0, \;\; j = 1,..,J; l = 1,..,L$$
$$V'_{jl} - \varsigma_0 + \varepsilon_{jl} < 0 \quad if \;\; t^*_{jl} = 0, \;\; j = 1,..,J; l = 1,..,L \tag{35}$$

where, $V'_{jl} = \mathbf{\Delta}'\mathbf{z}_j + \mathbf{\beta}'\mathbf{x}_{jl} - \ln\left(\left(t^*_{jl} / \gamma_j\right) + 1\right) - \ln\left(e_0^{*\rho-1} p_{jl}\right)$

$$= \mathbf{\Delta}'\mathbf{z}_j + \mathbf{\beta}'\mathbf{x}_{jl} - \ln\left(\left(t^*_{jl} / \gamma_j\right) + 1\right) + (1-\rho)\ln(e^*_0) - \ln(p_{jl}).$$

Note from the V'_{jl} term above that the $t_0^{*\alpha-1} q_{jl}$ term V_{jl} in Equation (25) dropped out because t^*_0 is infinity and $(\alpha - 1)$ is negative (hence $t_0^{*\alpha-1}$ becomes zero). Next, one can follow the same steps after Equation (25) and derive the following joint MDCEV-MNL probability for situations with a single linear budget constraint.

$$P\left(t^*_0, e^*_0, (0,..,t^*_{1l_1},..,0),...,(0,..,t^*_{jl_j},..,0),...,(0,..,t^*_{Ml_M},..,0),(0,..,0,..,0),...,(0,..,0,..,0)\right) =$$
$$P\left(t^*_0, e^*_0, t^*_1, t^*_2, ..., t^*_M, 0, ..., 0\right) \times \prod_{j=1}^{M} P(l_j \mid t^*_j > 0) =$$
$$\frac{1}{\sigma^M} \mid Jac' \mid \frac{\left\{\prod_{j=1}^{M} \exp\left(H'_j / \sigma\right)\right\} M!}{\left\{1 + \sum_{i=1}^{J} \exp\left(H'_i / \sigma\right)\right\}^{M+1}} \times \left\{\prod_{j=1}^{M} \frac{\exp\left(H'_{jl_j} / \sigma\theta\right)}{\sum_{l=1}^{L} \exp\left(H'_{jl} / \sigma\theta\right)}\right\} \tag{36}$$

where,

$$H'_j = \mathbf{\Delta}'\mathbf{z}_j - \ln\left(\left(t^*_j / \gamma_j\right) + 1\right) + \theta \ln \sum_{l=1}^{L} \exp\left(H'_{jl} / \sigma\theta\right)$$
$$H'_{jl} = \mathbf{\beta}'\mathbf{x}_{jl} + (1-\rho)\ln(e^*_0) - \ln\left(p_{jl}\right)$$
$$H'_{jl_j} = \mathbf{\beta}'\mathbf{x}_{jl_j} + (1-\rho)\ln(e^*_0) - \ln\left(p_{jl_j}\right)$$

and $\mid Jac' \mid$ is the determinant of a Jacobian matrix whose ih^{th} element is:

$$Jac'_{ih} = \partial(V'_{il_i} + \varsigma_0) / \partial t_{hl_h} \quad (i, h = 1, 2, ..., M)$$
$$= \frac{(1-\rho) p_{il_i}}{e^*_0} + \frac{Z_{ih}}{(t^*_i + \gamma_i)}$$

Equation (36) provides the MDCEV-MNL probability expression for situations with a single budget constraint (money constraint in this case). It is useful to note here that the determinant of the Jacobian matrix $\mid Jac' \mid$ has a compact form as in the standard MDCEV model.

4 DATA

4.1 Primary Data: The 1995 American Travel Survey (ATS)

The 1995 ATS is the source of household-level vacation destination and mode choice data used in this analysis. The 1995 ATS collected information from over 60 thousand American households on all long-distance trips each household made over an entire year to destinations farther than 100 miles (BTS 1995a). For each trip, the information on the purpose, mode, and destination of travel and other travel attributes such as the time spent (no. of days) on the trip and travel party size were collected. From this sample, 22,215 households reported making at least one trip over the year for one of the four leisure purposes – relaxation, sightseeing, entertainment, or outdoor recreation – by either the car mode or the air mode of travel. Out of these households, a random sample of 2000 households was selected for model estimation while another random sample of 500 households was selected for model validation.

4.2 Choice Alternatives

To define the destination choice alternatives, first each of the Metropolitan Statistical Areas (MSAs) from each of the 48 contiguous states in the US was identified as a potential vacation destination. Subsequently, the remaining non-MSA area in each state was counted as a single destination (one non-MSA area for each state. All together, the U.S. was divided into 210 destinations comprising 162 MSA destinations and 48 non-MSAs. For each destination, auto and air were considered as the two primary modes of travel.

