Multiple discrete-continuous choice models with additively separable utility functions and linear utility on outside good: Model properties and characterization of demand functions

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Abstract

In this paper, we enhance the current understanding of the properties of multiple discrete-continuous (MDC) choice models with additively separable (AS), independent and identically distributed (IID) utility functions, and linear utility form on the essential outside good. First, we highlight that the prior implementations of this model in the literature ignore primal feasibility conditions related to the budget constraint and the essential nature of outside good in formulating the model likelihood function. Second, we evaluate the suitability and performance of the model for alternative consumption patterns relative to the budget. In addition, we provide a systematic comparison of the performance of MDC choice models with the linear outside good utility form (*i.e.*, the $L\gamma$-profile model) and those with the non-linear outside good utility form (*i.e.*, the $NL\gamma$-profile model). Third, for the $L\gamma$-profile model with infinite budgets (*i.e.*, when a very small proportion of the budget is allocated to inside goods), we derive the distributions of the resulting optimal demand functions and analytic expressions for the corresponding first and second moments, and identify a property that makes it easy to estimate the utility function parameters of an inside alternative even when consumption data is not available for other alternatives. In addition, perhaps for the first time in the literature, we show how an independent system of Tobit models can be derived as a restricted version of the utility-theoretic $L\gamma$-profile MDC model structure. Finally, we apply the model for an empirical analysis of expenditure patterns of leisure trips from a domestic tourism survey sample of households in India.

1. INTRODUCTION

1.1. Background

Multiple discrete-continuous (MDC) choice models have now become the workhorse for analysis of consumer choices involving the allocation of resources such as time and money to a set of exhaustive discrete choice alternatives that are not mutually exclusive (see, for example, Bhat, 2005; von Haefen *et al.*, 2004; Kim *et al.*, 2002; Habib and Miller, 2009; Eluru *et al.*, 2010; Enam *et al.*, 2018; Calastri *et al.*, 2021). In such choice situations, consumers potentially choose multiple choice alternatives, but not necessarily all available alternatives; hence the term multiple discreteness (Hendel, 1999; Dubé, 2004). In addition, they make decisions on how much of the available resources to allocate to each of the chosen alternatives; hence the term multiple discrete-continuous (MDC) choices (Bhat, 2005).

Most of the MDC models in use today are based on the classical consumer choice theory of random utility maximization (RUM), subject to constraints on available resources for consumption and non-negativity of consumption amounts. Specifically, consumers are assumed to optimize a direct utility function $U(\boldsymbol{x})$ over a bundle of non-negative consumption quantities $\boldsymbol{x} = (x_1, ..., x_k, ..., x_K)$ subject to a linear budget, as:

$$Max\ U(\boldsymbol{x}) \text{ such that } \boldsymbol{x}.\boldsymbol{p} = E \text{ and } x_k \geq 0\ \forall\ k = 1, 2, ..., K \tag{1}$$

In the above equation, $U(\boldsymbol{x})$ is a quasi-concave, increasing and continuously differentiable utility function with respect to the consumption quantity vector $\boldsymbol{x}$, $\boldsymbol{p} = (p_1, ..., p_k, ..., p_K)$ is the vector of prices of unit consumptions for all goods, and E is the budget for total expenditure. The consumption quantity vector $\boldsymbol{x}$ may or may not include an outside good, which, when included, is a composite good that represents all goods other than the inside goods of interest to the analyst. Typically, the outside good is treated as a numeraire with unit price, based on the assumption that the prices of goods combined into the outside category do not influence the expenditure allocation among inside goods (Deaton and Muelbauer, 1980).

A commonly used approach to work with the above problem is based on the Karush-Kuhn-Tucker (KKT) conditions of optimality. Since the utility function is stochastic, the resulting KKT conditions are stochastic, which form the basis for deriving likelihood expressions for observed consumptions and deriving the demand function distributions. Due to the central role played by the KKT conditions, MDC choice models based on this approach are also called Kuhn-Tucker (KT) demand systems.

The form of the utility function $U(\boldsymbol{x})$ governs the characteristics of optimal consumptions resulting from the above utility maximization problem. A typical assumption in this context is that the contribution to $U(\boldsymbol{x})$ from different goods is additively separable (AS). Another typical assumption is that the utility $u(x_k)$ accrued from consuming a good k rises non-linearly with consumption to follow the law of diminishing marginal utility and allows corner solutions (zero consumptions) to the utility maximization problem. In most applications, however, the numeraire outside good is not associated with corner solutions in that the consumers are assumed to always allocate some part of the budget to it (*i.e.,* the outside good is an essential good). Further, while most model formulations in the literature specify a non-linear utility form for the outside good, Bhat (2018) relatively recently introduced the linear utility form for the outside good within a traditional γ-profile (see Bhat, 2008), which he labels as the $L\gamma$-profile model, with the additively separable (AS) utility form as below:

$$U(\boldsymbol{x}) = \psi_1 x_1 + \sum_{k=2}^{K} \gamma_k \psi_k \ln\left(\frac{x_k}{\gamma_k} + 1\right) \tag{2}$$

This formulation has also recently been considered by Palma and Hess (2020), Bhat *et al.*, (2020), and Mondal and Bhat (2021). These studies cite various advantages of using the linear utility form (as opposed to a non-linear form) for the outside good. These advantages are: (a) the ability to infer the influence of covariates on discrete choices separately from that on continuous choices (Bhat, 2018), and (b) the ability to model consumption data even if the budget information is unknown (Bhat *et al.,* 2020; Palma and Hess, 2020) and when the consumption data is grouped into intervals (Bhat *et al.,* 2020). However, the above papers do not undertake a detailed and systematic evaluation of the performance of the $L\gamma$-profile model for different consumption patterns. While the study by Palma and Hess (2020) presents a simulation evaluation of the $L\gamma$-profile model, none of the studies delve into the reasons for any differences in efficacy for different consumption patterns.

1.2. The Current Study

MDC model specifications and applications with an $L\gamma$-profile formulation will likely increase in the future, due to the potential advantages offered by a linear utility specification on the outside good. However, the implications of using a linear utility on the outside good vis-à-vis that of a non-linear utility function are not yet fully understood. The current study contributes in this direction along two primary research thrusts: (1) understanding the properties of RUM-

based $L\gamma$-profile MDC choice models, and (2) deriving analytic expressions for the distributions of the Marshallian demand functions for $L\gamma$-profile MDC models. Each of these issues is discussed briefly in turn in the next two sections.

1.2.1. Properties of RUM-based $L\gamma$-profile *MDC choice models*

The $L\gamma$-profile model has been motivated by its value for situations when the budget is unobserved. However, several applications (for example, modeling daily activity time-use) have a natural budget on the resource to be allocated. While such applications can be readily modeled through the MDC model with a non-linear utility function for the outside good (as proposed in Bhat (2008), which we will henceforth refer to as the $NL\gamma$-profile model[1]), a natural question is whether the $L\gamma$-profile can be used to model situations with finite and known budgets. Further, while it is the case that the $NL\gamma$-profile model cannot be used in situations with unobserved budgets (the $NL\gamma$-profile requires the budget information) and the $L\gamma$-profile model has been proposed for this purpose, it is important to investigate the suitability of the $L\gamma$-profile for different consumption patterns even when the budgets are unobserved. Furthermore, the current literature does not shed adequate light on which of the two specifications – $L\gamma$-profile and $NL\gamma$-profile – performs better and is suitable for different consumption patterns when the budgets are observed.

1.2.2. Distributions of Marshallian demand functions

Despite significant advances in the context of prediction with the MDC choice models, almost all work in this area resorts to using a combination of simulation (of the stochastic utility functions) and optimization for forecasting the Marshallian demand, policy analysis, and welfare calculations (for example, see von Haefen *et al.*, 2004; Pinjari and Bhat, 2021). Little work exists on characterizing the distributions of the Marshallian demand functions from MDC choice models and the corresponding elasticities. This is because it is not possible to analytically derive the distributions and moments of the distributions of the optimal demand functions for the $NL\gamma$-profile, due to the close tie between the continuous consumption value of any inside good with its corresponding discrete choice as well as the consumption value of the outside good. However, the $L\gamma$-profile of Eq. (2) breaks the strong linkage between the

[1] For completeness, the AS $NL\gamma$-profile utility function that we consider in this paper is as follows:
$U(\boldsymbol{x}) = \psi_1 \ln x_1 + \sum_{k=2}^{K} \gamma_k \psi_k \ln\left(\frac{x_k}{\gamma_k} + 1\right)$.

discrete choice of an inside good with the continuous consumption values for that good as well as the continuous consumption value of the outside good (Bhat, 2018). This should open up the possibility of deriving the distributions (and the corresponding moments) of Marshallian demand functions for such models, at least for specific situations. The ability to do so and having access to analytic expressions for the moments of the demand functions can help obviate the need for extensive simulations for prediction and policy analysis.

1.2.3. Research Objectives

The discussion above drives our efforts in this paper, with two primary objectives. The <u>first objective</u> is to elucidate the properties of MDC choice models with the $L\gamma$-profile utility function of Eq. (2). Specifically, the paper theoretically examines the suitability of the $L\gamma$-profile model for different consumption patterns (relative to the budget). In doing so, the paper highlights the importance of explicitly considering the primal feasibility conditions of the budget constraint and the essential nature of the outside good during parameter estimation. None of the implementations of the $L\gamma$-profile model in the literature consider these conditions during parameter estimation, which poses a risk of biased parameter estimation and erroneous prediction. In the current paper, we examine the theoretical appropriateness of the $L\gamma$-profile model[2] for the following three broad consumption patterns relative to the budget:

(i) A very small proportion of the budget is allocated to inside goods (in subsequent discussions, such a pattern is referred to as the *infinite* budget case),

(ii) A small (but significant) proportion of the budget is allocated to inside goods, and

(iii) A large proportion of the budget is allocated to inside goods.

An important clarification regarding the term *infinite budget* is in order here. Note that the term infinite budget does not necessarily imply that the budget is always literally infinite. It also applies to situations when a very small proportion of a finite budget is allocated to inside goods (or equivalently, the outside good allocation is very large compared to the allocation to inside goods). The use of the term infinite budget is justified for such situations because the budget amount does not have a bearing on the consumption patterns. More specifically, the optimal demand density, when integrated to the finite (but large) budget to obtain its moments, produces essentially the same results as one would get by integrating to infinity.

[2] In the rest of this paper, unless mentioned otherwise, we use the term $L\gamma$-profile model to refer to the additively separable (AS) model that does not consider the budget constraint and the essential nature of the outside good during parameter estimation. As discussed later, these issues do not arise with the AS $NL\gamma$-profile model, because both the budget constraint and the essential nature of the outside good are implicitly considered during parameter estimation (as with the $L\gamma$-profile, any reference to the $NL\gamma$-profile will refer to the additively separable form).

For each of the consumption patterns listed above, extensive simulations are conducted to confirm our theoretical expositions of the $L\gamma$-profile model vis-à-vis the $NL\gamma$-profile model, in the context of parameter bias and prediction performance. The paper proceeds to provide guidance on when to use the $L\gamma$-profile model vis-à-vis the $NL\gamma$-profile model (*i.e.*, which model is suitable for what type of consumption pattern).

The <u>second objective</u> of this paper is to derive analytic expressions for the distributions (and corresponding first and second moments) of the Marshallian demand functions resulting from $L\gamma$-profile utility function-based MDC choice models with infinite budgets, considering independent and identically distributed (IID) log-extreme value kernel error terms in the baseline utility (*i.e.*, $L\gamma$-profile MDCEV models) as well as IID log-normal kernel error terms in the baseline utility (*i.e.*, $L\gamma$-profile MDCP models). We also accommodate situations with choice alternative-specific upper bounds on consumptions, which result in a probability mass at the upper bound value. The analytic expressions we derive obviate the need to use simulation for prediction and policy analysis using MDC choice models with large budgets. Further, one can easily compute elasticities of the first and second moments of the demand distributions with respect to prices and covariates entering the utility functions.

