On the Nonlinearity of Response to Level of Service Variables
in Travel Mode Choice Models

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ABSTRACT

It is important to accommodate variations in responsiveness (or response heterogeneity) to level of service attributes in travel mode choice models. This response heterogeneity may be disaggregated into a systematic (observed) component and a random (unobserved) component. Earlier studies have typically considered systematic response heterogeneity by examining differences in LOS response sensitivities due to individual demographic and other attributes. In this research, our emphasis is on another element of systematic response heterogeneity – systematic response heterogeneity originating from nonlinear responsiveness to LOS attributes. Specifically, we consider both the components of systematic response heterogeneity (due to individual characteristics and due to nonlinear responsiveness) as well as unobserved response heterogeneity at the same time, and compare the empirical results of models that assume a traditional linear responsiveness to LOS attributes with those that adopt a nonlinear responsiveness to LOS attributes.

The empirical analysis uses the Austin Commuter Stated Preference Survey data to examine commute travel mode choice. The nonlinear specifications for travel time and travel time unreliability indicate that commuters place a small value to travel time, and a very high value to travel time reliability, in the first 15 minutes. Beyond 15 minutes, however, the valuation of travel time increases rapidly, while that of travel time reliability drops dramatically. In addition to clearly indicating the nonlinear nature of responsiveness to travel time and travel time unreliability, the results indicate that ignoring nonlinear responsiveness leads, in the current empirical context, to (a) biased parameter estimates, (b) an inflated estimate of unobserved heterogeneity, (c) counterintuitive signs on the LOS variables for a high fraction of individuals, (d) inaccurate estimates of willingness-to-pay measures, and (e) loss in model fit.

1. INTRODUCTION AND MOTIVATION

1.1 Mode Choice Models and Level of Service Attributes

Random utility-based discrete choice models have been used extensively over the past three decades in travel demand, economics, marketing and other fields to analyze choice-making behavior. In the travel demand field, discrete choice models[1] have been used to analyze behaviors associated with several choice dimensions, including (but not limited to) car ownership, activity participation location, travel route, and travel mode choice. Among these dimensions, the travel mode choice dimension has been the one most extensively studied, mainly because shifting individuals away from driving alone to other forms of travel, if feasible, is a very effective and efficient way of alleviating traffic congestion and related problems. Discrete mode choice models serve as the vehicle to undertake an *a priori* evaluation of the effectiveness of alternative traffic congestion alleviation strategies by estimating the potential shifts in mode shares.

Discrete mode choice models involve the estimation of a latent indirect utility function associated with each mode, based on the observed mode choice decisions of individuals. The indirect utility for each mode is usually specified as a function of a mode-specific preference term and level-of-service (LOS) attributes of the mode. The typical LOS attributes include travel time and travel cost, each of which may further be disaggregated into several components (for example, travel time may be broken down into in-vehicle time and out-of-vehicle time). More recently, a few research efforts have also recognized and accommodated the important effect of travel time (un)reliability in travel mode choice decisions [see, for example, Abkowitz (*1*), Bhat and Sardesai (*2*), Ghosh (*3*), König and Axhausen (*4*) and Lam and Small (*5*)].

The LOS variables discussed above are the policy-sensitive variables in mode choice models. The estimated response sensitivities to the LOS variables inform the analyst about the trade-offs decision-makers are willing to make, in the form of such measures as the implied value of time (VOT) and the implied value of reliability (VOR). These willingness-to-pay (WTP) measures, in turn, aid in designing appropriate transportation policy strategies to alleviate traffic congestion and in cost-benefit calculations to prioritize transportation planning projects.

The preceding discussion highlights the importance of estimating accurate mode choice models in general, and the sensitivities (or response) to LOS variables in particular, from both a transportation planning/policy perspective as well as a project evaluation perspective. This has led to the use of behaviorally realistic model structures in the past decade that relax the restrictive multinomial logit (MNL) model assumptions of (a) independent and identically distributed (IID) random utility components (error terms) across alternatives and (b) the absence of heterogeneity (across decision-makers) in preference to the alternatives (preference homogeneity assumption) and in response to LOS attributes (response homogeneity assumption). In this research, our specific objective is on contributing to the literature on relaxing the response homogeneity assumption. In doing so, we also relax the IID error structure and preference homogeneity assumptions of the MNL structure, lest these restrictive assumptions should interfere with our efforts to obtain accurate LOS response estimates.

1.2 Heterogeneity in Response to LOS Attributes

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[1] In the rest of this paper, we will use the term discrete choice models to refer to random utility-based discrete choice models.