4.3 Secondary data sources

In addition to the 1995 ATS, several secondary data sources were utilized to compile other required information such as: (1) the transportation level of service variables, including the travel times and costs between each origin–destination pair via air and auto modes, (2) lodging prices, and non-lodging (dining, entertainment/recreation, and other) prices at each of the 210 destinations, (3) the destination size and attraction variables for the year 1995, including land area, number of employees in different sectors (leisure and hospitality, retail, etc.), total population, and gross domestic product, and (4) the destination climate variables, including mean monthly temperatures for different months in a year, miles of coastline at the destination, and the annual number of freezing days experienced at the destination.

Travel distances and travel times between each origin–destination pair (210 x 210 pars) by the auto mode were obtained from the Microsoft MapPoint software in conjunction with its Mile Charter add-on (Microsoft 2009; Winwaed Software Technology 2009). Travel costs by the auto mode for the year 1995 were derived as a function of travel distance, average fuel efficiency per gallon (Grush 1998), and gasoline prices in different Census regions from the Energy Information Administration (EIA 1995). Travel times and costs by the air mode for the year 1995 were obtained from the Airline Origin and Destination Survey sample provided by the BTS (1995b). Destination employment and population data was obtained from the BLS (1995) and the 2000 Census data (U.S. Census Bureau 2000). Other destination data including the gross domestic product of the destinations were obtained from the US Bureau of Economic Analysis (Bureau of Economic Analysis 1995). Climate data for the destinations, including the mean monthly temperatures for both January and June months (i.e., winter and summer months) and the annual number of freezing days were obtained from the Places Rated Almanac (Savageau and Loftus 1997) for the year 1995. Length of coastline for each destination was obtained from the National Oceanic and Atmospheric Administration's Ocean and Coastal Resource Management (2011).

Gathering and assembling data from all the above sources required a significant and painstaking amount of effort. Details on how the above data sources were used to create the specific variables of interest are documented in the master's thesis report by (Van Nostrand 2011). Since a focus of the paper is on accommodating multiple budget constraints along with the price variation across the different vacation destinations and travel modes, the procedures and assumptions used to construct the price variables are described next.

4.4 Unit Prices

There are two types of unit prices for each vacation destination and travel mode alternative – time-prices and money-prices (i.e., the q_{jl} and p_{jl} variables in the time and money constraints of equations (22) and (23)). For the current analysis, the time-prices (q_{jl}) are considered to be unity in that the amount of time needed to spend one day of vacation time is equal to one day.[16] The synthesis of money-prices (p_{jl}), on the other hand, required several assumptions and significant data gathering and processing, as described below.

The money-price p_{jl} is the monetary expenditure a household needs to incur to spend unit time (i.e., a day) at a vacation destination *j* traveled by mode *l* (note that the subscript for the household is suppressed for simplicity in notation). These prices comprise two components – (a) destination prices that do not depend on the mode of travel and (b) travel prices that depend on the mode of travel. The destination prices, in turn, have two components – (a1) lodging prices (i.e., lodging costs per day) and (a2) non-lodging prices (i.e., costs per day for dining, recreation, entertainment, etc.). The process used to synthesize the information on money-prices for each household to travel to each available destination by each available travel mode is described below.

First, the lodging costs and non-lodging costs per day at each destination were synthesized from the 1995 Consumer Expenditure Survey (CEX) data using a two stage process. In the first stage, the per-day lodging costs for each household was derived using a regression equation relating the per-day costs to the household's socio-demographic characteristics (income, household size, and residential Census region). This regression equation was estimated using household-level microdata on annual vacation expenditures (and the annual number of days spent on vacation) from the 1995 Consumer Expenditure Survey (CEX) data. Similarly, the per-day non-lodging costs were derived using another regression equation estimated with the CEX data on non-lodging vacation expenditures. Both the above mentioned regression equations recognize the variation in per-day costs by household characteristics. Thus, this approach recognizes that not every household incurs the same costs at a destination. Rather, households make the lodging choices and other expenditure choices according to their income and other

[16] This makes an implicit assumption that the time spent traveling to a destination *j* is part of the vacation time t_j. That is, households derive utility not only from the time spent at a vacation destination, but also from the time spent traveling to the destination. This is a reasonable because traveling for vacation might not be as onerous (it might in fact be fun) as compared to commuting. However, doing so makes it difficult to account for the possibility that households tend to prefer to visit closer destinations as opposed to farther destinations. To account for such preferences, the mode choice utility functions incorporate the travel time to the destination (by the corresponding mode) as an explanatory variable. One would expect a negative coefficient on this variable. And the log-sum variable from the mode choice model component would feed into the destination choice component to account for the influence of travel times on household preferences toward the destinations.

characteristics. However, the regression equations do not recognize the variation in the lodging and non-lodging prices across the different destinations (because the CEX data does not provide information on which destinations were visited by the households). To accommodate the price-variation across destinations, in the second stage, the regressed per-day costs for each household were scaled by a factor capturing how pricy (or less expensive) each destination is compared to an average destination (as measured by the median per-day costs at different destinations. To implement this second state strategy, the median values of lodging and other costs of vacationing at each of the 210 destinations were obtained from a database made available by VisitUSA.com (2011). The lodging prices and non-lodging prices obtained in the above manner were added up to obtain the destination prices. Call such destination price as p_j, where *j* is the index for destination.