In pursuing the two objectives identified above, we identify an important theoretical property of IID $L\gamma$-profile utility-based MDC models for the infinite budget case – the optimal consumption value of an inside good or its corresponding distribution does not depend on the presence or attributes of other choice alternatives. This is because the budget amount does not have a bearing on the consumption patterns, thereby the inside goods do not compete with each other for resources from the budget. This property, referred to as the *irrelevance of other alternatives* (IOA) property, allows estimation of the utility function parameters of any alternative as long as consumption data is available for that alternative even if information is not available on other alternatives in the choice set. In addition, we shed light on the relationship between the scale parameter in $L\gamma$-profile utility-based MDC models and the existence of finite moments for the Marshallian demands for the case of infinite budgets. Furthermore, we clarify identification issues in the $L\gamma$-profile MDC model (building on Bhat 2018 and Bhat *et al.*, 2020). In addition, we show that the Tobit models typically used to model censored demand data are a restricted version of the utility theoretic $L\gamma$-profile MDC model with infinite budget. We also develop a forecasting algorithm for the case when an $L\gamma$-profile utility-based MDC model is implemented for a situation with an observed and finite budget.

The rest of the paper is structured as follows. Section 2 presents a theoretical analysis for the case when $L\gamma$-profile utility function-based models are applied to the case of the three broad types of consumption patterns identified earlier. It also delves into issues associated with the estimation of scale parameter in the $L\gamma$-profile model. Section 3 involves simulation experiments to confirm the theoretical analysis in Section 2. Section 4 delves into $L\gamma$-profile models with infinite budgets and derives the distributions (and corresponding first and second moments) of optimal demand functions resulting from such models. It also discusses the relationship between the $L\gamma$-profile model with infinite budgets and Tobit models. Section 5 demonstrates the applicability of the $L\gamma$-profile models with infinite budgets for an empirical analysis of tourism travel expenditures in India. Section 6 summarizes and concludes the paper.

2. $L\gamma$-profile Utility Functions, Finite Budgets, and Essential Outside Good

2.1. Primal feasibility conditions in $L\gamma$-profile models

Consider the constrained utility maximization problem in Eq. (1) with an $L\gamma$-profile utility function as in Eq. (2). Assuming that the first M of the K available alternatives are chosen (*i.e.*, $x_k^* > 0, \forall\, k = 1, 2, ..., M$), the KKT conditions of optimality are as below (where, λ is the Lagrange multiplier):

$$\psi_1 = \lambda, \text{ since } x_1^* > 0$$

$$\frac{\psi_k}{\left(\frac{x_k^*}{\gamma_k} + 1\right)} = \lambda p_k, \text{ since } x_k^* > 0 \;\forall\; k = 2, 3, ..., M \tag{3}$$

$$\psi_k < \lambda p_k, \text{ since } x_k^* = 0 \;\forall\; k = M+1, ..., K$$

From the above KKT conditions, we obtain the optimal consumption values as follows for the consumed goods:

$$x_k^* = \left(\frac{\psi_k}{\psi_1 p_k} - 1\right)\gamma_k \;,\; k = 2, 3, ...M, \text{ with } \frac{\psi_k}{\psi_1 p_k} > 1. \tag{4}$$

Thus, the KKT conditions ensure that, for any consumed inside goods, the optimal continuous value of consumption will exceed zero. But the optimal consumptions should also satisfy the primal feasibility condition given by the budget constraint $x_1^* = E - \sum_{k=2}^{M} p_k x_k^*$, with $x_1^* > 0$.

Unfortunately, the budget E does not enter anywhere in the KKT conditions for the $L\gamma$-profile utility, except in its primal form that needs to be honoured but is not during estimation. When the data is such that the budget is very large relative to the expenditure on the consumed inside goods as given by $\sum_{k=2}^{M} p_k x_k^*$, the estimation will proceed such that, even without expressly imposing the condition that $x_1^* = E - \sum_{k=2}^{M} p_k x_k^* > 0,$ the condition may get met. Of course, when $E \to \infty,$ the condition $x_1^* > 0$ will automatically get satisfied during estimation. Therefore, in situations with a very large (or infinite) budget (relative to the allocation to inside goods), it is safe to assume that the outside good consumption will always be positive (i.e., the outside good acts as an essential good).

It is worth noting that all applications of $L\gamma$-profile utility models hitherto estimate model parameters using likelihood functions that ignore the primal feasibility condition during estimation. Such an approach might seem innocuous as it is similar to how a likelihood function is developed for $NL\gamma$-profile utility models. However, a subtle but important difference between the likelihood functions of $L\gamma$-profile utility models and $NL\gamma$-profile utility models is that the latter models implicitly embed the budget, and guarantee that $x_1^* > 0$[3].

2.2. An alternate $L\gamma$-profile model with finite budgets and essential outside good

During parameter estimation, the condition $x_1^* > 0$ can be maintained. Specifically, for x_1^* to be always positive, one can insert Equation (4) for x_k^* in the equation $x_1^* = E - \sum_{k=2}^{M} p_k x_k^* > 0$ to obtain the following condition:

[3] The KKT condition for the essential outside good in $NL\gamma$-profile utility models, whose utility function is of the form given by $\psi_1 \ln(x_1^*)$, is $\psi_1 / x_1^* = \lambda$, due to which the likelihood function includes x_1^* and, therefore, the budget constraint (since $x_1^* = E - \sum_{k=2}^{M} p_k x_k^*$). The optimal consumption of the consumed inside goods in the $NL\gamma$-profile is $x_k^* = \left(\frac{\psi_k x_1}{\psi_1 p_k} - 1\right)\gamma_k$ with $\frac{\psi_k x_1}{\psi_1 p_k} > 1,$ and $x_1^* = \left(E + \sum_{k=2}^{M} p_k \psi_k\right) \Big/ \left(1 + \sum_{k=2}^{M} \frac{\psi_k \gamma_k}{\psi_1}\right)$. x_1^* is always positive here, given ψ_k, p_k, γ_k are all positive for all inside goods, $\psi_1 > 0,$ and E >0.

$$\psi_1 > \frac{\sum_{k=2}^{M} \psi_k \gamma_k}{E + \sum_{k=2}^{M} \gamma_k p_k} \tag{5}$$

The above condition implies a truncation on the inside good's baseline preference based on the budget and other inside good parameters. Importantly, Eq. (5) ensures that the primal conditions corresponding to the budget constraint and positivity of the outside good consumption are satisfied. Therefore, the likelihood function of a model with $L\gamma$-profile utility functions should not only consider the KKT conditions identified in Eq. (3), but also include the condition in Eq. (5). Of course, when the budget tends to infinity, the exponential form for ψ_1 immediately guarantees the condition in Eq. (5), and no additional truncation needs to be considered in model estimation. That is, the likelihood function used in the literature so far (which does not include the budget information) for $L\gamma$-profile models work well for situations when the budget is large relative to the consumption on inside goods. One can estimate the model parameters and carry out predictions without knowledge of the budget (*i.e.*, even when the budgets are not observed).

Unfortunately, however, in the general case, the truncation condition in Eq. (5) will need to be considered, which leads to likelihood functions with multidimensional integrals and complicates the estimation of such a model. If the truncation condition is not considered in the likelihood function (as done by earlier studies), the resulting parameter estimates would be biased toward situations when the budget is very large compared to the allocation to inside goods (aka, the infinite budget case). And such a bias in estimation can potentially make it difficult to interpret covariate effects solely from the parameter estimates. Therefore, it is useful to assess if embedding the truncation condition during prediction (without doing so during estimation) can help mitigate forecasting errors. Such a prediction algorithm is presented in Section 2.4. However, before delving into the prediction algorithm, a specific issue related to the estimation of scale parameter in $L\gamma$-profile MDC choice models is worth discussing.

2.3. Estimation of scale in the $L\gamma$-profile model

Bhat (2018) discusses the fact that the scale of the error term in the logarithm of the baseline parameters of the inside and outside goods (assuming constant scale across all goods) is estimable in the $NL\gamma$-profile model. In that paper and in Bhat *et al.* (2020), issues arising with the estimation of scale in the $L\gamma$-profile model are discussed. These earlier papers do not

indicate a definitive theoretical identification problem that precludes the estimation of the scale when there is no price variation across goods. However, the papers raise issues related to empirical identification that may make it difficult to estimate a scale when there is no price variation and distinct γ_k parameters are estimated for each inside good. Here, we further study this issue and add some additional nuanced perspectives related to the identification of the scale parameter in the $L\gamma$-profile model.

Consider the following KKT conditions from Eq. (3).

$$\psi_1 = \lambda, \text{ since } x_1^* > 0$$

$$\frac{\psi_k}{\left(\frac{x_k^*}{\gamma_k}+1\right)} = \lambda p_k, \text{ since } x_k^* > 0 \ \forall \ k = 2,3,...,M \tag{6}$$

$$\psi_k < \lambda p_k, \text{ since } x_k^* = 0 \ \forall \ k = M+1,...,K$$

To complete the model specification, the utility function parameters (ψ_k, ψ_1, and γ_k) can be expressed as a function of observed and unobserved attributes of decision-makers and choice alternatives as: $\psi_k = \exp(\boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_k)$, $\psi_1 = \exp(\varepsilon_1)$, and $\gamma_k = \exp(\boldsymbol{\theta}'\mathbf{w}_k)$, where $\mathbf{z}_k$ and $\mathbf{w}_k$ are vectors of decision-maker and alternative attributes that influence ψ_k and γ_k, respectively, and ε_k are random utility terms specific to each good $k(k = 1,2,...,K)$. Note that ψ_1 is normalized to $\exp(\varepsilon_1)$, and does not include observed explanatory variables. Such a specification allows identification of parameters on attributes that do not vary across alternatives. While one can think of attributes that are specific to alternatives, it is unlikely for the outside good to have its own attributes as it typically represents a numeraire composite good (i.e., it represents all the other goods that are not of interest to the analysis). However, if there are such attributes that are specific to the outside good, an interpretable specification would include difference between the corresponding attribute value for the kth inside good and the outside good in $\mathbf{z}_k$.

With the above discussed statistical specification, and defining $\varepsilon_{k1} = \varepsilon_k - \varepsilon_1$, the KKT conditions can be rewritten as:

$$\boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_{k1} - \ln p_k = \ln\left(\frac{x_k^*}{\gamma_k}+1\right) \text{ when } x_k^* > 0;\ k = 2,3...,K$$
$$\boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_{k1} - \ln p_k < 0 \text{ when } x_k^* = 0;\ k = 2,3...,K \tag{7}$$

Consider the situation with no price variation (i.e., $p_k = 1\ \forall\ k = 2,3,...,K$), and write the above equations using standardised error terms (assuming homoscedastic error terms),

$$\sigma\left(\boldsymbol{\beta}'^*\mathbf{z}_k + \varepsilon^*_{k1}\right) = \ln\left(\frac{x_k^*}{\gamma_k}+1\right) \text{ when } x_k^* > 0;\ k = 2,3...,K$$
$$\sigma\left(\boldsymbol{\beta}'^*\mathbf{z}_k + \varepsilon^*_{k1}\right) < 0 \text{ when } x_k^* = 0;\ k = 2,3...,K \tag{8}$$

where, ε^*_{k1} is the standardised error difference, and $\boldsymbol{\beta}'^* = \frac{\boldsymbol{\beta}'}{\sigma}$.