The need to accommodate response heterogeneity arises from the possibility that decision-makers may be differentially sensitive to LOS attributes. The neglect of such heterogeneity, in general, leads to inconsistent parameter estimates and severely inconsistent probability estimates (*6*). The heterogeneity in response to the LOS variables may be disaggregated into a systematic (observed) component and a random (unobserved) component. Systematic response heterogeneity, as has been typically considered in the literature, is the variation in response due to observed (to the analyst) individual factors (for example, the response to travel cost may be a function of the income earnings of the individual). Random response heterogeneity is the variation in response due to unobserved (to the analyst) individual factors (for example, a dynamic, go-getter, personality may be associated with higher sensitivity to travel time, while a patient, carefree, personality may be associated with lower sensitivity to travel time). Systematic response heterogeneity, as considered in the extant literature, is included in mode choice models by interacting individual characteristics with LOS attributes, while random response heterogeneity is normally captured through estimation of parameters characterizing an assumed distribution for response heterogeneity.

In the context of accommodating response heterogeneity, it is important, first and foremost, to comprehensively accommodate systematic response heterogeneity. In particular, and as emphasized by Bhat (*7*), the fundamental idea of discrete choice modeling will always remain the identification of systematic preference and response variations in the population. Random response heterogeneity cannot be in lieu of poor systematic specifications, but should be included in the spirit of obtaining accurate coefficients characterizing systematic variations in the potential presence of unobserved heterogeneity across individuals. In fact, and as empirically documented by Warburg *et al.* (*8*), inadequately considering systematic variations when introducing unobserved heterogeneity can lead to biased parameter estimates as well as an inflated estimate of the magnitude of unobserved heterogeneity (since the ignored systematic variations get manifested incorrectly as unobserved heterogeneity). Brownstone and Small (*9*) also make a similar point about response heterogeneity when they indicate that further advances in understanding heterogeneity in response have to come from isolating sources of observable variation in response rather than "sophisticated random term specifications". Additionally, as stated by Train (*10, p.145*), there is a "natural limit" on the extent to which we can understand a phenomenon if a substantial amount of heterogeneity is due to unobservable elements.

1.3 The Current Research: A Case for Nonlinear Specification of the LOS Effects
As indicated in the previous section, earlier studies have typically considered systematic response heterogeneity by examining differences in LOS response sensitivities due to individual demographic and other attributes [see, for example, Bhat (*11,12*), Bhat and Sardesai (*2*), Cherchi and Ortúzar (*13*), and Morera *et al.* (*14*)]. In this research, our emphasis is on another element of systematic response heterogeneity – systematic response heterogeneity originating from a nonlinear response to LOS attributes. The underlying argument is that just as ignoring LOS sensitivity variations due to individual characteristics can lead to biased model estimates and an inflated estimate of unobserved heterogeneity across individuals, so can ignoring sensitivity variations due to non-linear responsiveness to LOS attributes. More precisely, different individuals encounter different values of the LOS attributes. Therefore, a nonlinear response sensitivity profile to LOS attributes would imply different sensitivities to LOS measures across individuals. If ignored, this nonlinear response-induced heterogeneity will, in general, lead to (a) biased parameter estimates, (b) an inflated estimate of unobserved heterogeneity, (c)

counterintuitive signs on LOS variables for a higher fraction of individuals if an unbounded distribution, such as the normal distribution, is used to characterize unobserved heterogeneity, (d) inaccurate estimates of willingness-to-pay measures, and (e) loss in model fit.

Interestingly, the issue of nonlinear response to LOS attributes has received very scant attention in the literature. Ben-Akiva and Lerman (*15, p. 174-176*) suggested piecewise linear (splines) and power series functions to test for the presence of nonlinear responsiveness, while Gaudry and Wills (*16*), Mandel *et al.* (*17*), and Lapparent and de Palma (*18*) used a Box-Cox functional form for introducing nonlinear responsiveness to travel time in their empirical studies. However, these studies have been in the context of traditional MNL models with no consideration of unobserved response heterogeneity. Besides, to our knowledge, no mode choice model in the literature has examined the potential repercussions of ignoring nonlinear responsiveness on model parameters, value of travel time/reliability estimates, model fit, and unobserved response heterogeneity, all at the same time. In this paper, we consider all three sources of response heterogeneity (nonlinear responsiveness, variations in responsiveness due to observed individual characteristics, and unobserved individual heterogeneity), and compare the empirical results of models that assume the traditional linear responsiveness to LOS attributes and that accommodate nonlinear responsiveness to LOS attributes.

The rest of the paper is structured as follows. The next section discusses the data source and sample formation for the empirical mode choice models. Section 3 presents the empirical results. Section 4 concludes the paper with a summary of important findings and avenues for further research.