Second, using the 1995 ATS data, the number of days spent at a destination were regressed, using an ordered logit model, as a function of the household characteristics (age of householder, household size, income, presence of children), distance between origin and destination, and an indicator if the destination is an MSA. The resulting ordered response model estimates were used, for each household in the estimation sample, to estimate the expected number of days (n_j) that the household would spend at each destination ($j = 1,2,\ldots,J$) if the household visited that destination. Third, the money-price p_{jl} of spending a day visiting a destination *j* by mode *l* was computed as: $p_{jl} = \frac{p_j n_j}{(n_j + tt_{jl})} + \frac{tc_{jl}}{(n_j + tt_{jl})}$, where tc_{jl} and tt_{jl} are the round trip travel cost and travel time, respectively, to travel to a destination *j* (from the household's origin) by travel mode *l*. The first component of this money-price formula can be viewed as the destination price, while the second component can be viewed as the travel price. Further details on the above-described process, including the regression estimates used to synthesis the money-prices, are suppressed here to conserve space but available from the authors. But note from the formula for p_{jl} that the money-prices are computed assuming that the travel costs (tc_{jl}) can be amortized over the duration spent visiting a destination ($n_j + tt_{jl}$). As discussed earlier, formulating the model to relax this assumption and consider travel costs as *fixed* is an important avenue for further research.

4.5 Sample Description

Table 1 presents the descriptive statistics from the estimation sample used in this analysis. Keeping in view the space limitations, they are discussed here very briefly. Further details are available from the authors.

The households in the estimation sample have an average size of 2.81 persons/household, householder age of 46.2 years, and an average income of $48,913 per annum. 32% of them have at least one child of age less than 16 years, and 14% of them have householders who are retired.

About 52% of the households in the estimation sample made multiple long-distance leisure trips over a year. Further, a significant proportion ($\sim$39%) of the households visited multiple destinations. Furthermore, a large percentage (about 79%) of households visited a destination (if they did so) only once. This suggests multiple discreteness (or variety-seeking) in households' annual destination choices. Even if households visited a destination more than once a year, a vast majority of the times (99.5% of the time in the data; not shown in the table) the same mode was used to travel across all the different trips made by a household to that same destination. This suggests perfect substitution (or single discreteness) among the mode choice alternatives to a destination. The total annual household time spent on vacation ranged from a day to about 60 days, with an average of 9 days. The total annual vacation expenditure ranged from $153 to $5566 (not shown in table), with an average of $958 per household.

The trip level characteristics suggest 84.9% of the trips in the estimation sample were by auto mode and the remaining 15.1% were by the air mode. The average round trip distance was about 1027 miles. On average, the households in the sample spent about 5 and a half days and about $600 on each trip.

The last set of descriptive statistics is for the characteristics of the 210 destinations and level of service variables (between the 210 x 210 OD pairs) used as explanatory variables in the model.

5 EMPIRICAL ANALYSIS

5.1 Model Specification

A variety of model specifications were estimated before arriving at the final empirical model. The estimated models and their model fit measures are shown in Table 2. For each model estimated, a variety of model evaluation measures, including log-likelihood (*LL*) at convergence, Rho-square (ρ^2), Akaike Information Criterian (AIC), Bayesian Information Criterian (BIC), and predictive log-likelihood (*PLL*) on a validation sample of 500 households, are presented.

The first set of rows (numbered 1 to 4) is for model formulations for choice situations with only imperfect substitutes in the choice set. In the current empirical context, these are models for annual destination choice(s) without considering the mode choice to the travelled destination(s).[17] The first two of these models are standard MDCEV models with a single budget constraint as in Bhat (2008). The next two models consider both time and money constraints simultaneously whose formulations are provided in Section 3.2. Specifically, the third model does not consider the normalization that the baseline utility parameters of the constraint-specific Hicksian composite outside goods should be constrained to be equal (i.e., $\psi_0 = \phi_0$). To be more precise, it considers the first normalization discussed in Section 3.2.3 that the coefficients of the explanatory variables have to be normalized to zero in the baseline utility parameters of the outside goods, but it allows the random error components of ψ_0 and ϕ_0 to be different. As a result, the likelihood function for this model is a double integral, as in Equation (13), approximated using numerical quadrate. For a variety of different sets of starting values of the parameters, the model either did not converge or converged to different sets of parameter estimates with covariance matrices that couldn't be inverted. Besides, while the model seemed to converge (although without standard errors) when the