It can be seen from the above set of equations that the identification of the scale parameter (i.e., σ parameter) is possible only through the first set of equations that are written for positive continuous consumption values (i.e., when $x_k^* > 0$). As discussed in Bhat (2018) and Bhat *et al.* (2020), empirical identification issues may arise with the estimation of σ for the model and a separate satiation parameter γ_k for each inside good. However, our explorations regarding estimation of scale in the absence of price variation (as will be evident in Section 3) suggest that such identification issues are not as severe as they were previously thought. In most situations (as our simulation experiments in Section 3 suggest), the scale parameter σ is estimable, even when there is no price variation. To understand the reason behind the estimability of the scale parameter, divide the first set of KKT conditions in Eq. (8) by σ on both sides. The resulting KKT conditions show $\frac{1}{\sigma}$ as a coefficient of $\ln\left(\frac{x_k^*}{\gamma_k}+1\right)$, which makes it possible to estimate σ in addition to γ_k ($k = 2,3...,K$). That said, a specific situation in which we noticed the estimation of scale parameter became difficult was when the allocation to inside goods constituted a large proportion of a limited budget, as discussed next.

When the allocation to an inside good is large but the budget is limited (even if the budget is unobservable), the corresponding γ_k parameter tends to be large due to the low satiation rate of consuming that good. However, the scale parameter of such an $L\gamma$-profile model becomes small so as to compensate for not recognizing the budget constraint in model estimation; lest the model should imply a large probability of unreasonably large consumption

values (greater than what can be afforded by the budget) for the good. With some datasets where the allocation to the inside goods is large but the budget is limited, the scale parameter keeps decreasing in magnitude during estimation, to an extent that it becomes difficult to estimate it. Such datasets with high consumption of inside goods (while the budget is limited) further exacerbate the difficulty in estimating scale due to low satiation rate, or, high values of γ_k. Specifically, the high values of γ_k parameters can potentially result in situations where $\frac{x_k^*}{\gamma_k}$ values become sufficiently smaller than 1, such that $\ln\left(\frac{x_k^*}{\gamma_k}+1\right)$ approximates to $\frac{x_k^*}{\gamma_k}$. In such a case, the first set of equations from Eq. (8) can be written as:

$$\begin{aligned} &\sigma\left(\boldsymbol{\beta}'^{*}\mathbf{z}_k+\varepsilon^{*}{}_{k1}\right)=\frac{x_k^*}{\gamma_k} \text{ when } x_k^*>0, \text{ or,} \\ &x_k^*=\sigma\,\gamma_k\left(\boldsymbol{\beta}'^{*}\mathbf{z}_k+\varepsilon^{*}{}_{k1}\right) \end{aligned} \tag{9}$$

In the above-discussed case, one can only estimate $\pi_k=\sigma\gamma_k$, and to estimate σ, at least one of the γ_k parameters must be fixed. However, when there is price variation, despite the scale being small in value, it may be generally possible to estimate it. To see this, Eq. (9), in the case of price variation, can be written as:

$$\begin{aligned} &\sigma\left(\boldsymbol{\beta}'^{*}\mathbf{z}_k+\varepsilon^{*}{}_{k1}-\frac{1}{\sigma}\ln p_k\right)=\frac{x_k^*}{\gamma_k} \text{ when } x_k^*>0, \text{ or,} \\ &x_k^*=\sigma\,\gamma_k\left(\boldsymbol{\beta}'^{*}\mathbf{z}_k+\varepsilon^{*}{}_{k1}\right)-\gamma_k\ln p_k \end{aligned} \tag{10}$$

In the above equation, γ_k for each inside good can be identified form the coefficient on $\ln p_k$ in addition to the scale parameter σ, as long as the unit price p_k is not equal to one.

In summary, the estimation of scale parameter in the $L\gamma$-profile MDC model is generally possible even in situations without price variation. However, in situations with limited budgets (even if unobservable) and relatively high allocations to inside goods, the estimation of the scale parameter together along with all the other satiation parameters (i.e., $\gamma_k\ \forall\ k=2,3,...,K$) may not be possible. As we establish later in this paper, the $L\gamma$-profile MDC model is anyway not suitable to model such consumption patterns and the $NL\gamma$-profile MDC model is more appropriate in such cases.

2.4. $L\gamma$-profile model forecasting with finite budgets and essential outside good

Given the data on model covariates $(\mathbf{z}_k)$, budget (E), and the model parameters $(\boldsymbol{\beta}', \gamma_2, \gamma_3, ..., \gamma_K, \sigma)'$, the following procedure is proposed:

Step 1: Draw K independent realizations of ε_k (say η_k), one for each good k $(k = 1, 2, ..., K)$ from the corresponding distribution with a location parameter of 0 and a scale parameter equal to the estimated σ $\left(or \ \frac{1}{\mu} \right)$ value.

Step 2: Compute $H_{k,0} = \eta_k - \tilde{V}_{k,0}$ for each inside good k=2,.3,…,K using the inputs, and set $H_{1,0}$ for the outside good to be an arbitrary value higher than the maximum of the $H_{k,0}$ values across the inside goods, where $\tilde{V}_{k,0} = V_1 - V_k$, and $V_k = \boldsymbol{\beta}'\mathbf{z}_k - \ln p_k$ (k = 2, 3,…, K) . $H_{k,0}$ is same as $\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k + \varepsilon_k$ (since V_1 corresponds to the outside good and is equal to zero).

Step 3: Re-order the goods in descending order of $H_{k,0}$; let **G** be the vector of the re-ordered indices of the inside goods (with the outside good appearing as the first entry and the ordering of the inside goods starting from position 2); set a new index m (m=1,2,…,K) for this new ordering of the outside and inside goods. Let $\tilde{\mathbf{H}}_0$ be the re-ordered vector of values of $H_{k,0}$ so that $\tilde{\mathbf{H}}_0 = (H_{1,0}, \tilde{H}_{2,0}, ..., \tilde{H}_{m,0}, ...\tilde{H}_{K-1,0})$, where $\tilde{H}_{m,0} = \underset{\substack{k \\ k \neq G[m]}}{Max}(H_{k,0})$ for m=2,3,…,K.

Step 4: Set M = 2.

Step 5: If $\eta_1 > \tilde{H}_{M,0}$, set the consumptions of all the re-ordered inside goods m=M to m=K to zero. STOP.

Step 6: If $\eta_1 < \tilde{H}_{M,0}$, compute $\psi_M = \exp(\boldsymbol{\beta}'\mathbf{z}_M + \eta_M)$.

Step 7: If $\eta_1 > ln\left(\frac{\sum_{m=2}^{M} [\gamma_m \psi_m]}{E + \sum_{m=2}^{M} p_m \gamma_m} \right)$, declare the inside good M as being selected for consumption and forecast the continuous value of consumption as follows: $x_M^* = \left[\exp(\eta_M - \eta_1 - \tilde{V}_{M0}) - 1 \right] \gamma_M$.

Set $M = M + 1$. Go to Step 5.

Step 8: If $\eta_1 < ln\left(\frac{\sum_{m=2}^{M}[\gamma_m \psi_m]}{E+\sum_{m=2}^{M} p_m \gamma_m}\right)$, declare the inside good M as not being selected for consumption (*i.e.,* $x_m^* = 0 \ \forall \ m \geq M$) and $x_1^* = E - \sum_{m=2}^{M} p_m x_m^*$. STOP.

The above forecasting procedure is similar to the forecasting approach for the $NL\gamma$-profile in that the resources are allocated in the decreasing order of the baseline preferences. However, in steps 7 and 8 of the procedure, the truncation condition in Eq. (5) is verified to ensure that the primal feasibility conditions are met. Specifically, if the truncation condition in Eq. (5) is violated (step 8), the allocation of all the subsequent goods, including the good under consideration, is made equal to zero, and the remaining budget is allocated to the outside good. This is akin to truncating the baseline preference parameters of the inside goods in accordance with the truncation condition of Eq. (5) such that they are not chosen. This ensures that the essential outside good is always chosen with a positive consumption and the budget constraint is always met.

3. Simulation Experiments

In this section, simulation experiments are presented to evaluate the suitability of the $L\gamma$-profile utility model (that does not consider in its likelihood function the truncation condition) vis-à-vis the suitability of the $NL\gamma$-profile utility model for different consumption patterns – both in terms of the model's ability to recover true parameters and its predictive performance.

3.1. Simulation experiment design

We performed simulation experiments on synthetic data with four choice alternatives. Specifically, the following $L\gamma$-profile utility structure was considered, with the first alternative as the essential Hicksian outside good and three other alternatives as inside goods:

$$U = \psi_1 x_1 + \psi_2 \gamma_2 \ln\left(\frac{x_2}{\gamma_2}+1\right) + \psi_3 \gamma_3 \ln\left(\frac{x_3}{\gamma_3}+1\right) + \psi_4 \gamma_4 \ln\left(\frac{x_4}{\gamma_4}+1\right), \text{ where} \tag{11}$$

$$\psi_1 = \exp(\varepsilon_1)$$
$$\psi_2 = \exp(ASC_2 + \beta_{2,z_1} z_1 + \beta_{2,z_2} z_2 + \varepsilon_2)$$
$$\psi_3 = \exp(ASC_3 + \beta_{3,z_1} z_1 + \beta_{3,z_2} z_2 + \varepsilon_3)$$
$$\psi_4 = \exp(ASC_4 + \beta_{4,z_1} z_1 + \beta_{4,z_2} z_2 + \varepsilon_4)$$

The model covariates influencing the baseline preferences were simulated as $z_1 \sim Normal(4,4)$ and $z_2 \sim Bernoulli(0.5)$. The satiation parameters were expressed as $\gamma_2 = \exp(w_2)$, $\gamma_3 = \exp(w_3)$, and $\gamma_4 = \exp(w_4)$. In all the simulations, we assumed ε_k to be IID extreme value distributed with scale parameter $\sigma = 0.4$ (equivalently $\mu = 2.5$). Also, in all subsequent data generation, we assume that unit prices for all goods are unity (i.e., no price variation).

We simulated optimal consumptions for the following three budget and consumption pattern scenarios:

(a) Scenario 1: Infinite budget scenario, where the budget is set to 50,000 units and the total allocation to inside goods is very small (less than 1% of the budget, on average),

(b) Scenario 2: A small but significant proportion (on average, 16%) of the budget is allocated to inside goods, with the budget amount as 1,000 units, and

(c) Scenario 3: A large proportion (on average, 43%) of the budget is allocated to inside goods, with the budget amount as 1,000 units.

The parameter values used to simulate optimal consumptions for each of the above scenarios are presented in Table 1.

We simulated 50 datasets (for different simulated values of ε_k), each of 10,000 individuals (for different values of z_1 and z_2) for each of the above scenarios using the forecasting algorithm discussed in the previous section. Subsequently, we estimated the MDC choice model (one that ignores the truncation condition) to assess the ability of the model to retrieve parameters. In addition, we evaluated the prediction accuracy using the estimated parameters (average of the parameter estimates across the 50 datasets) and the forecasting algorithm discussed in the previous section.

3.2. Parameter recovery for the $L\gamma$-profile model

As can be observed from the results for Scenario 1 in Table 1, for situations with very large (that is, infinite) budgets, the $L\gamma$-profile model (that ignores the truncation condition) is able to recover the true parameters very well, with a very low average absolute percentage bias (APB) of 0.22 and small finite sample standard errors (FSSE) that are close to the asymptotic standard error (ASE) values.