2. DATA SOURCE AND SAMPLE FORMATION

The Austin Commuter Stated Preference Survey (ACS), administered through a web-based survey conducted by the Department of Civil Engineering at The University of Texas at Austin, is the data source used in the current study [see Morris and Adler (*19*) and Bhat and Sardesai (*2*) for a review of the advantages and limitations of a web-based survey]. The focus of the stated preference (SP) experiments was to acquire data to facilitate the efficient estimation of the trade-offs among travel time reliability, usual commute travel time, and travel cost. The stated choice experiments and the overall survey instrument were carefully designed and refined through several successive pilot surveys. The final SP survey presented four choice occasions per individual, with four possible alternatives in each choice occasion; Drive Alone, Shared Ride, Bus and Commuter Rail Transit (or rail for short). Five attributes were used to characterize each choice occasion: (1) Usual door-to-door commute travel time (in minutes), defined as the door-to-door commute travel time typically experienced (2) Additional possible door-to-door commute travel time (minutes) due to the uncertainty in traffic conditions, (3) Travel cost (in dollars), including any parking costs, (4) Availability of a grocery store near the CRT station, and (5) Presence of a child care place near the CRT station. The attributes were framed in the context of the one-way direct home-to-work commute trip, and the levels for the first three of the five attributes above were based on pivoting off the current commute trip characteristics reported by the individual. Such a customized presentation in the stated choice experiments increases the realism of the choice scenarios. To assess the effect of land-use design around the proposed rail system in Austin, we included "on" and "off" switches (levels) for the presence of a grocery store (fourth attribute) and a child care facility (fifth attribute) close to the proposed rail stations. Comprehensive details of the stated preference choice experimental design and survey content

are provided in Bhat and Sardesai (*2*) and Bhat (*20*). The survey instrument is available at http://www.ce.utexas.edu/commutersurvey/index.htm.

The data from the completed web surveys were downloaded in ASCII format, and then imported into SPSS (a data management and statistical software program) to label and code the variables appropriately. The commute and individual characteristics were then supplemented with the commute level of service characteristics, which were obtained by overlaying the geocoded home and work addresses of individuals onto a Geographic Information System (GIS)-based skim network provided by the Capital Area Metropolitan Planning Organization (CAMPO). Finally, several cleaning and screening steps were undertaken to ensure consistency in the records, and records with missing network level-of-service, location, or demographic information were deleted.

The final stated preference sample used for estimation consists of 317 individuals, each of whom responded to four stated choice questions, for a total of 1268 choice occasions. The mode choice shares in the sample are as follows: Drive alone (45.5%), shared ride (13.6%), bus (5.7%), and rail (35.2%).[2]

3. EMPIRICAL ANALYSIS

3.1 Model Structure
As just indicated, the sample for analysis is obtained from a stated preference exercise that includes repeated choices from 317 individuals. Since the intrinsic preferences, as well as the LOS sensitivities, of each individual apply to all choices made by that individual, we use a "panel" mixed logit model in the estimations. That is, we include individual-specific observed and unobserved factors in both the intrinsic preference term and the sensitivity to the LOS variables. In the next section, we discuss the individual-specific observed variables considered in the analysis. In Section 3.1.2, we describe the procedure to accommodate individual-specific unobserved factors in the analysis. Section 3.1.3 presents the procedures used to introduce nonlinearity in responsiveness to the level of service variables.

3.1.1 Individual-Specific Observed Factors
The individual-specific observed factors considered in the analysis as direct determinants of modal utilities (through the mode preference terms) and/or moderating determinants of modal utilities (through interaction effects with response sensitivities) include several groups of variables. These are household socio-demographics (household size, number of adults, number of workers, number of vehicles, vehicle availability per licensed adult, household income and family type), household residential characteristics (population density of residence zone; whether the individual resides in a core central business district, fringe central business district, urban business, urban, suburban, or rural area; and whether the individual lives in a single family detached unit, duplex unit, multifamily unit, or other type of housing unit), individual socio-demographic and employment characteristics (age, personal income, personal income as a

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[2] There is clear evidence of individuals overstating their tendency to use the rail mode, as pointed out and appropriately controlled for by Bhat and Sardesai (*2*) in their joint RP-SP study. However, since the emphasis of this paper is on LOS sensitivities and trade-offs, we do not delve into this issue in the current paper. Rather, we focus only on the SP sample to focus attention on the issue of non-linearity in responsiveness. Besides, the LOS sensitivities and trade-offs obtained in Bhat and Sardesai from the RP and SP samples were not statistically different. Thus we feel comfortable undertaking the analysis regarding response sensitivities using only the SP data, while also circumventing the additional econometric complications arising from RP-SP data fusion.

proportion of total household income, sex, race, work duration, work flexibility, work place employment density and the work place area type), the commute and midday stop-making attributes of the individual, both at a weekly level and on the day of the survey, and the land use design attributes around rail stations as presented in the stated choice experiments.