[17] A challenge in modeling destination choices without simultaneously considering mode choice is in determining the time-prices and money-prices to alternative destinations (since the travel times and travel costs to travel to a destination depend on the mode of travel). Indirect utility-based discrete choice models use log-sum variables as a composite measure of the travel times and travel costs by all available travel modes. This approach, however, does not work with the proposed, direct utility-based Kuhn-Tucker (KT) model with explicit money and/or time constraints. This is due to the need to explicitly consider the time-prices and money-prices (that a household would incur to visit alternative destinations) into the time and money budget constraints of a direct-utility based KT model. To address this issue, we used *expected* time-prices and *expected* money-prices to travel to a destination. The expected time-price (or money-price) to travel to a destination is a weighted average of the time-prices (money-prices) to travel to a destination by different modes; weighted by the probabilities of traveling by the corresponding modes, obtained from a separate mode choice model. Such expected time-prices and money-prices can be directly used in the time and money budget constraints to represent the unit prices faced by a household to travel to alternative destinations.

integral was approximated with either three or four support points for the quadrature, increasing the support points lead to significant instability in estimation. This experience corroborates the discussion in Section 3.2.3 that the model is unidentified when $\psi_0 \neq \phi_0$. The fourth model, on the other hand, considers the normalization discussed in Section 3.2.3 that $\psi_0 = \phi_0$ and employs the closed-form probability expression in Equation (19) to construct the likelihood function. This model converged to a same set of parameter estimates with different sets of starting values, an indication of appropriate parameter identification and stability in estimation. It is worth noting here that, for none of the seven models reported in the table, the satiation parameters on the outside goods, neither α nor ρ could be estimated. Both of these parameters were fixed to zero in all cases. These results suggest the importance of the normalizations discussed in Section 3.2.3 for parameter identification and stability in model estimation.

The second set of models (numbered 5 to 7) corresponds to the MDCEV-MNL structure described in Section 3.3 for choice situations with both imperfect and perfect substitutes in the choice set. In the current empirical context, these are joint destination and mode choice models. Specifically, the fifth and the sixth models consider a single budget constraint, while the last model considers both time and money budget constraints. The log-likelihood value of the time- and money-constrained model (i.e., model #7) is better than that of the time-constrained (money-constrained) model by 428 (484) points. It can be observed from all other goodness of fit measures in the table (Rho-square, AIC, BIC, and predictive log-likelihood on a sample of 500 households) that the time- and money-constrained model performed better than the two single-constrained models. The same can be observed in the context of the first four models (numbered 1 to 4), where the time- and money- constrained model of destination choices (model #4) performed better than the single-constrained models (models #1 and #2). These results suggest the need to consider both the constraints.[18]

5.2 Model Coefficients

Table 3 reports the model parameter estimates from three different joint models for annual vacation destination and mode choices – (1) The time-constrained model (model #5), (2) The money-constrained model (model #6) , and (3) The time- and money-constrained model (model #7). As discussed earlier, the time- and money-constrained model performs better than the two single-constrained models in terms of model goodness of fit as well as predictive ability (log-likelihood) on a validation sample. Thus, we use the parameter estimates from the time- and money-constrained model to discuss the influence of different factors on households' annual destination and mode choices. Wherever appropriate, we discuss the differences in the interpretations from the single-constrained models. The specification of the baseline utility function (ψ_{jl}) is discussed first, followed by the specification of the translation function (γ_j).

The first set of variables in the baseline utility function has common coefficients across all destination-mode combinations (i.e., inside goods) with the outside goods as the base

[18] A non-nested likelihood ratio test was also conducted to compare the model fit of the time- and money-constrained model with that of the time-constrained model. To do so, a naïve time constrained model with only constants in it (with a log likelihood value of -29,188) was considered as the base. The rho-square values for time-constrained model and the time- and money- constrained models are 0.1382 and 0.1529, respectively with respect to the naïve, time constrained model. The difference between the above adjusted rho-squared values is 0.0147. The probability that this difference could have occurred by chance is less than $\Phi(-\sqrt{-2 \times 0.0147 \times -29,188})$. This value is almost zero, suggesting that the time- and money-constrained model has a better data fit compared to the time-constrained model.

category (for normalization). Among these variables, the alternative specific constant is negative suggesting that households spend a smaller proportion of the year (365 days) on vacation at long-distance destinations compared to all other purposes captured in the outside goods (such as work, sleep, leisure activities pursued closer to the household). This is reasonable because, as suggested by the descriptive statistics of the model estimation data, the amount of annual time that a household typically spends on vacation is much less compared to the other time investments to be made in the year. Households with retied householder are likely to spend less time on vacation to long-distance destinations compared to other households. This is perhaps because of the physical limitations as well as financial constraints faced by such households to travel longer distances. The next variable, leisure employment per capita at the household location captures the influence of opportunities for leisure activities within a closer vicinity of the household (as opposed to long-distance destinations). The negative coefficient suggests that households living in places with greater leisure opportunities within a shorter distance are likely to spend less time on long-distance vacation. This result suggests higher substitution between the time spent locally and the time spent on long-distance vacation for households in locations with greater leisure opportunities. While the result is intuitive, the corresponding coefficient is not statistically significant when only the time constraint in considered, suggesting the need to consider the money constraint as well.