For Scenario 2 where a small (but significant) proportion of the budget is allocated to inside goods, ignoring the truncation condition in Eq. (5) in model estimation has resulted in a

non-negligible bias in parameter estimates, even if the FSSE continues to be close to the ASE values. In this scenario, the average APB rises to 25.58 %. For Scenario 3 where the proportion of the budget allocated to inside goods becomes large, the bias in parameter estimates becomes large (overall APB is 83.53 %). Overall, the results from Table 1 confirm our theoretical observation earlier that, as the proportion of the budget allocated to inside goods increases, the need for accommodating the truncation condition increases, and ignoring it will increase bias in parameter estimation. In fact, in Scenario 3, some of the recovered parameter values are even different in sign from that of the true parameter values. Further, as discussed in Section 2.3, the scale parameter estimate in Scenario 3 is very small compared to the corresponding true parameter value. Such a large bias in parameters makes it difficult to rely on the parameter estimates for interpretation of covariate effects on consumer preferences and will likely cause several other issues, including inferior model fit and erroneous forecasts.

3.3. Predictive performance of the $L\gamma$-profile model vis-a-vis the $NL\gamma$-profile model

To examine the effect of bias in the estimated parameters on the predictive performance of $L\gamma$-profile models (in which we expressly recognize the need for the outside good consumption to be positive, using the procedure in Section 2.4), we used the estimated parameters from Table 1 to predict consumptions and compared the predicted values with the 'true' consumption values simulated using the $L\gamma$-profile (that is, the truncation condition is ignored in estimation but recognized in prediction). In addition, we compared the predictive performance of the $L\gamma$-profile model with that of the $NL\gamma$-profile model estimated on the same data. Similarly, data were generated using $NL\gamma$-profile utility functions (using the Pinjari and Bhat, 2021 approach) and the utility function parameters of both $L\gamma$-profile and $NL\gamma$-profile models were estimated on such data. The resulting parameter estimates were used to assess predictions from both the $L\gamma$-profile and $NL\gamma$-profile models against data simulated using $NL\gamma$-profile utility functions. All these assessments were performed for two consumption patterns: (1) when a small (but significant) proportion of the budget is allotted to inside goods and (2) when a large proportion of the budget is allotted to inside goods. In summary, the following data generation processes (DGP) are considered:

(i) DGP 1: Data simulated using $L\gamma$-profile utility functions such that a small proportion (16%) of the budget is allocated to inside goods.

(ii) DGP 2: Data simulated using $NL\gamma$-profile utility functions such that a small proportion (16%) of the budget is allocated to inside goods.

(iii) DGP 3: Data simulated using $L\gamma$-profile utility functions such that a large proportion (43%) of the budget is allocated to inside goods.

(iv) DGP 4: Data simulated using $NL\gamma$-profile utility functions such that a large proportion (43%) of the budget is allocated to inside goods.

For each of the above DGPs, 50 datasets of sample size 10,000 were simulated assuming a budget of 1,000 units. Next, the predictive performance of both $L\gamma$-profile and $NL\gamma$-profile models was assessed for all the above cases.[4] Table 2 presents the results of the above discussed predictive assessments, which are discussed next.

3.3.1 When a small (but significant) proportion of the budget is allocated to inside goods

As can be observed from the first set of rows in Table 2 (for DGP1), the $L\gamma$-profile model provides accurate forecasts, with very small weighted mean absolute percentage error (weighted MAPE) [5] values of 1.25% for discrete choice and 1.65% for continuous choice. Further, these forecasts are better than those from the $NL\gamma$-profile model (6.08% for discrete choice and 10.08% for continuous choice). These results suggest that although the parameter estimates are biased due to ignoring the truncation condition, accommodating the condition during forecasting helps in obtaining accurate forecasts. This is perhaps because, in situations when a small (but significant) proportion of the budget is allocated to inside goods, the condition in Eq. (5) requires truncation of a small part of the distribution of the baseline preference parameters, ignoring which during estimation does not harm model predictions.

The results in the second set of rows in Table 2 (for DGP2, where the $NL\gamma$-profile utility function is used to simulate data) suggest that the predictions of the $NL\gamma$-profile model are superior to that of the $L\gamma$-profile model. Together, the results for DGP1 and DGP2 imply that when a small (but significant) proportion of the budget is allocated to inside goods, the underlying DGP determines which model to work with. In empirical contexts where the DGP is unknown, the analyst should try both the models and select the one that provides better fit, predictions, and interpretation.

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[4] Predictive performance of the $L\gamma$-profile model for the infinite budgets case is presented in the next section, where the model is analyzed in greater detail for the infinite budgets case. Further, we do not present the predictions of the $NL\gamma$-profile models for the infinite budgets case because, as discussed in Bhat (2018), it is not easy to estimate $NL\gamma$-profile models for situations with very large budgets.

[5] $Weighted\ MAPE = \dfrac{\sum_k \left|(O_k - P_k)/O_k\right| \times 100 \times O_k}{\sum_k O_k}$. In this expression, O_k and P_k are the observed and predicted discrete choice shares (or average expenditures), respectively, for the kth choice alternative.

3.3.2 When a large proportion of the budget is allocated to inside goods

The results for DGP3 and DGP4 (third and fourth set of rows in Table 2) suggest that the $L\gamma$-profile model predictions are worse than the predictions of the $NL\gamma$-profile model in situations when a large proportion of the budget is allocated to inside goods, regardless of the underlying DGP. These results highlight that the large bias in the parameter estimates of the $L\gamma$-profile model due to ignoring the truncation condition harms the model predictions, even if the truncation condition is enforced at the prediction stage. The impact of this bias grows as the proportion of the budget allocated to inside goods increases. Therefore, when the proportion of the budget allocated to inside goods is high, it is better to work with the $NL\gamma$-profile model than the $L\gamma$-profile model that ignores the truncation condition, even if the underlying DGP is that of the $L\gamma$-profile utility functions.

3.4. What proportion of the budget is very small, small, and large for $L\gamma$-profile models?

The preceding discussion sheds light on the suitability of the $L\gamma$-profile model (that ignores the truncation condition during estimation) for different consumption patterns. While the simulation experiments of the preceding section considered the proportional allocations of around 1%, 16%, and 43% as very small, small, and large, respectively, these definitions are subjective. In this context, a pertinent question is what proportion of the budget allocation to inside goods can be considered very small, small, and large.

To determine what proportion of the budget allocation to inside goods can be considered 'very small' (such that the budget can be considered very large or infinite), we conducted additional experiments by increasing the percentage of the budget allocated to inside goods from 1% to 5% and then to 10% for the $L\gamma$-profile utility functions. To generate data where up to 5% of the budgets were allocated to inside goods, the assumed budget value was 50,000 units, and to generate data where 10% or more of the budgets were allocated to inside goods, the assumed budget value was 1000 units. Using each of these simulated consumption data, we estimated $L\gamma$-profile models and applied the parameter estimates to forecast consumptions using two different approaches. One approach considers the budget constraint as in the forecasting procedure described in Section 2.3. The other approach does not consider the budget constraint (*i.e.*, skips the verification of the truncation condition in steps 7 and 8 of the forecasting procedure of Section 2.4). While detailed results of these experiments are not presented here to conserve space, for consumption data with about 5% of the total budget allocated to inside goods, the bias in parameter estimates was small (overall APB was 1.5%).

Also, the predictions from not considering the budget constraint were close enough to those obtained after considering the budget constraint (a difference of 0.5 % in discrete choices and 11.2% in continuous choices between the consumptions forecasted with and without the budget constraint). Increasing the inside good allocation to 10% of the budget resulted in an overall bias (APB) of 4.5% in the parameter estimates. While this bias might appear low, the predictions from not considering the budget constraint were not close to those obtained after considering the budget constraint (a difference of 1.3% in discrete choices and 34.7% in continuous choices). These results suggest that an average allocation of less than 5% of the total budget can be considered 'very small'. In such situations, it is not necessary to consider the budget information during estimation or forecasting, for the budget can be treated as very large (essentially infinite) in such situations.[6] However, for situations with more than 5% of the budget allocated to inside goods, it becomes important to consider the budget constraint, at least during forecasting.

Next, to differentiate between small (but significant) and large proportional allocations to inside goods, in addition to the already considered cases of 5%, 10%, 16%, and 43% allocation to inside goods, an additional consumption pattern was simulated, where about 24% of the budget was allocated to inside goods. The predictive performance of both the $L\gamma$-profile and $NL\gamma$-profile models estimated on all these data are shown in Fig. 1. In all these predictions, the budget constraint was considered through the truncation condition (as described in Section 2.4). Clearly, as the proportion of the budget allocated to inside goods increases (on the *X*-axis), the errors (plotted on the *Y*-axis as weighted MAPE) in both discrete predictions (figure on the left side) and continuous predictions (figure on the right side) for the $L\gamma$-profile model increases. The same trend is not observed with the $NL\gamma$-profile model. While the trends presented in the figure might change with the parameter estimates, variable specification in the utility functions, and choice set size, the results suggest that any allocations with more than 35% of the budget allocated to inside goods can be considered large. In such cases, the $NL\gamma$-profile model should be preferred over the $L\gamma$-profile model. And anywhere

[6] Our experience with simulated datasets suggests that estimation of the $NL\gamma$-profile models for situations with 'very small' allocations to inside goods is not easy, which makes the $L\gamma$-profile model as the preferred alternative. This is because data with large allocations to the outside good do not reflect a discernible bend in the utility profile (or diminishing marginal utility) to be suitable for a non-linear utility form for the outside good. As a result, such data pose difficulty in estimation of $NL\gamma$-profile models. Another related reason is that in situations when inside goods are chosen relatively frequently but with low continuous consumptions, the satiation parameters are close to zero (to reflect high satiation). This causes too dramatic a bend in the utility profiles of the inside goods, adding to estimation difficulties.

between 5% and 35% of the budget can be considered small (but significant). In such cases with a small (but significant) proportion of the budget allotted to inside goods, it is recommended that both the $L\gamma$-profile and $NL\gamma$-profile models are estimated to select the one that offers better fit, interpretation, and forecasts.

3.5. Summary of findings from the simulation experiments

The findings from the simulation experiments in this section can be summarized as follows:

(i) The $L\gamma$-profile model (that ignores the truncation condition during estimation) is ideally suited for situations when a very small proportion (<5%) of the budget is allocated to inside goods. The budget in such situations can be treated as very large or infinite. In such cases, the $L\gamma$-profile model can be used for both parameter estimation and predictions without budget information. The $NL\gamma$-profile models are less likely to suit these situations, for it would be difficult to estimate the model parameters. Of course, if the budget information is available, the analyst may use the information during predictions to reduce prediction errors.

(ii) When a small (but significant) proportion (5% to 35%) of the budget is allocated to inside goods, if the DGP is unknown, the analyst should estimate both the $L\gamma$-profile and $NL\gamma$-profile models and select the one that offers better fit, interpretation, and forecasts. If the underlying DGP involves $L\gamma$-profile utility functions, estimating the $L\gamma$-profile model will yield biased parameter estimates attributable to the truncation condition. Despite the bias, accommodating the truncation condition during prediction will help improve the accuracy of predictions. Note that prediction with the $L\gamma$-profile model in such situations will require the analyst to use the budget information. That is, one cannot apply the model if the budgets are unobserved.

(iii) When a large (>35%) proportion of the budget is allocated to inside goods, using the $L\gamma$-profile model will result in a high bias in parameter estimates and poor predictive performance. Therefore, regardless of the DGP, it is preferable to use the $NL\gamma$-profile model. Another implication is that one cannot use constrained utility maximization models to analyze consumption data with unobserved budgets if it is known that a large proportion of the budget is allocated to inside goods (or if the analyst does not have any idea of the proportional allocation to inside goods).

In addition to the above important points, it is worth mentioning that the scale parameter is generally estimable across the three consumption patterns (and well recovered in situations when a small or very small proportion of the budget is allocated to inside goods; i.e., Scenarios

1 and 2) along with all the γ_k parameters, despite the absence of price variation. However, issues related to estimation of scale are likely to crop up in situations when the budget is limited and the allocation to inside goods is large. As discussed in Section 2.3, in such situations, the scale parameter takes small values to compensate for not accommodating the budget constraint during model estimation. This can be observed from the parameter recovery results reported Table 1 for Scenario 3, where the estimated scale parameter is very small. Sometimes the scale parameter might become so small during estimations that the model estimation breaks down. In such situations, as already discussed above, the $L\gamma$-profile model that ignores the budget constraint is anyway not appropriate, and the $NL\gamma$-profile model is better suited.