3.1.2 Individual-Specific Unobserved Factors

The variables listed above are associated with the observed characteristics of individuals. However, there are also likely to be several individual-specific unobserved factors influencing the intrinsic mode preferences and the sensitivity to LOS variables. It is natural to consider the combined effects of these individual-specific unobserved factors to be randomly distributed in the population, leading to the panel version of the mixed logit model [see Bhat (*12*) and Revelt and Train (*21*)]. The reader will note that the use of random parameters immediately captures the correlation across mode choice observations of the same individual. In the current analysis, we use a normal distributional assumption to characterize the variation across individuals in the random preference parameters and the sensitivity to the LOS variables. However, we do not allow random variation in the response to the cost variable because of the following reasons. First, fixing the coefficient provides stability to the convergence in the maximum likelihood estimation [see Bhat and Sardesai (*2*), Brownstone and Small (*9*), Revelt and Train (*21*), and Train (*22*)]. Second, using a fixed coefficient makes the willingness-to-pay computations convenient, since these values take a simple normal distribution. Third, as indicated by Train (*22*), typical distributional assumptions for the cost coefficient (such as the normal or lognormal distributions) raise the concern of the cost coefficient being positive and/or extremely small or large in magnitude for some individuals. This can substantially complicate the distribution of willingness to pay measures. Of course, fixing the cost coefficient also has the downside of imposing an assumption of the absence of unobserved cost sensitivity variations. Thus, exploring distributional assumptions for the LOS variables, including the cost coefficient, while circumventing the problems associated with a random cost coefficient remains an important avenue for further research.

In addition to capturing unobserved individual-specific effects in preference and LOS sensitivities, we control for covariance in individual-specific unobserved factors across alternatives. For example, an individual who is environmentally conscious may have a higher preference for bus as well as rail relative to her/his observationally identical peer group. Similarly, an individual who prefers the privacy and convenience of a car may have a higher preference for driving alone as well as shared ride relative to her/his observationally identical peer group. While the specific focus of the current study is not expressly on the competitive correlation patterns among the modes, it is important to consider these patterns because they could get incorrectly manifested in the unobserved preference and response sensitivity variations across individuals.

3.1.3 Linear versus Non-linear Specification of Level of Service Effects

The level of service (LOS) variables considered in the analysis include total travel time, out-of-vehicle-time, distance, travel cost, and travel time unreliability (travel time unreliability is defined as the additional travel time that may be needed to reach the work place because of uncertainty in traffic conditions; we will refer to travel time unreliability simply as unreliability in this paper).

We estimate panel mixed multinomial logit models using both a linear as well as a nonlinear specification for the non-cost level of service variables. In the nonlinear specification, a spline function (or piece-wise linear) approach is used. A systematic model specification procedure is followed in which finer spline segments are collapsed into more aggregate segments based on statistical fit, parsimony, and intuitive considerations, while also ensuring an adequate number of data points in each spline segment. The precise definition of the spline variables is discussed below in the context of total travel time, with non-linear "kinks" at 15 minutes and 30 minutes.

T_(0 to 15)	*= travel time*	*if*	*travel time < 15*
	= 15	*if*	*travel time ≥ 15*
T_ (15 to 30)	*= 0*	*if*	*travel time < 15*
	= travel time - 15	*if*	*15 ≤ travel time < 30*
	= 15	*if*	*travel time ≥ 30*
T _(30 plus)	*= 0*	*if*	*travel time < 30*
	= travel time - 30	*if*	*travel time ≥ 30*

Thus, if the travel time is 17 minutes, *T_(0 to 15)* would take a value 15 and *T_ (15 to 30)* would take a value 2. The utility contribution of a travel time of 17 minutes is equal to β_{tt1} *15 + β_{tt2} *2, where β_{tt1} is the marginal disutility for every minute in the 0-15 minute range and β_{tt2} is the marginal disutility in the 15-30 minute range.

An additional point to note here about the non-linear specification is that we considered only a single random response heterogeneity term for each of the non-cost LOS parameters. This implies that, though the mean of the response to LOS variables varies by the LOS value, the random distribution around the mean (due to unobserved factors affecting response sensitivity) is identical (that is, homoscedastic) for different LOS values. There are at least two reasons to maintain this assumption. First, we did not have any strong and clear *a priori* behavioral reasons to believe that the response sensitivity would vary by attribute value. Second, from a practical standpoint, allowing different variances for different ranges of LOS values would lead to an explosion in the number of random parameters whose variances have to be estimated, complicating the estimation process and potentially leading to convergence problems. However, allowing for variation in response heterogeneity by attribute value is an avenue for further research with large samples, since such an exercise can inform future efforts in the area of non-linear responsiveness to LOS variables.

In the rest of the paper, we will refer to the mixed logit model with a linear specification on the LOS parameters as the linear model, and the one with a nonlinear specification on the non-cost LOS parameters as the nonlinear model.