The second set of variables consists of destination-specific characteristics. The interpretations of these variables have reasonable and expected substantive interpretations that are similar to those discussed in Van Nostrand et al. (2012) who considered only the time constraints. Specially, MSA destinations (as opposed to Non-MSA destinations), destinations with greater leisure opportunities and longer coastlines, and destinations with moderate temperatures are more attractive to households for vacation purposes. While the substantive interpretations of the parameter estimates are similar across the three models, the differences in the magnitudes and t-statistics of the estimates are not negligible. For example, the t-statistics on the coefficients of the dummy variables indicating if the destination is in the same (or adjacent) state as the household residence are much higher in the time-constrained model compared to the other two models. This is because the time-constrained model doesn't consider the influence of travel costs on destination choices as it ignores the influence of the money constraint (which incorporates travel costs) on households' choices. Since destinations in the same or adjacent state as the household residence state are less expensive to travel to (when compared to other destinations), ignoring the money constraint lead to an over-estimated influence of the variables under consideration. Once the influence of travel cost is considered through the money constraint, the estimated influence of the dummy variables can be interpreted as the influence of households' familiarity with (hence greater preference to) destinations in same or adjacent states. In summary, these results suggest that ignoring the influence of a constraint (when it is present) can lead to the cofounding of its influence into household preferences in the utility function.

The third set of variables is specific to the travel modes under consideration. The alternative specific constant has no interpretation but reflects that households have a general preference to travel by car even after considering the time- and money-constraints and other mode-specific variables in the model. The next two variables indicate if the origin (i.e., household residential location) is an MSA and if the destination is an MSA, respectively. As expected, MSA origins and destinations are more attractive for the air mode of travel than the non-MSA origins or destinations because of the greater access to the air travel mode in the MSAs. The last variable in this category is the round trip travel time by the alternative modes of travel, whose negative coefficient suggests that households prefer to travel by faster modes of travel. In addition to its influence on mode choice, this variable helps in accommodating (through the log-sum variable described in the context of Equation 29) that farther destinations are less attractive for vacation compared to closer destinations. Note that mode-specific travel costs are not included as explanatory variables in the model, while the travel times are included. This is because the travel costs are already incorporated into the money-budget constraint through the money-prices (p_{jl}) of travel to the destinations. Such money-prices help in incorporating that farther destinations are more pricy to travel to and hence less likely to be chosen because of the monetary constraint. On the other hand, as discussed earlier, the travel times were not incorporated into the time-prices (q_{jl}). This is because the time-price (q_{jl}) of allocating unit time for a destination has been set to unity assuming that traveling also contributes to the utility derived from vacation (in addition to the utility due to the time spent at the destination).[19]

The next parameter is the scale (σ) of the error terms (ε_{jl}) in the baseline utility parameters (ψ_{jl}). This parameter is measure of the magnitude of the variation in the household preferences due to unobserved factors. The parameter was fixed to 1 in the time-constrained model as it could not be estimated due to the absence of price variation. In the other two models, the parameter could very well be estimated and is significantly different from 1. Specifically the estimate is 0.754 in the money-constrained model and 0.432 in the time-constrained model. These estimated suggest that the magnitude of variation in the household preferences due to unobserved factors is lower in the time- and money-constrained model than that in the two single-constrained models. This suggests that accounting for both the time and money constraints together helped in capturing a greater proportion of the variation in household preferences. Ignoring either of the two constraints resulted in a greater proportion of unexplained variation in household preferences.

The next parameter is the dissimilarity parameter (θ). The estimate for this parameter is significantly different from 1 (in all three models) suggesting the significant presence of destination-specific unobserved factors inducing correlations between the baseline utility parameters of the destination-mode combination alternatives that share the same destination. Neglecting such correlations and estimating the destination and mode choice models separately would result in significantly inferior model fit.