4. MDC Choice Models with $L\gamma$-profile Utility Functions and Infinite Budgets

Consider the constrained utility maximization problem in Eq. (1) with an $L\gamma$-profile utility function as in Eq. (2) and the budget (*E*) being very large (infinite). As discussed earlier, such a formulation is applicable for situations where the total expenditure on inside goods is a very small (<5%) proportion of the available budget. In such situations, one can assume safely that the outside good will always be consumed, and as a result, the KKT conditions in Eq. (3) are sufficient for optimality and feasibility without the need for the truncation condition of Eq. (5). Therefore, the likelihood function can be formed by anchoring the value of the Lagrange multiplier using the outside good ($\lambda = \psi_1$) and substituting it into the KKT conditions for inside goods. Assuming that the first M $(M \le K)$ goods are chosen and that the corresponding consumption values are $x_2^*, ..., x_M^*$, the KKT conditions are given by:

$$\begin{aligned} &\frac{\psi_k}{\left(\frac{x_k^*}{\gamma_k}+1\right)} = \psi_1 p_k \text{, since } x_k^* > 0 \ \forall \ k = 2,3,...,M \\ &\psi_k < \psi_1 p_k \text{, since } x_k^* = 0 \ \forall \ k = M+1,...,K \end{aligned} \tag{12}$$

With statistical specification for the above KKT conditions as discussed in Section 2.3, where $\psi_k = \exp(\boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_k)$, $\psi_1 = \exp(\varepsilon_1)$, and $\gamma_k = \exp(\boldsymbol{\theta}'\mathbf{w}_k)$, different assumptions on the random error terms (ε_k) give rise to different models. Independent and identically distributed (IID) Gumbel error terms give rise to the $L\gamma$-profile MDCEV model with the following log-likelihood function (Bhat, 2018):

$$P(x_2^*,...,x_M^*,0,...,0) = |J| \times \mu^{(M-1)} \times (M-1)! \times \frac{\prod_{k=2}^{M} e^{\mu(V_k)}}{\left(1+\sum_{k=2}^{K} e^{\mu(V_k)}\right)^M} \tag{13}$$

where, $V_k = \boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k^*}{\gamma_k}+1\right) - \ln p_k$, $|J| = \prod_{k=2}^{M}\left(\frac{1}{x_k^*+\gamma_k}\right)$, and μ is inverse of the scale parameter of the distribution of ε_k (*i.e.*, $\mu = \frac{1}{\sigma}$). Note that $V_k = \boldsymbol{\beta}'\mathbf{z}_k - \ln p_k$ for non-chosen goods (*i.e.*, $\forall\ k > M$), because $x_k^* = 0$. Further, the likelihood function can also be interpreted as the MDC density function for the $M-1$ dimensional vector $X = \{X_1, X_2, ..., X_K\}$ of random variables representing optimal consumptions from the $L\gamma$-profile MDCEV model. Rather than IID Gumbel error terms for ε_k, if IID normally distributed random error terms are used, the $L\gamma$-profile MDC Probit **(**MDCP) model results, whose likelihood involves a product of normal probability density and cumulative density functions (non-IID normally distributed error terms may also be used within the MDCP framework, as discussed in Bhat *et al.*, 2013, but we confine attention here to the IID-normal case here within the MDCP framework). Furthermore, as demonstrated in Saxena *et al.* (2021), it is straightforward to accommodate upper bounds on the consumption of individual goods in either the MDCEV or the MCDP models. For example, consider $x_k^{\max}$ as the upper bound on the consumption of a good k. One can accommodate such an upper bound via modifying the KKT conditions for the optimal consumption (x_k^*) of that good accordingly, as below:

$$\begin{aligned}
&\psi_k < \psi_1 p_k, \text{ if } x_k^* = 0\ \forall\ k = 2,3,...,K \\
&\frac{\psi_k}{\left(\frac{x_k^*}{\gamma_k}+1\right)} = \psi_1 p_k, \text{ if } 0 < x_k^* < x_k^{\max}\ \forall\ k = 2,3,...,K \\
&\frac{\psi_k}{\left(\frac{x_k^{\max}}{\gamma_k}+1\right)} > \psi_1 p_k, \text{ if } x_k^* = x_k^{\max}\ \forall\ k = 2,3,...,K
\end{aligned} \tag{14}$$

Saxena *et al.* (2021) provide a detailed derivation of the likelihood for MDC models from the above KKT conditions that accommodate upper bounds on optimal consumptions.

4.1. Distribution of demand functions with $L\gamma$-profiles and infinite budgets

One can express the optimal consumptions for any inside good k from the KKT conditions of Eq. (14) as:

$$x_k^* = 0 \text{ if } \frac{\psi_k}{p_k} \le \psi_1$$

$$x_k^* = \left(\frac{\psi_k}{\psi_1 p_k} - 1\right)\gamma_k \text{ if } \psi_1 < \frac{\psi_k}{p_k} \le \psi_1\left(\frac{x_k^{\max}}{\gamma_k} + 1\right) \tag{15}$$

$$x_k^* = x_k^{\max} \text{ if } \frac{\psi_k}{p_k} > \psi_1\left(\frac{x_k^{\max}}{\gamma_k} + 1\right)$$

Note from the above expressions that x_k^* is left-censored at 0 and right-censored at $x_k^{\max}$. Let the uncensored version of the optimal consumption be represented as: $x_k = \left(\frac{\psi_k}{\psi_1 p_k} - 1\right)\gamma_k$. It is easy to see from this expression that the minimum possible value for x_k is $-\gamma_k$. Further, these expressions can be used to derive the discrete-continuous density function and the corresponding first and second moments of optimal consumptions from MDC choice models with AS $L\gamma$-profile utility functions involving IID random error terms and infinite budgets. In this section, we derive the distributions of the demand functions and their first and second moments for the $L\gamma$-profile MDCEV model and the $L\gamma$-profile MDCP model.

4.1.1. Demand functions from the $L\gamma$-profile MDCEV model with infinite budgets

One may utilize the expression $x_k = \left(\frac{\psi_k}{\psi_1 p_k} - 1\right)\gamma_k$ for the uncensored version X_K of the optimal consumptions provided in Eq. (15) and apply the change of variables technique to derive the distribution of X_k. Given the IID type-1 extreme value distributional assumptions on ε_k and ε_1 in ψ_k and ψ_1, the marginal density of X_k in its uncensored form where $-\gamma_k \le X_k < \infty$ may be derived as follows (see **Appendix A** for the derivation):

$$f_{X_k}(x_k) = \frac{\mu}{(x_k + \gamma_k)} \frac{e^{\mu(V_k)}}{\left(1 + e^{\mu(V_k)}\right)^2}, \text{ where } V_k = \boldsymbol{\beta}'\mathbf{z}_k - \ln\left(\frac{x_k^*}{\gamma_k} + 1\right) - \ln p_k \tag{16}$$

The above density function is the same as that of a three-parameter log-logistic (or, shifted log-logistic) distribution. Using this distribution for the uncensored variable X_k, one can derive the discrete-continuous distribution for the censored optimal consumption variable X_k^* as follows:

$$P(X_k^* = 0) = F_{X_k}(0) = \frac{1}{\left(1 + e^{\mu(\boldsymbol{\beta}'\mathbf{z_k} - \ln p_k)}\right)} \tag{17}$$

$$f_{X_k^*}(x_k) = \frac{\mu}{\left(x_k + \gamma_k\right)} \times \frac{e^{\mu(V_k)}}{\left(1 + e^{\mu(V_k)}\right)^2} \quad \text{for } 0 < x_k < x_k^{\max} \tag{18}$$

$$P(X_k^* = x_k^{\max}) = 1 - F_{X_k}(x_k^{\max}) = 1 - \frac{1}{1 + \exp\left\{\mu\left(\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k - \ln\left(\frac{x_k^{\max}}{\gamma_k} + 1\right)\right)\right\}} \tag{19}$$

The above density function for X_k^* reflects that the optimal consumption is left-censored at zero and right-censored at $x_k^{\max}$ with probability masses $F_{X_k}(0)$ and $F_{X_k}(x_k^{\max})$, respectively, and has a continuous distribution $f_{X_k^*}(x_k)$ between zero and $x_k^{\max}$. A special case of this distribution is when there is no right-censoring or when there is no upper bound on the optimal consumption of good k (*i.e.*, $x_k^{\max} \to \infty$).

4.1.2. Demand functions from the $L\gamma$-profile MDCP model with infinite budgets

Assuming that the random error terms ($\varepsilon_k \ \forall\, k = 1,...,K$) are IID normal distributed with a scale parameter σ, the marginal density for the uncensored variable X_k ($-\gamma_k \le X_k < \infty$) representing the optimal consumption of good k from the MDCP model may be derived as (see **Appendix A** for the derivation):

$$f_{X_k}(x_k) = \frac{1}{(x_k + \gamma_k)} \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{-V_k}{\sqrt{2}\sigma}\right)^2}, \text{ where } V_k = \boldsymbol{\beta}'\mathbf{z_k} - \ln\left(\frac{x_k}{\gamma_k} + 1\right) - \ln p_k \tag{20}$$

The derived density is the same as that of a shifted log-normal density function.

Using the above distribution for uncensored variable X_k, one can derive the discrete-continuous distribution for the optimal consumption variable X_k^* from an MDCP model as:

$$P(X_k^* = 0) = F_{X_k}(0) = \Phi\left(\frac{-\boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sigma\sqrt{2}}\right) \tag{21}$$

$$f_{X_k^*}(x_k) = \frac{1}{(x_k + \gamma_k)} \times \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{-V_k}{\sqrt{2}\sigma}\right)^2} \text{ for } 0 < x_k < x_k^{\max} \tag{22}$$

$$P(X_k^* = x_k^{\max}) = 1 - F_{X_k}(x_k^{\max}) = 1 - \Phi\left(\frac{\ln\left(\frac{x_k^{\max}}{\gamma_k} + 1\right) - \boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sigma\sqrt{2}}\right) \tag{23}$$

4.1.3. Moments of demand distributions from $L\gamma$-profile models with infinite budgets

Table 3 provides expressions for the first and second moments for the continuous conditional distribution $f_{X_k^*|0<X_k<x_k^{\max}}(x_k)$ for the case without an upper bound on the consumptions for both MDCEV and MDCP models with $L\gamma$-profile utility functions, infinite budgets, and IID error terms. Appendix B derives these expressions. Note that these expressions are not fully closed-form and involve open integrals for both MDCEV and MDCP models. However, the open integrals are easy to solve using off-the-shelf functions for integration. For the MDCP model, for example, the open integral is a univariate CDF of standard normal distribution, which is a commonly available function in most spreadsheet and data analysis software platforms.

The ability to easily compute these expressions obviates the need for simulation to derive distributions from the corresponding MDC choice models. This becomes helpful when implementing such models in large-scale travel demand micro-simulation systems, where one can draw from the distributions given in Sections 4.1.1 and 4.1.2 as opposed to implementing the consumer's utility optimizing program. Besides, now one can not only investigate the influence of prices or other covariates on the expected value of the distribution of optimal consumptions but also on the variance of the distribution. Furthermore, one can easily compute elasticities and other such metrics by simply computing the moments for the base case and a policy case (of 1% change in the variable for which elasticity is computed). In this context, we derived expressions for elasticities of the first and second moments of optimal consumptions with respect to unit prices and covariates in the baseline utility and satiation functions. However, those expressions are not reported here as they become quite cumbersome. It might be easier to compute the moments for the base and policy cases and use them to compute the

elasticities. It should also be possible to use the density function derived in the earlier section to derive different quantile values of the distribution.