3.2 Model Results
The final specifications for the linear and nonlinear models were determined after systematic testing of several alternative specifications. The only LOS variables that turned out to be statistically significant in the final specifications were travel cost, total travel time, and travel time unreliability. Also, the only correlation in individual-specific unobserved factors across alternatives that emerged to be important in the final specifications was the one corresponding to the utilities of the drive alone and shared-ride modes. The final model specification results are presented in Table 1. The log-likelihood function value at convergence for the linear model is

-1118.49 and for the nonlinear model is -1093.60 (see last few rows of Table 1). The two models are not comparable using a nested likelihood ratio test since the preferred specification of the nonlinear model does not nest the preferred specification from the linear model. However, a non-nested likelihood ratio test is appropriate. Specifically, the probability that the difference in the adjusted rho-bar squared ($\bar{\rho}_c^2$) values between the two models of 0.0135 (= 0.2427–0.2292), as reported in Table 1 could have occurred by chance is less than $\Phi\left\{-[-2\times 0.0135\times L(C)+(20-15)]^{0.5}\right\}$ [see Ben Akiva and Lerman (*15, p. 172*)]. This value is almost zero, indicating that the difference in adjusted rho-bar squared values between the two models is highly statistically significant and that the nonlinear model is to be preferred over the linear model in terms of model fit.[3]

3.2.1 Overall Modal Preferences

The variables influencing overall modal preferences in Table 1 are similar in both the linear utility and non-linear utility models. The results indicate that individuals from high income-earning households are unlikely to use the bus mode. We did not find any corresponding negative income effect for the rail mode, suggesting that, while individuals from high income households perceive traveling on the bus as not being consistent with their social status, they may have a more positive image of the rail mode. The effect of personal income (as a fraction of total household income) indicates that high income earners in the household avoid the shared ride and bus modes. Again, there is no negative impact of personal income on the rail mode. The workplace employment density variable has a positive impact on the use of the bus mode, perhaps due to the quality attributes of bus service in such areas that are not captured by costs, times, and reliability measures. Finally, the commute and mid-day stop-making variables have the expected effect of increasing the use of the auto modes of travel. These results are consistent with those found by Bhat and Sardesai (*2*), who also used the ACS survey in their analysis.

The mean value of the alternative specific constants in Table 1 does not have any substantive interpretation because of the presence of continuous explanatory variables in the preferred specifications. The standard deviations characterizing the unobserved preference heterogeneity across individuals are highly statistically significant for all modes, except the bus mode [note that we are able to estimate all the modal standard deviations because the preference heterogeneity is captured across individuals, based on repeated choice data from each individual; see Bhat and Lockwood (*23*) for a more extended discussion].

3.2.2 Response to LOS Variables

3.2.2.1 Mean Response The mean response coefficients on cost, travel time, and unreliability are statistically significant in both the models and show the expected negative signs. Rather surprisingly, we did not find any moderating effects of individual characteristics on the response to the LOS variables in the current empirical context.

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[3] One can also compute a nested likelihood ratio test by including the unobserved travel time and travel time unreliability standard deviations in the non-linear model (even though these are statistically insignificant at the 5% level). Such an expanded non-linear model had a convergent log-likelihood value of -1091.90. The linear model is now a restricted version of this expanded non-linear model with 7 restrictions. The corresponding nested likelihood ratio test statistic for the presence of nonlinearity in travel time and travel time unreliability effects is 53.2, which is substantially larger than the chi-squared table value with 7 degrees of freedom at any reasonable level of significance. Thus, the nonlinear model is to be preferred over the linear model.

In the non-linear model, the spline effects show that an increase in travel time is not very onerous at low time ranges (0 to 15 minutes), perhaps because of the overall low disutility when travel times are very short. Similarly, at very high travel time values (greater than 60 minutes), individuals appear to get relatively insensitive to an additional minute of travel time. The sensitivity is highest in the middle 30-60 minutes range. Interestingly, the response to unreliability in the non-linear model shows a monotonic decreasing trend compared to the non-monotonic travel time trend just discussed. In particular, the effect of one minute of unreliability is highest up to 15 minutes, and starts fading rapidly after that. This is quite intuitive. For instance, one would expect that a mode that is generally "reliable" will more easily become the target of disappointment when it is unreliable compared to a mode that is perennially unreliable

3.2.2.2 Unobserved Heterogeneity Across Individuals The standard deviation parameters in the linear utility model indicate significant random response heterogeneity across individuals to both travel time and unreliability, As indicated earlier in the paper, the random coefficients on travel time and reliability variables were specified as normally distributed, which necessarily implies that a certain percentage of decision makers have positive coefficients on the travel time and unreliability variables. The linear model indicates a small percentage (4.14%) of the population with positive sensitivity to travel time, but a rather high percentage (20.45%) with positive sensitivity on unreliability.