The last set of variables is related to the translation parameters which allow for corner solutions as well as differential satiation effects across different vacation destinations. Households are likely to allocate greater amount of time for vacation destinations that are farther (than those that are closer). This is perhaps because it takes greater amount of time to travel to those destinations. Further, households might want to spend more time at a destination that is farther from home (if they chose to visit the destination) because it would require significant amount of time and money to make another visit to that destination. Larger households are likely to allocate more time to a vacation destination (than smaller households), if they choose to visit

[19] In the absence of this assumption, the time-price would be equal to the time spent at the destination added to the time spent traveling to (and from) the destination divided by the time spent traveling. Such a time-price would always be greater than unity and automatically account that farther destinations are more time-pricy and hence less likely to be chosen compared to closer destinations. But the downside of not making the assumption is one cannot accommodate that traveling while on vacation can itself contribute to the utility derived from vacation.

the destination. Households with retired householders are likely to allocate more time to a destination (than other households), if they chose to visit the destination.

Overall, the model estimation results are all reasonable and shed light on the various factors influencing households' annual vacation destination and mode choices and related time and money allocations. Further, the model results show that ignoring either the time constraint or the money constraint would lead to the confounding of the neglected constraint into household preferences. In addition, the time- and money-constrained model demonstrated a greater capture of the variation in household preferences than the models that ignored one of the two constraints. All these results, combined with the superior performance of the time- and money-constrained model (over the single constrained models) in terms of goodness of fit and predictive log-likelihood, suggest the need to consider both time and money constraints simultaneously in analyzing households' vacation travel choices.

6 SUMMARY AND CONCLUSIONS

This study formulates and applies a joint model of annual vacation destination and mode choices to simultaneously analyze the vacation destinations that a household visits over an entire year, along with the time and money allocations and mode of travel to each of the visited destinations. The formulation assumes that, over a year, households allocate a part of the total time (365 days) and money (annual income) available with them to one or more vacation destinations and make the mode choices in such a way as to maximize the utility derived from their choices. This formulation enhances the recently emerging Multiple Discrete-Continuous Extreme Value (MDCEV) model structure in several ways. First, an extended MDCEV framework is proposed to simultaneously consider the influence of both time and money budget constraints in household vacation travel decisions, as opposed to most previous MDCEV applications that consider only a single budget constraint. Second, the time- and money-constrained MDCEV framework of vacation destination choices is integrated with a multinomial logit (MNL) model of travel mode choice. The integrated framework recognizes that households make decisions on where to travel (i.e., vacation destinations) and how to travel (i.e., travel mode) in a joint fashion. Specifically, the framework recognizes that the vacation destinations are *imperfect substitutes* in that a household can potentially choose to visit multiple destinations over a year, while the travel mode alternatives to a destination are *perfect substitutes* in that only one mode of travel is chosen. Third, the proposed time- and money-constrained MDCEV-MNL framework not only accommodates multiple budget constraints and a mix of imperfect and perfect substitutes in the choice set, but also recognizes the possibility of price variation across both imperfect and perfect substitutes. Finally, in developing the above-described framework, the paper highlights and resolves certain identification issues related to the specification of MDCEV models with multiple budget constraints. Simple normalizations are proposed that not only help with model identification but also facilitate the derivation of closed form probability expressions for the proposed formulation. To our knowledge, this is the first formulation in the econometric literature to account for multiple linear budget constraints and price variation to model discrete-continuous choices with a combination of perfect and imperfectly substitutable choice alternatives.

The proposed modeling framework is applied to the 1995 American Travel Survey (ATS) data to estimate the empirical model parameters, with the United States divided into 210 alternative long-distance vacation destinations. The ATS data provides information on the different vacation destinations visited (and the time spent on each vacation) by the surveyed households over the time-frame of an entire year. Along with this information from the ATS data, a variety of other data sources, including the Consumer Expenditure Survey (CEX) are used to synthesize information on destination attributes, and lodging costs and other costs of vacation for each of the 210 destinations. The empirical analysis demonstrates the importance of considering both time and money budget constraints simultaneously and that of modeling both destination and mode choices jointly. Specifically, considering the time and money constraints simultaneously lead to a significant improvement of the model goodness of fit in the estimation sample as well as the predictive performance (as measured by predictive log-likelihood) on a validation sample. As importantly, analysis of the parameter estimates suggested that ignoring either the time constraint or the money constraint would lead to a confounding of the neglected constraint into household preferences. In addition, the time- and money-constrained model demonstrated a greater capture of variation in household preferences than the models that ignored one of the two constraints.

The parameter estimates of the time- and money-constrained MDCEV-MNL model shed several insights into the determinants of households' vacation destination and mode choices and related time allocation behavior. It could be used to analyze the influence of changes in household socio-demographics, transportation level-of-service (travel times and costs), and destination characteristics (lodging and recreational costs, and recreational opportunities) on household vacation travel behavior. In addition, the model can be incorporated into a larger national travel modeling framework for predicting the national-level origin-destination flows for vacation travel. Thus, in addition to providing the methodological model formulations, the study contributes to the long-distance travel modeling literature.