For the MDCEV model, it is important to note that the first and second moments of optimal consumption of a good might not always be finite when there are no upper bounds on that good. This is due to the fat tail of the Gumbel distribution used in the underlying utility functions and the availability of an infinite budget. To examine this, we derived the conditions under which the moments from the distribution of optimal consumptions from the MDCEV model are finite. As derived in Appendix C, the first moment is finite only when the scale parameter $\sigma < 1$, and the second moment is finite only when the scale parameter $\sigma < 0.5$. For other values of the scale parameter, the corresponding moments become infinite. These findings are in line with the properties of the three parameter (or shifted) log-logistic distribution, which resembles closely with the distribution of the censored variable X_k^*. As a result, it becomes important to verify that the scale parameter estimate σ is less than 0.5. This issue is demonstrated in Appendix C (see Table C.1 and the discussion associated with it in the appendix) with simulated data from an MDCEV model (with $\sigma = 1$), where the first and second moment values fluctuate substantially as the seed for the simulation of the error term distribution is varied.

Interestingly, the above-discussed problem does not arise with the MDCP model in that the first and second moments are always finite (despite infinite budgets and no upper bound on the inside good's consumption value). This can be verified from a visual examination of the expressions provided in Table 3 for the MDCP model. Specifically, the moments exist for all finite values of σ . Table C.1 of Appendix C also demonstrates this using simulated data from an MDCP model.

To verify the correctness of the expressions derived in this section, we used the same utility functions assumed in Section 3 (and the parameters in the first row of Table 1) to simulate the first and second moments of optimal consumptions. To do so, the Pinjari and Bhat (2021) algorithm was used to find the optimal consumptions at each of several simulated values of the error terms, and then the first and second moments (*i.e.,* mean and variance) were computed from these simulated consumptions. It can be observed from Table C.2 in the appendix that the simulated values for both the first and second moments are close to the corresponding values computed using the expressions shown in Table 3 for the MDCEV model. The same can be verified for the MDCP model by comparing the simulated and analytical moments reported in Table C.1.

4.1.4. *Irrelevance of other alternatives (IOA) property for MDCEV $L\gamma$-profile model with infinite budgets*

Interestingly, the density function for X_k in Eq. (16) is a special case of the likelihood expression in Eq. (13) when good k is chosen along with the outside good, while no other inside good is available in the choice set. This result highlights that $L\gamma$-profile MDC choice models with infinite budgets and IID error terms exhibit a property according to which the optimal consumption amount and the corresponding density function of an inside alternative do not depend on the presence or the attributes of other alternatives. We refer to this property as the *irrelevance of other alternatives* (IOA) property for discrete-continuous choice models. Note that this property holds true for the MDCP model (with IID error terms) as well. Intuitively, the marginal density of optimal consumption for any good *k* depends only on the parameters of good *k* and is same as the likelihood expression when good *k* is the only available inside good. To see this for the MDCP model with IID error terms, the likelihood expression for a *K*-good case (with the first good as the essential outside good) can be written as below:

$$f_{X_2,X_3,\ldots,X_K}(x_2,x_3,\ldots,x_k) = |J| \frac{1}{\sigma^K} \int_{\varepsilon_1=-\infty}^{\infty} \prod_{k=2}^{K} \left(\phi\left(\frac{V_1 - V_k + \varepsilon_1}{\sigma} \right) \right) \phi\left(\frac{\varepsilon_1}{\sigma} \right) d\varepsilon_1 \tag{24}$$

where, $\phi(\cdot)$ is the standard normal probability density function. When good *k* is the only available inside good, the above likelihood expression simplifies to the following expression:

$$f_{X_k}(x_k) = \frac{1}{x_k + \gamma_k} \frac{1}{\sigma^2} \int_{\varepsilon_1=-\infty}^{\infty} \phi\left(\frac{V_1 - V_k + \varepsilon_1}{\sigma} \right) \phi\left(\frac{\varepsilon_1}{\sigma} \right) d\varepsilon_1 \tag{25}$$

It can be shown that the above expression is the same as the marginal density function for X_k in Eq. (20). This indicates that the IOA property holds for the MDCP model as well.

Note that the IOA property does not imply independence across consumptions of different alternatives (since the error term corresponding to the outside good induces correlations across consumptions of different inside alternatives). Yet, due to this property, such dependencies do not affect parameter estimation and model predictions. This is because the inside goods do not compete among each other due to the presence of a relatively large budget to draw from. Thanks to this property, for situations with infinite budget and IID $L\gamma$-profile utility functions, one can use discrete-continuous consumption data of only one good (or a subset of goods) to estimate the utility function parameters of that good (that subset

of goods); there is no need to observe or collect demand data of other goods. To verify this, we used the simulated MDCEV choice data (with infinite budgets) described in Section 3 to estimate binary discrete-continuous (BDC) choice models separately for each of the three inside goods. As can be noted from the results in Table 4, such BDC choice models for each inside good retrieve the parameters equally well as that of the MDC choice models that consider all inside goods at a time. We verified the IOA property in a similar manner using simulated data for the $L\gamma$-profile MDCP model as well, although we do not report the results here to conserve space.

As discussed above, the IOA property implies that the optimal allocation to a good depends only on the attributes of that good (i.e., its baseline preference and satiation parameters). However, an implication of this property is that the cross elasticities with respect to price or any other alternative attributes are zero, which essentially is a result of the availability of a large budget. While this may be seen as a limitation of the $L\gamma$-profile utility MDC models with infinite budgets (unlike the traditional MDC choice models which can accommodate non-zero cross elasticities since the inside goods compete amongst each other due to a finite budget), an empirical strategy to accommodate non-zero cross price elasticities is to include prices and attributes of other alternatives as explanatory variables in the baseline preference function (ψ_k) of the good under consideration. Such an approach is useful in situations when consumption data of other goods is not available but information on prices is available to the analyst. However, this is only an empirical strategy and not a rigorous, utility theoretic approach to accommodate non-zero cross elasticities in $L\gamma$-profile MDC model (this approach is akin to the use of the mother logit model in the single discrete choice case; see McFadden, 1975). As such, when finite budgets create competition among goods and result in non-zero cross elasticities, the analyst should consider using a non-linear utility form for the outside good or find a way to accommodate the budget constraint within the context of the linear form for the outside good.

4.2. Value of MDC models with infinite budgets and their relationship with a system of Tobit models

It may appear that the proposed models with infinite budgets are not useful for practical applications since infinite budgets are rare in practice. However, as we observed from the simulation experiments in Section 3, the models are applicable in situations when the budgets are very large in comparison to the total expenditure on inside goods. Applications involving such situations are abundant. For example, households' expenditures on tourism travel in a year

are typically very small compared to overall household expenditures in that year. Similarly, households' expenditures at a shopping occasion are generally much smaller compared to their overall expenditures. In both these cases, households' disposable incomes and overall expenditures can be assumed as large enough (relative to the total expenditure on inside goods) to be considered infinite. If the utility functions are well-specified with the important factors influencing the consumptions as explanatory variables entering the baseline utility and the satiation parameters, and if the scale parameter is estimated, the random components of the utility functions can be expected to be tight enough (*i.e.*, of small variance) to limit the likelihood of predicting unrealistically large consumption values for the inside goods. Also, as discussed earlier, it is possible to impose upper bounds on the consumption values of individual goods if such information is available with the analyst.

Further, the $L\gamma$-profile MDC model with infinite budgets leads to RUM-based discrete-continuous choice models that have similarities with the traditional discrete-continuous mixture regression models for censored data such as the Tobit model (Tobin, 1958). To see this specifically, consider the utility function from Eq. (2) and restrict all γ_k parameters to one (i.e., $\gamma_k = 1$ for $k = 2,3,...,K$). Such restrictions result in the following utility profile:

$$U(\boldsymbol{x}) = \psi_1 x_1 + \sum_{k=2}^{K} \psi_k \ln\left(x_k + 1\right) \tag{26}$$

Note that in the above utility function, the satiation effects are still included for the goods, independent of γ_k (γ_k provides additional satiation effects and allows for corner solutions; see Bhat, 2008 for more details) This satiation is introduced through the log-transformation, (the α_k satiation parameter in Bhat's (2008) formulation is normalised to zero, which results in the log-transformed utility profile) and can be verified from the marginal utility for an inside good *k* given by:

$$\frac{\partial U(\boldsymbol{x})}{\partial x_k} = \frac{\psi_k}{x_k + 1} \tag{27}$$

The above marginal utility decreases as consumption (x_k) increases. Also, for any two inside goods with same baseline preferences (ψ_k), the utility profiles are identical, with equal satiation. This implies that ψ_k determines not only the discrete consumption, but also is the sole determinant of the continuous consumptions, thus imposing tight tie between the discrete

and continuous consumptions. However, allowing free estimation of γ_k provides additional flexibility to the models in that it loses the strong tie between the discrete and continuous consumptions to a larger degree. Besides, by allowing γ_k to be a function of individual characteristics, it is possible to accommodate heterogeneity in satiation level across individuals.

Interestingly, the KKT conditions of the optimization model (as in Eq. (1)) for the utility function of Eq. (26) (that restricts $\gamma_k = 1 \,\forall\, k = 2,3,...,K$) result in a sequence of Tobit models. To see this, with $\gamma_k = 1$ for $k = 2,3,...,K$, the resulting KKT conditions for the utility function in Eq. (26) are:

$$\begin{aligned} &\boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_{k1} - \ln p_k = \ln\left(x_k^* + 1\right) \text{ when } x_k^* > 0;\ k = 2,3...,K \\ &\boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_{k1} - \ln p_k < 0 \text{ when } x_k^* = 0;\ k = 2,3...,K \end{aligned} \tag{28}$$

The above set of equations can be equivalently written as:

$$y_k = \begin{cases} y_k^* \text{ when } y_k^* > 0 \\ 0 \text{ when } y_k^* < 0 \end{cases} \text{ for } k = 2,3,...,K \tag{29}$$

where, y_k^* is the latent propensity and is given by $y_k^* = \boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_{k1} - \ln p_k$, and y_k is the observed variable (which is same as the log-transformed consumption value, i.e., $\ln\left(x_k^* + 1\right)$).

The above system of equations (in Eq. (29)) is exactly same as a Tobit model, with a Tobit equation for each alternative. This implies that a Tobit model is in fact a restricted version of the $L\gamma$-profile MDC model and is consistent with utility maximization. To the best of our knowledge, this is the first attempt to show the derivation of the Tobit model form a utility maximization approach, albeit with several restrictions as discussed above.

5. Empirical Application

5.1. Empirical data

The empirical data for this analysis comes from the Domestic Tourism Expenditure survey lead by the National Sample Survey Office (NSSO) of India in the years 2014 and 2015 (survey of domestic tourism in India, NSS 72nd round). For every respondent household, the survey recorded data on domestic trips that involved at least an overnight trip in 365 days prior to the date of the survey. For each such domestic trip made for the primary purpose of (a) leisure, holiday and recreation, (b) shopping, or (c) health and medical, the survey recorded expenditures across six expenditure classes: transportation, accommodation, food and

beverages (f&b), shopping, recreation and leisure, and health/medical. For this analysis, only the trips made for the purposes of leisure, holiday and recreation were considered (referred to as leisure trips from now on). Most households reported at most one overnight domestic leisure trip over the timeframe of a year. For households that reported multiple trips, we considered the most recent leisure trip made by the household. Among these, only those trips that were not reimbursed by an employer or other sources were considered. Also, package-deal trips were not included in the analysis since the data did not contain information on expenditure by category for these trips. After further cleaning and processing, the final empirical dataset used in this study had expenditure information on the recent leisure trip made by 4981 households, along with other details such as household socio-demographics and trip-level variables. These 4981 trips were split into two samples – an estimation sample with 3500 trips and a validation sample with 1481 trips.