In contrast to the linear model, the standard deviations corresponding to the unobserved individual heterogeneity for travel time and unreliability turned out to be statistically insignificant. There are two points to note based on this result. First, the significant unobserved individual heterogeneity in response to travel time and unreliability in the linear model appears to be an artifact of not considering the nonlinearity in LOS responsiveness. That is, different individuals are presented with different values of LOS variables in the stated choice experiments based on a pivot design off their current actual commute LOS variables. To the extent that the current commute LOS variables will have a distribution, the stated choice LOS variables also exhibit systematic variations across individuals (with some individuals being presented with LOS variables in the lower end of the spectrum than others). The responsiveness of the individuals will consequently vary because of the nonlinearity in responsiveness. If this non-linearity is not considered, it will be manifested in the form of "spurious" unobserved individual heterogeneity effects. In the current empirical context, the results indicate that the "spurious" unobserved individual heterogeneity in response to the travel time and unreliability variables in the linear model can be explained entirely by the nonlinear responsiveness. A second point to note from the statistical insignificance of the unobserved individual heterogeneity terms in the nonlinear model is that travel time and unreliability are perceived negatively by all individuals (unlike in the linear model case). This cautions against interpretations of positive utility of travel time and deliberate risk-taking behavior [see Mokhtarian and Redmond (*24*), Brownstone and Small (*9*), and Senna (*25*)] if a nonlinear specification is not first attempted and appropriately accommodated.

3.2.3 Implied Money Values of Travel Time and Unreliability
The LOS response coefficients can be translated to equivalent money values of travel time and unreliability, as shown in Figures 1.1 and 1.2. In each of these figures, the solid line indicates the linear model result, while the dashed profile indicates the nonlinear model result.

The value of time (VOT) results in Figure 1.1 show a flat money value of $11.64 per hour from the linear model. On the other hand, the VOT profile for the nonlinear model shows substantial variation (nonlinearity) in the willingness to pay for travel time improvements. Specifically, the VOT value for the nonlinear model is about $4.42 per hour in the first 15 minutes, increases to about $18.74 per hour in the 40-50 minute range, and then dips down to $3.29 per hour beyond 60 minutes.

The value of reliability (VOR) results in Figure 1.2 indicate the flat money value of $4.49 per hour from the linear model. Again, there are significant variations in the VOR values based on the level of existing unreliability in the nonlinear model. The VOR profile shows the following values: $8.88 per hour (0-15 minutes), $2.85 per hour (15-30 minutes), and $1.39 per hour (30-40 minutes)[4]. A comparison of the VOR values between the linear and non-linear models indicates that the VOR obtained from the linear model is severely underestimated in the first 15 minute range, and overestimated beyond 15 minutes of unreliability in travel time. In fact, it is the severe bias toward zero of the mean response estimate on unreliability in the first 15 minutes in the linear model, along with the overestimation of unobserved heterogeneity, that is leading to the large percentage (20.45 %) of the population with positive values on unreliability in the linear utility model. It is interesting to note that other studies have also reported linear models with a relatively high percentage of the population with positive values on unreliability [for example, see Brownstone and Small (*9*) and Bhat and Sardesai (*2*)]. Our results suggest that these large percentages, like in the linear model of the current study, may be spurious manifestations of ignoring nonlinearity in the unreliability responsiveness.

The mean VOT and VOR estimates from the linear model indicate that the mean VOR is about 39 % of the mean VOT. The VOR to VOT percentage is about the same even in the linear models of Bhat and Sardesai (*2*) and Brownstone and Small (*9*). However, the situation is quite different when the nonlinear model is used to compute the VOR to VOT ratio. The results indicate a very high VOR to VOT percentage of 201 % in the first 15 minutes, indicating that commuters consider unreliability to be much more onerous than travel time when both are within 15 minutes. However, the percentage drops dramatically beyond 15 minutes, with a value of 46% between 15-30 minutes, 9% between 30-40 minutes, 7.5 % between 40-50 minutes, 11 % between 50-60 minutes, and 42 % beyond 60 minutes.[5] An important implication of these VOR to VOT percentages is that transportation services that get people from their origin to their destination quickly on average should also be designed to be very reliable. Overall, the results once again suggest the kind of biases that can potentially occur in VOT and VOR computations if nonlinearity in response to LOS variables is ignored.

4. CONCLUSIONS AND FUTURE WORK
This research paper has identified and demonstrated the importance of incorporating nonlinear responsiveness to travel time and travel time unreliability in travel mode choice modeling. This is achieved by comparing the distributions of the travel time and unreliability response coefficients from a model with linear responsiveness and another model with nonlinear responsiveness, while controlling for preference heterogeneity and correlations between modal

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[4] We are unable to estimate the profile beyond 40 minutes because no individual in the stated choice experiments is presented with an unreliability value beyond 40 minutes. For the Austin area, unreliability values of more than 40 minutes would be difficult to comprehend.

[5] The VOR to VOT percentages beyond 40 minutes are computed by maintaining the VOR value of $1.39 per hour in the 30-40 minutes range to beyond 40 minutes.

utilities. The linear and non-linear models take the form of a mixed multinomial logit structure, The empirical analysis uses the Austin Commuter Stated Preference Survey data to examine commute travel mode choice.