This study can be extended in several important directions. First, development of efficient forecasting procedures for the proposed formulation will enable the use of the estimated model for practical forecasting and policy analysis purposes. von Haefen et al. (2004) and Pinjari and Bhat (2010) propose efficient forecasting algorithms for discrete-continuous choices in situations with a single budget constraint and only imperfect substitutes in the choice set. The challenge will be in extending these algorithms to the proposed formulation for forecasting discrete-continuous choices in situations with multiple budget constraints and a mix of imperfect and perfect substitutes in the choice alternatives. Second, the proposed formulation assumes that travel costs can be amortized into a constant price per unit consumption (or time allocation to the destination). Relaxing this assumption and treating fixed costs separately from variable costs is an important avenue for future research. Third, on the empirical front, the current study does not consider visiting friends and family as part of vacation travel (or long-distance leisure travel). This is because of the lack of data on social networks that influence vacation destination choices. Finally, from a theoretical perspective, the current formulation considers time as the only entity that is consumed for deriving utility during vacation. However, as long-been described in economic theories of time allocation (Becker, 1965; De-Serpa, 1971; Jara-Diaz, 2007), goods and services in the market place are also consumed to derive utility. In the current context, consumption of services such as movies and theme parks and commodities such as souvenirs can provide utility. In fact, time is invested to consume such services and commodities, which in turn provide utility (Becker, 1965). Considering the consumption of both time and other commodities will make the formulation more behaviorally realistic.

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Table 1: Descriptive Statistics of the Estimation Data

Household Socio-demographic Characteristics (in the estimation sample of 2000 households)			
Household size	Average: 2.81		Std. Dev: 1.34
Age of householder (years)	Average: 46.2		Std. Dev. : 1.34
Household yearly income	Average: $48,913		Std. Dev: $29,306
Presence of Kids		38.2%	
Householder is Retired		14.0%	
Household Leisure Travel Characteristics (in the estimation sample of 2000 households)			
Number of long distance leisure trips	Average: 2.53		Std. Dev: 2.97
1		48.1%	
2 or more		51.9%	
Number of destinations visited	Average: 1.59		Std. Dev: 0.93
1		61.1%	
2		24.2%	
3or more		14.7%	
Number of trips made to a destination	Average: 1.59		Std. Dev: 2.10
1		79.0%	
2 or more		21.0%	
Total Annual Vacation Time (Days)	Average: 9.04		Std. Dev: 16.62
Total Annual Expenditure on Vacation	Average: $958.30		Std. Dev: $1,488.41
Trip-level Characteristics (for 3183 leisure trips made by the 2000 households)			
Primary mode of transportation	Auto: 84.9%		Air: 15.1%
Round trip Ground Distance(miles)	Average: 1,027		Std. Dev:1,193
No. of nights away from home on trip	Average: 5.69		Std. Dev: 12.76
Monetary Expenditure	Average: $602.13		Std. Dev:$1,123.15
Destination Characteristics (for 210 Destinations)			
Destination is an MSA		76.7%	
Ln (LandArea in square miles)	Average: 5.92		Std. Dev: 2.88
Leisure Employment (100's /Sq.Mile)	Average: 4.84		Std. Dev: 7.07
Leisure Employment Per Capita	Average: 0.12		Std. Dev: 0.14
Length of Coastline (Miles)	Average: 2,310		Std. Dev: 3,898
Winter Temperature (Fahrenheit)	Average: 42.3		Std. Dev: 16.53
Summer Temperature (Fahrenheit)	Average: 82.03		Std. Dev: 8.79
Level of Service Characteristics (between 210 x 210 OD pairs)			
Highway Distance (Roundtrip)	Average: 2,622		Std. Dev: 1,749
Auto Travel Time (hours)	Average: 23		Std. Dev: 12.25
Air Travel Time (hours)	Average: 4.5		Std. Dev: 2.96
Auto Travel Cost (US dollars)	Average: $154.24		Std. Dev: $105.74
Air Travel Cost (US Dollars)	Average: $437.76		Std. Dev: $301.69

Table 2: Goodness of Fit Measures for the Models Estimated in the Study

	Log-likelihood at model convergence (LL)	No. of Parameters (K)	Rho-square (ρ^2) $1-\frac{LL}{LL(C)}$	AIC $-2LL+2K$	BIC $-2LL+\ln(N).K$	Predictive LL for 500 households (PLL)
1. Time-constrained MDCEV model for destination choices	-22,115	23	0.1545	44,277	44,405	-5,840
2. Money-constrained MDCEV model for destination choices	-22,500	24	0.1398	45,048	45,182	-5,938
3. Time- and Money-constrained MDCEV model (as in Equation 13) for destination choices	--	--	--	--	--	--
4. Time- and Money-constrained MDCEV model proposed in this paper (as in Equation 19) for destination choices	-21,893	24	0.1630	43,835	43,969	-5,816
5. Time-constrained MDCEV-MNL model for destination and mode choices (no money constraint)	-25,123	28	0.1383	50,302	50,459	-6,780
6. Money-constrained MDCEV-MNL model for destination and mode choices (no time constraint)	-25,179	29	0.1364	50,416	50,579	-6,799
7. Time- and Money-constrained MDCEV-MNL model for destination and mode choices	-24,695	29	0.1529	49,449	49,612	-6,730