For the above-described leisure trips considered in this study, we analyzed expenditures in the following categories: (a) accommodation, (b) food and beverages (f&b), (c) shopping, and (d) recreation and leisure (or recreation for brevity). Recall from the discussion in Section 4 that parameter estimation on a subset of alternatives does not affect the parameter estimates, as long as the utility functions belong to the $L\gamma$-profile class and have IID error terms. Further, since the expenditures incurred on health and medical services and transportation may not be fully discretionary, and transportation expenses include fixed costs of travel to and from the destination, we did not consider these expenditures in the current study.

Table 5 reports the aggregate expenditure patterns of the trips in the estimation sample. As can be observed, a large majority of households expended in f&b and shopping categories, but only 46.9 percentage of households spent on accommodation and 37.4 percentage spent on recreation and leisure. However, if households spent on accommodation, their average expenditure in that category (3200 Indian rupees (₹)) was higher than that for other expense categories. Interestingly, despite trips being for leisure, holiday, or recreation purposes, the expenditures in the recreation and leisure category are smaller than those in other categories. These statistics have implications for the tourism industry in India. Specifically, recreation does not appear to be the primary revenue-generating activity by leisure travelers in India. Their expenditures on accommodation, shopping, and food and beverages generate more revenues than their leisure activities.

Apart from the information on spending across the four classes, the sample has information on households' socio-demographic attributes, such as household type (whether

urban or rural), income level, and the composition of the group undertaking the trip (such as gender ratio). Also, trip-level attributes such as the trip duration (number of days), travel group size, and the destination of the trip aggregated at a state level were available. All these variables were explored as exogenous variables in the empirical model. Note here that since the model framework employs $L\gamma$-profile utility functions and assumes infinite budgets, the information on the total budget (and therefore, the outside good expenditure) is not necessary for our analysis.

5.2. Estimation results

We estimated both MDCEV and MDCP models with $L\gamma$-profile utility functions. While the scale parameter in the MDCEV model was estimated to be 0.46 which ensured existence of the first and second moments, the MDCP model provided a better statistical fit than the MDCEV model, both in term of AIC and BIC values (log-likelihood of -38563.70, and the corresponding AIC and BIC values of 77237.40 and 77576.22, respectively for the MDCEV model as compared to log-likelihood of -38302.25, and the corresponding AIC and BIC values of 76714.50 and 77053.32, respectively for the MDCP model). The subsequent results and findings are reported for the MDCP model owing to its superior statistical fit, albeit the MDCEV model also provided similar interpretations.

This empirical analysis intends to demonstrate the applicability of such models despite the assumption of infinite budgets, more than contributing substantially to understanding tourism expenditures in India. Nonetheless, we explored several alternative empirical specifications and arrived at the final empirical specification based on statistical significance and substantive interpretation of the parameter estimates. The model estimation results are presented in Table 6 and discussed next.

5.2.1. Baseline marginal utility and satiation functions

The constants in the baseline preferences and the satiation parameters are estimated for each alternative, except for the outside good. The constants themselves do not have a substantive interpretation. However, the constants in the baseline preference functions adjust for the bias in the location of the baseline preference distributions, and therefore, are important in the specification regardless of their statistical significance.

In the context of attributes that represent household demographics, the parameter estimates on urban dummy in the baseline preference highlight that urban households are more likely to spend in accommodation than their rural counterparts (Shucksmith *et al.,* 2009). On the other hand, rural households are more likely to spend in shopping, possibly due to lack of

retail infrastructure in rural regions (Gupta, 2011). Unsurprisingly though, the positive coefficient on urban dummy in the satiation function indicates that urban households typically spend more across all expenditure categories (Pal and Ghosh, 2007). The effects of households' monthly income (considered through a surrogate variable that represents household usual monthly expenditure) indicates that low-income households (whose usual monthly expenditure is less than ₹10,000) and medium-income households (whose usual monthly expenditure is between ₹10,000 and ₹20,000) are less likely to spend than high-income households in all expenditure categories except shopping. Income did not have a significant effect on the likelihood of a household expending on shopping. In the context of income effects on the satiation parameters, high-income households tend to spend more on shopping and recreation than low- and medium-income households, and more on food and beverages than low-income households. Overall, the parameter estimates of income variables in the baseline preference and satiation functions reflect the higher spending capabilities of the medium- and high-income households than low-income households.

In the context of group and trip specific attributes, larger groups are less likely to spend in accommodation category. Intuitively though, such groups spend more in each of the categories as compared to a smaller group. Interestingly, groups with higher proportion of women are less likely to spend in accommodation and food and beverages. However, groups that are constituted primarily of women spend more in accommodation and food and beverages category, indicating that women prioritize safety, health, and hygiene (Herter *et al.*, 2014; (Zemke *et al.*, 2015; Meng and Uysal, 2008; Hao and Har, 2014). Also, such groups are more likely to spend in shopping category (Herter *et al.*, 2014).

Finally, the duration of the trip did not influence the likelihood of spending in any category, except in accommodation, where longer duration trips resulted in less likelihood of opting for a paid accommodation (see Pellegrini *et al.*, 2021 for a similar finding). However, as expected, the extent of expenditure across all categories increases with the duration of stay. Similarly, farther destinations are associated with higher likelihood of spending in each of the spending categories, though trip destination does not influence the satiation parameter of expenditure in any category, except shopping.

5.2.2. Scale parameter

Based on the discussion in Section 2.3, we attempted to estimate the scale parameter, which was estimated with a value equal to 0.66 (despite the absence of price variation). Another empirical specification that fixed the scale parameter to 1 was of substantially inferior fit.

Therefore, we retained the model with the scale parameter estimate of 0.66. As discussed later, the relatively low variance of the error terms results in thin right tails for the implied distributions of the expenditures. This, in turn, implies very low probabilities for predicting unrealistically high expenditures, and obviates the need for imposing finite budgets. Overall, the estimation results provide intuitive and behaviourally plausible insights on leisure travelers' expenditure patterns on overnight leisure trips in accommodation, food and beverage, shopping, and leisure categories.

5.3. Characterization of the distributions of empirical demand

Figure 2 presents the trip-level probability density function (PDF) plots for the expenditures implied by the empirical model for a randomly selected rural household in the data for a four-day trip made by one of the household's male members to a destination within the same state as that of the UPR. For this household, each of the four panels in the figure presents expenditures in each of the four expenditure categories – accommodation, food and beverages, shopping, and recreation. In each panel, there are three PDF plots – one for each of low-, medium-, and high-income scenarios. Each density function plot is for values of expenditures above zero, based on the expression given in Eq. (22). The probability mass values for zero expenditure, computed using the expression in Eq. (21), are reported in the legends of the panels in the figure.

As discussed earlier, the empirical model implicitly assumes infinite budgets. This is reflected in the expressions for the PDF expression of Eq. (22) as well. Specifically, the PDF expression is defined for all values of x_k above zero (*i.e.*, $x_k^{\max} = \infty$). Therefore, in theory, the predicted values of optimal expenditures can be unreasonably large. This possibility raises questions on the practical applicability of the model with $L\gamma$-profile utility functions. However, the plots in Figure 2 illustrate that the PDF values for high expenditure values are very small and close to zero. This is because the scale parameter estimate (0.66) of the model is small enough to render a thin right tail to the implied probability distributions of expenditures. As income increases, all the PDF plots skew to the right, indicating an increase in spending in all expenditure categories. Nonetheless, the right tails of the distributions are not fat enough to result in a high likelihood of large expenditures.

We plotted the PDF functions for a few more randomly selected households (not shown in Figure 2) and observed similar trends. Therefore, despite the assumption of infinite budgets, MDC choice models with $L\gamma$-profile utility functions can be used to model consumption data (in situations with large budgets). The model does not necessarily lead to high likelihoods of

unreasonably large consumption values. Of course, there is no guarantee that such an observation will hold across all empirical contexts or for all empirical specifications. Therefore, it is useful to plot the model-implied PDF functions for at least a few individuals in the data. Further, if the analyst has access to information on upper bounds (*i.e.,* $x_k^{\max}$) for any of the expenditures, it is straightforward to apply the bounds to avoid the prediction of unrealistically large values (Saxena *et al.,* 2021).

5.4. Aggregate predictive performance assessment

This section evaluates the aggregate predictive performance of the estimated MDC choice model with $L\gamma$-profile utility functions, IID log-normal error terms, and infinite budgets. The evaluation was undertaken for both the estimation sample (N = 3,500) and the holdout validation sample (N = 1,481). Table 7 provides the results for both the discrete choice decision to spend in each of the four expense categories and the corresponding continuous expenditure amounts. The observed aggregate discrete choice shares for an expense category are reported as the percentage of individuals in the sample who spent in that category. The observed aggregate expenditures for an expense category are the average of observed expenditures across all those who spent in that category. These metrics from the observed data are compared with two types of model predictions – analytic predictions and simulated predictions. The analytic predictions were obtained using the expressions derived in this paper. Specifically, Eq. (21) was used to compute the discrete choice probability of each household in the sample expending in each category. Such choice probabilities for a given category were summed across all individuals in the sample to predict the percentage of individuals spending in that category. Similarly, the analytic expression reported in Table 3 (for the MDCP model) was used to compute the expected value of the continuous probability distribution for the expenditure by each household in each category. The average values (across the sample) of such expected values are reported as average expenditures for each expense category. Simulated predictions were obtained by solving each household's utility maximization for each of the 50 sets of simulated values of the models' error term draws. The aggregated predictions across all 50 sets of error draws for all households are reported as simulated predictions.

To compare model predictions with observed aggregate values, the last row of Table 7 reports weighted MAPE values for both analytic and simulated predictions. Two important observations can be made from the Table. First, despite making an implicit assumption of infinite budgets, the aggregate predictions from the model (both analytic and simulated predictions) are fairly accurate (*i.e.,* close to observed aggregate values). The weighted MAPE

values are small for both discrete and continuous choice components. The same pattern may be observed for predictions in both the estimation and validation samples. These results highlight that the assumption of infinite budgets is innocuous in the current empirical application. Second, the simulated predictions are generally close to the analytic predictions in almost all cases, except for the average expenditures in the accommodation category. The weighted MAPE values for the simulated predictions are 13.82 and 11.82 in the estimation and validation sample, respectively, and are higher relative to the more accurate analytic predictions, with a weighted MAPE of 7.18 and 4.65 for the estimation and validation sample, respectively. We noticed that increasing the number of error draws from 50 to 100 helped reduce the weighted MAPE for simulated predictions, albeit with an increase in the computation time needed. These results suggest the benefit of using the analytic methods proposed in this paper for prediction and policy analysis with MDC choice models with $L\gamma$-profile utility functions and infinite budgets.

6. Summary and Conclusions

This paper sheds light on the properties of MDC choice models with a linear utility profile for the outside good and an additively separable specification for the inside goods (*i.e.,* $L\gamma$-profile models). Specifically, the paper examines the suitability of the $L\gamma$-profile models for different consumption patterns (relative to the budget), using both a theoretical examination of the model and extensive simulations. In doing so, the paper highlights the importance of explicitly considering the budget constraint and the essential nature of the outside good during parameter estimation. Doing so requires the likelihood function to accommodate a truncation condition on the baseline marginal utility parameters of the model. Prior implementations of the $L\gamma$-profile model in the literature do not consider the truncation condition during parameter estimation, which causes a risk of biased parameter estimation and erroneous prediction.