The nonlinear specifications for travel time and travel time unreliability indicate that commuters place a small value to travel time, and a very high value to travel time reliability, in the first 15 minutes. Beyond 15 minutes, however, the valuation of travel time increases rapidly, while that of travel time reliability drops dramatically. In addition to clearly indicating the nonlinear nature of responsiveness to travel time and travel time unreliability, the results indicate that ignoring nonlinear responsiveness leads to (a) an inflated estimate of unobserved heterogeneity, (b) counterintuitive signs on LOS variables for a higher fraction of individuals, (c) inaccurate estimates of willingness-to-pay measures, and (d) loss in model fit.

While most of the findings from the study are plausible and intuitive, it is important to note that the results may be different in other empirical settings. In addition, data from stated preference (SP) and revealed preference (RP) surveys may provide a better idea about the specific trends in the value of time (VOT) and the value of unreliability (VOR).

ACKNOWLEDGEMENTS

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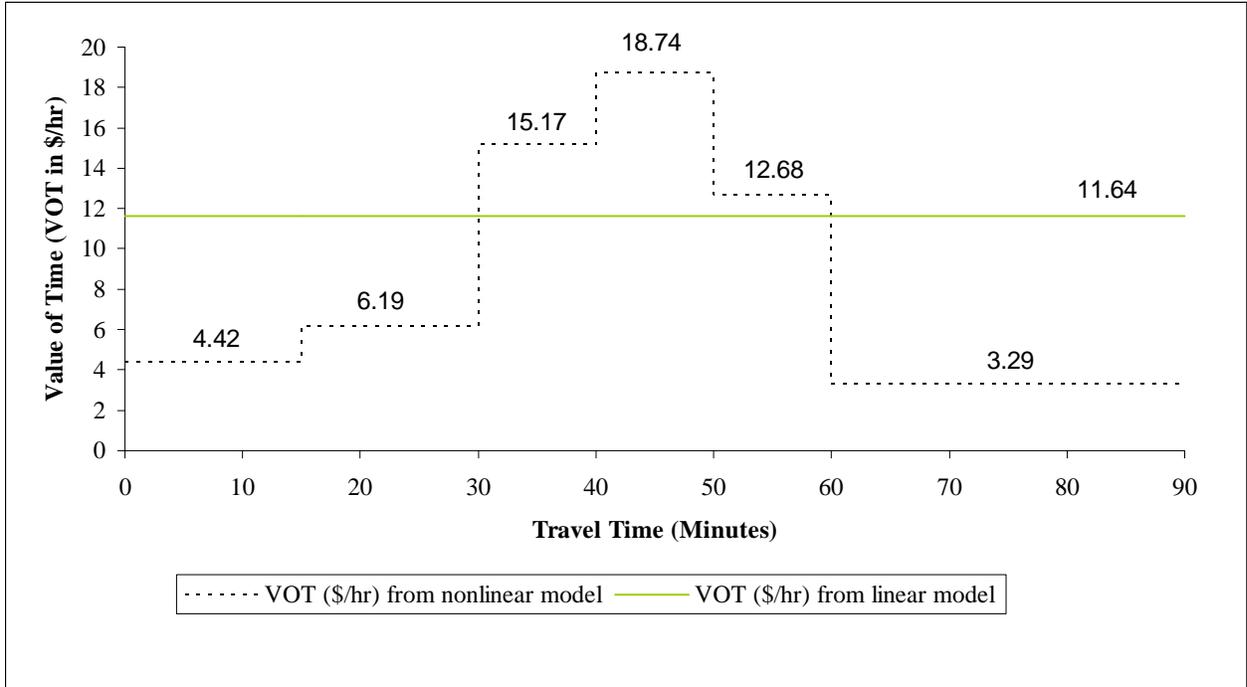**LIST OF FIGURES AND TABLE**