Note:
LL = Log-likelihood at model convergence
LL(C) = Log-likelihood with only constants in the model
K = No of parameters in the model
Rho-square (ρ^2) = $1-\{LL / LL(C)\}$
Akaike Information Criterion (AIC) = $-2LL+2K$
Bayesian Information Criterion (BIC) = $-2LL+\ln(N).K$

Table 3: Model Estimation Results

	Time-Constrained MDCEV-MNL		Money-Constrained MDCEV-MNL		Time & Money Constrained MDCEV-MNL	
Baseline Utility Function (ψ_{jl}) Specification	**Coeff**	**t-stat**	**Coeff**	**t-stat**	**Coeff**	**t-stat**
Variables common to all destination-mode combinations (outside goods are in the base category)						
Alternative specific constant	-12.172	-39.06	-10.821	-37.97	-7.083	-44.37
Householder is retired	-0.200	-1.43	-0.230	-2.78	-0.079	-1.75
Leisure employment per capita at the HH MSA/non-MSA	-0.041	-0.09	-0.816	-2.69	-0.340	-2.05
Destination-Specific Characteristics (Z_j)						
Log (Land area of the destination in sq. miles)	0.458	18.20	0.331	16.24	0.192	16.63
Destination is an MSA (dummy variable)	0.843	5.76	0.616	5.62	0.350	5.57
Leisure employment density in 100's of jobs/sq. mile	0.089	25.60	0.071	22.91	0.040	22.91
Length of coastline in 1000's of miles	0.07	6.13	0.069	7.84	0.038	7.58
Difference in no. of freezing days (destination – origin)	0.007	8.96	0.005	8.77	0.003	9.04
Winter (January) temperature. 65^o-75^o Fahrenheit is base						
55^o-65^o Fahrenheit	-0.574	-6.66	-0.270	-4.17	-0.184	-4.97
45^o-55^o Fahrenheit	-1.046	-10.30	-0.595	-7.70	-0.375	-8.50
35^o-45^o Fahrenheit	-1.484	-11.60	-0.934	-9.55	-0.560	-9.98
$< 35^o$ Fahrenheit	-1.518	-10.14	-0.975	-8.62	-0.585	-9.02
Summer (June) temperature. 65^o-75^o Fahrenheit is base						
60^o to 65^o Fahrenheit	-3.184	-7.28	-2.965	-8.78	-1.645	-8.66
75^o to 80^o Fahrenheit	-0.460	-6.37	-0.375	-6.87	-0.223	-7.13
80^o to 85^o Fahrenheit	-0.258	-3.50	-0.266	-4.82	-0.147	-4.65
85^o to 90^o Fahrenheit	-0.616	-7.30	-0.498	-7.76	-0.290	-7.97
$> 90^o$ Fahrenheit	-0.292	-3.41	-0.267	-4.12	-0.151	-4.07
Dummy if destination in same state as HH residence	3.297	55.88	2.426	30.48	1.425	33.68
Dummy if destination in adjacent state to HH residence	1.997	39.21	1.452	26.73	0.853	28.46
Mode-Specific Variables (X_{jl})						
Alternate specific constant for air mode (auto is base)	-2.223	-14.36	-1.086	-11.64	-0.696	-12.28
Origin is an MSA – on the air mode (auto is base)	0.394	6.70	0.233	5.86	0.127	5.63
Destination is an MSA – on the air mode (auto is base)	0.792	9.46	0.537	10.49	0.315	10.30
Round-trip travel time in days	-0.051	-15.30	-0.038	-17.10	-0.021	-15.65
Scale parameter σ (t-stats are against a value of 1)	1.000	fixed	0.754	13.12	0.432	57.91
Dissimilarity Parameter θ (t-stats are against a value of 1)	0.489	14.24	0.411	21.18	0.413	20.04
Translation Function (γ_j) Specification						
Highway distance to destination (100's miles)	0.052	6.85	0.056	10.72	0.103	17.85
Household Size	0.352	11.94	0.42	18.81	0.599	32.48
Distance*Presence of children in HH	-0.026	-1.78	-0.033	-3.08	-0.075	-8.66
Householder is retired	0.450	1.96	0.579	3.12	0.678	4.91

Note: Winter (summer) temperatures are monthly averages of maximum daily temperatures over a month