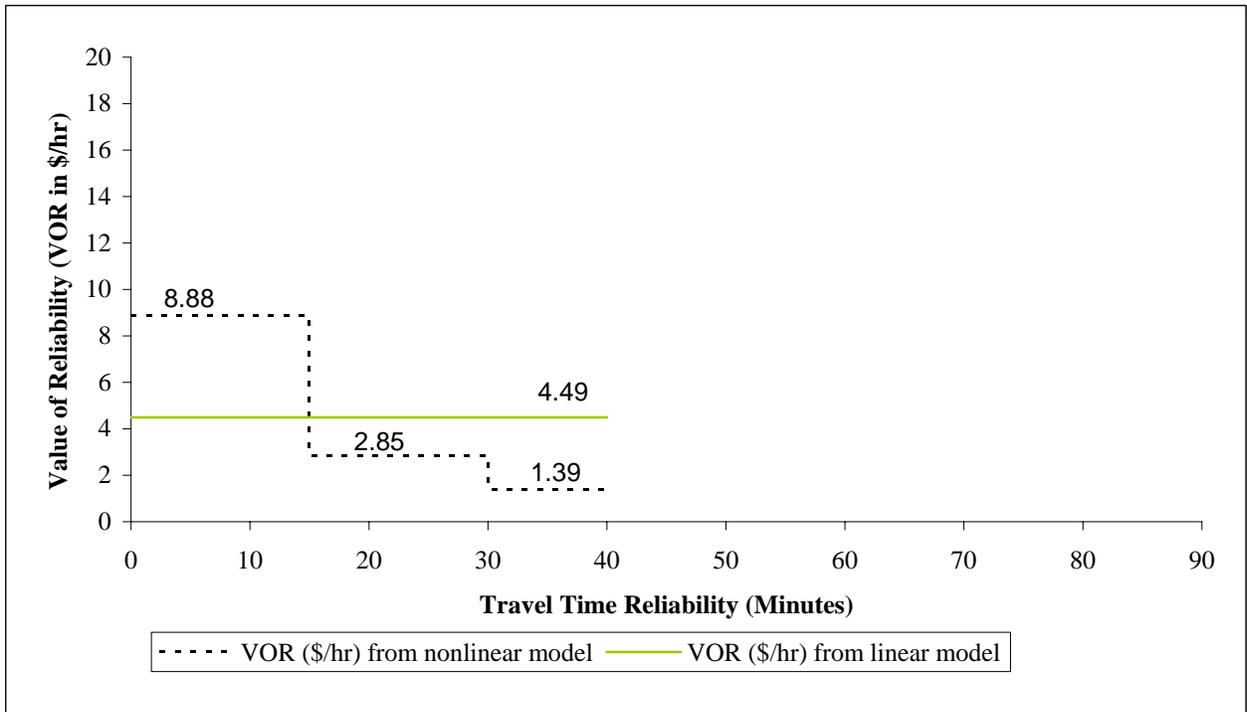**FIGURE 1.1 Trends in Value of Time (VOT).**

FIGURE 1.2 Trends in Value of Reliability (VOR).

TABLE 1 Final Specification of Linear and Nonlinear Models

Variable	Linear Model		Nonlinear Model	
	Estimate	t-stat	Estimate	t-stat
Preference				
Observed preference heterogeneity				
Household income (10^5 \$/year) (specific to bus)	-1.8126	-3.15	-1.5503	-3.11
Personal income as a fraction of HH income (specific to shared ride and bus)	-1.4374	-1.81	-1.5072	-2.09
Work place employment density (specific to bus)	0.0059	2.36	0.0053	2.10
Makes commute stop(s) on at least one day per week (specific to shared ride)	1.3467	2.44	1.0799	1.77
Makes mid-day stop(s) on every day of the week (specific to drive alone)	2.7437	2.45	3.3965	2.63
Mean preference				
Constant (specific to drive alone)	0.0000	Fixed	0.0000	Fixed
Constant (specific to shared ride)	-2.8457	-3.72	-2.4240	-3.03
Constant (specific to bus)	-0.6616	-0.86	-0.7938	-1.02
Constant (specific to rail)	-1.3237	-3.00	-1.6635	-3.36
Unobserved preference heterogeneity				
Standard deviation_ Constant (specific to drive alone)	3.5063	7.97	3.3052	8.97
Standard deviation_ Constant (specific to shared ride)	1.0194	1.57	0.8263	2.64
Standard deviation_ Constant (specific to bus)	0.2481	0.48	0.0191	0.09
Standard deviation_ Constant (specific to rail)	2.8557	7.53	2.6826	7.28
Response				
Mean response				
Linear Model				
Cost(\$)	-0.6010	-6.15		
Travel time in minutes	-0.1055	-7.54		
Unreliability of travel time in minutes	-0.0407	-4.28		
Nonlinear Model				
Cost (\$)			-0.5884	-6.38
Travel time spline effects				
0 – 15 minutes			-0.0433	-1.00
15 – 30 minutes			-0.0607	-2.78
30 – 40 minutes			-0.1488	-5.30
40 – 50 minutes			-0.1838	-6.06
50 – 60 minutes			-0.1243	-3.88
60 minutes and above			-0.0323	-1.91
Unreliability spline effects				
0 – 15 minutes			-0.0871	-3.78
15 – 30 minutes			-0.0279	-2.06
30 – 40 minutes			-0.0136	-0.50
Unobserved response heterogeneity				
Standard deviation_Travel time	0.0608	3.97		
Standard deviation_Unreliability of travel time	0.0493	3.73		
Standard deviation of common error-component between drive alone and shared ride utilities	1.1853	3.08	1.6741	3.43
Log likelihood (Constants) -- L(C)	-1470.50		-1470.50	
Log likelihood (Convergence) -- L($\hat{\beta}$)	-1118.49		-1093.60	
Adjusted Rho-bar squared-- $\overline{\rho}_c^{\,2}$	0.2292		0.2427	

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* The adjusted rho-bar squared value with respect to the constants only model is computed as

$\overline{\rho}_c^{\,2} = 1 - [(L(\hat{\beta}) - K)/L(C)]$, where K is the number of parameters excluding the constants.