The paper demonstrates that the $L\gamma$-profile models (that do not account for the truncation condition during parameter estimation) are best applied for situations when the total expenditure on inside goods is very small compared to the budget (<5% of the budget). The $NL\gamma$-profile models are less likely to suit these situations, for it would be difficult to estimate model parameters. As the proportion of the budget allocated to inside goods increases (beyond 5%), the bias in parameter estimates from the $L\gamma$-profile model increases. However, the model may still be used as an alternative to the $NL\gamma$-profile models only in situations when a small proportion of the budget is allocated to inside goods (<35% percent of the budget), as long as

the prediction algorithm considers the truncation condition. In such situations, it is advisable to try both the $L\gamma$-profile and $NL\gamma$-profile models and prefer the one that offers better interpretations and predictive ability. However, in situations when a large proportion of budget (>35% of the budget) is allocated to inside goods, it is imperative to explicitly incorporate the budget constraint in model estimation. Unfortunately, doing so becomes complicated with the usual assumptions made for the stochastic error terms. Thus, the $L\gamma$-profile model should not be used as it can result in substantial bias in parameter estimates and poor predictions. In such situations, the $NL\gamma$-profile model should be preferred, and information on the budget amount becomes necessary for parameter estimation and prediction. In addition, the paper highlights that the issues related to estimation of scale parameter in the absence of price variation are not as severe as previous studies pointed out. The paper indicates that the scale parameter is indeed estimable in the absence of price variation, except in some situations with limited budgets and large allocation to inside goods.

Importantly, the paper characterizes the stochastic distributions of the optimal demand functions resulting from MDC models for both $L\gamma$-profile MDCEV and $L\gamma$-profile MDCP models that assume additively separable and IID utility functions in situations when the budget is sufficiently large relative to the expenditure on the inside goods. In addition to the density functions of optimal demands, the paper derives expressions for the first and second moments of the distributions, thereby obviating the need for extensive simulations to forecast the expected consumption values. Based on these derivations, the paper sheds light on two important properties of these models. <u>First</u>, the $L\gamma$-profile MDCEV models yield demand functions with finite first moments only when the scale of the utility function $\sigma < 1$ (and finite second moments when $\sigma < 0.5$). However, this is not the case with $L\gamma$-profile MDCP model as the moments of the corresponding demand functions exist for all values of the distribution's parameters. <u>Second</u>, these models exhibit an *irrelevance of other alternatives* (IOA) property which implies that the discrete-continuous demand of an alternative is a function of the attributes of only that alternative and does not depend on the attributes (or availability) of other inside goods. This property allows the estimation of the utility function parameters of any alternative as long as the consumption data is available for that alternative even if information is not available on other alternatives in the choice set. However, the IOA property also results in a possible limitation of the $L\gamma$-profile MDC choice model in that the model exhibits zero cross elasticities with respect to price and other alternative attributes. Finally, perhaps for the first time in literature, the paper highlights the relationship between the $L\gamma$-profile MDC

models and Tobit models and shows that the Tobit model is a restricted version of the $L\gamma$-profile MDC models and is consistent with utility maximization.

The above discussed derivations and properties of the demand density functions from $L\gamma$-profile MDC choice models (for the infinite budgets case) are verified using simulated as well as empirical data. The empirical analysis was undertaken to analyse expenditure patterns of domestic tourism trips from a sample of households in India. Despite the assumption of infinite budgets, the empirical model provides plausible forecasts without a high likelihood of unreasonably large expenditures.

Future research should consider formulating $L\gamma$-profile models that make it easy to consider the budget constraint and essential nature of the outside good during parameter estimation. Doing so might need a departure from the typically used stochastic distributions on the error terms in the utility functions. Another useful avenue is to derive the marginal distributions of demand functions for non-IID error terms, since all the derivations in the current paper are for IID error terms. In addition to non-IID error terms, it will be useful to accommodate dependence among utility functions through non-additive utility forms (Bhat *et al.*, 2015). The empirical analysis presented in the study does not consider travel costs to and from the destination as fixed costs. In this context, the development of MDC models that recognize fixed costs (which can lead to non-smooth budget constraints) is an important area for future research.

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Table 1. Simulation experiment results: Parameter recovery for the $L\gamma$-profile model for different consumption patterns

Scenario 1: A very small proportion of the budget is allocated to inside goods (infinite budget case)
Data generation process: $L\gamma$-profile model, Budget = 50,000 units, Sample size = 10000 individuals.

Parameter	ASC_2	ASC_3	ASC_4	β_{2,z_1}	β_{3,z_1}	β_{4,z_1}	β_{2,z_2}	β_{3,z_2}	β_{4,z_2}	w_2	w_3	w_4	σ
True parameter value	-1.800	-1.200	-1.500	0.500	0.700	0.800	-1.000	-1.200	0.600	1.000	1.800	2.500	0.400
Parameter estimate with $L\gamma$-profile	-1.809	-1.202	-1.495	0.502	0.700	0.799	-1.006	-1.201	0.600	0.998	1.799	2.498	0.399
Absolute percentage bias (APB)	0.500	0.17	0.32	0.46	0.05	0.09	0.65	0.10	0.12	0.18	0.04	0.04	0.50
Asymptotic standard errors (ASE)	0.031	0.02	0.019	0.007	0.006	0.005	0.02	0.016	0.013	0.025	0.025	0.027	0.002
Finite sample standard errors (FSSE)	0.020	0.021	0.021	0.005	0.004	0.003	0.021	0.017	0.012	0.026	0.022	0.026	0.002
Overall Average APB	0.22												

Scenario 2: A small but significant proportion of the budget is allocated to inside goods
Data generation process: $L\gamma$-profile model, Budget = 1000 units, Sample size = 10000 individuals.

Parameter	ASC_2	ASC_3	ASC_4	β_{2,z_1}	β_{3,z_1}	β_{4,z_1}	β_{2,z_2}	β_{3,z_2}	β_{4,z_2}	w_2	w_3	w_4	σ
True parameter value	-1.800	-1.200	-1.500	0.500	0.700	0.800	-1.000	-1.200	0.600	1.000	1.800	2.500	0.400
Parameter estimate with $L\gamma$-profile	-1.597	-0.730	-0.826	0.423	0.501	0.467	-1.146	-1.132	0.352	0.833	2.087	3.378	0.490
Absolute percentage bias (APB)	11.26	39.17	44.92	15.38	28.40	41.57	14.63	5.63	41.16	16.65	15.96	35.13	22.61
Asymptotic standard errors (ASE)	0.027	0.018	0.017	0.005	0.005	0.005	0.018	0.015	0.012	0.024	0.025	0.028	0.003
Finite sample standard errors (FSSE)	0.014	0.014	0.015	0.006	0.005	0.005	0.022	0.015	0.010	0.022	0.024	0.039	0.002
Overall Average APB	25.58												

Scenario 3: A large proportion of the budget is allocated to inside goods
Data generation process: $L\gamma$-profile model, Budget = 1000 units, Sample size = 10000 individuals.

Parameter	ASC_2	ASC_3	ASC_4	β_{2,z_1}	β_{3,z_1}	β_{4,z_1}	β_{2,z_2}	β_{3,z_2}	β_{4,z_2}	w_2	w_3	w_4	σ
True parameter value	0.500	0.800	1.800	0.200	-0.149	-0.212	0.700	0.600	0.500	5.059	3.713	4.641	0.265
Parameter estimate with $L\gamma$-profile	0.078	0.150	0.285	0.003	-0.045	-0.055	-0.125	-0.104	-0.100	7.950	5.736	7.380	0.085
Absolute percentage bias (APB)	84.40	81.25	84.16	98.50	69.79	74.05	117.85	117.33	120.00	57.14	54.48	59.01	67.92
Asymptotic standard errors (ASE)	0.006	0.010	0.018	0.001	0.003	0.003	0.009	0.007	0.008	0.072	0.070	0.074	0.005
Finite sample standard errors (FSSE)	0.008	0.012	0.020	0.001	0.003	0.004	0.009	0.008	0.008	0.074	0.083	0.077	0.005
Overall Average APB	83.53												

Table 2. Simulation experiment results: Predictive accuracy of the $L\gamma$-profile model and its comparison with the $NL\gamma$-profile model

DGP 1: $L\gamma$-profile utility function. A small proportion of the budget is allocated to inside goods. Budget = 1000 units, 50 datasets of sample size 10000.

	Discrete choice (% of times an alternative is chosen)			Average continuous consumption choice		
	Simulated %	$L\gamma$-model prediction	$NL\gamma$-model prediction	Simulated	$L\gamma$-model prediction	$NL\gamma$-model prediction
Alternative 1	100	100	100	836.3	840.3	783.1
Alternative 2	31.0	31.7	35.5	4.9	5.3	8.8
Alternative 3	62.3	64.4	68.7	46.7	49.9	65.5
Alternative 4	77.9	78.5	83.5	170.9	161	201.8
Weighted MAPE (%)	--	1.25	6.08	--	1.65	10.08

DGP 2: $NL\gamma$-profile utility function. A small proportion of the budget is allocated to inside goods. Budget = 1000 units, 50 datasets of sample size 10000.

	Discrete choice (% of times an alternative is chosen)			Average continuous consumption choice		
	Simulated %	$L\gamma$-model prediction	$NL\gamma$-model prediction	Simulated	$L\gamma$-model prediction	$NL\gamma$-model prediction
Alternative 1	100	100	100	783.1	847.5	781.7
Alternative 2	35.5	29.1	35.5	8.8	6.9	8.8
Alternative 3	68.7	61.4	68.6	65.5	52.6	66.9
Alternative 4	83.5	77.2	83.5	201.8	153.0	202.8
Weighted MAPE (%)	--	6.94	0.06	--	12.1	0.4

DGP 3: $L\gamma$-profile utility function. A large proportion of the budget is allocated to inside goods. Budget = 1000 units, 50 datasets of sample size 10000.

	Discrete choice (% of times an alternative is chosen)			Average continuous consumption choice		
	Simulated %	$L\gamma$-model prediction	$NL\gamma$-model prediction	Simulated	$L\gamma$-model prediction	$NL\gamma$-model prediction
Alternative 1	100	100	100	569.9	704.7	536.6
Alternative 2	55.3	47.3	58.3	463.5	338.4	440.6
Alternative 3	32.9	25.5	32.6	43.6	41.1	79.4
Alternative 4	54.1	46.8	58.0	296.3	266.5	311.4
Weighted MAPE (%)	--	9.36	2.98	--	21.3	7.8

DGP 4: $NL\gamma$-profile utility function. A large proportion of the budget is allocated to inside goods. Budget = 1000 units, 50 datasets of sample size 10000.

	Discrete choice (% of times an alternative is chosen)			Average continuous consumption choice		
	Simulated %	$L\gamma$-model prediction	$NL\gamma$-model prediction	Simulated	$L\gamma$-model prediction	$NL\gamma$-model prediction
Alternative 1	100	100	100	536.63	760.53	533.27
Alternative 2	58.3	40.0	58.6	440.60	309.70	445.10
Alternative 3	32.6	22.3	32.7	79.43	54.80	78.35
Alternative 4	58.0	41.3	58.1	311.40	250.37	310.28
Weighted MAPE (%)	--	18.20	0.22	--	32.19	0.74

Table 3. Density functions and the corresponding moments for consumptions arising out of the $L\gamma$-profile MDCEV and MDCP models

	$L\gamma$-profile *MDCEV*	$L\gamma$-profile *MDCP*
Probability mass at 0	$\dfrac{1}{1+e^{\mu(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)}}$	$\Phi\left(\dfrac{-\boldsymbol{\beta}'\mathbf{z}_k+\ln p_k}{\sigma\sqrt{2}}\right)$
PDF for positive consumption values	$\dfrac{\mu}{(x_k+\gamma_k)}\dfrac{e^{\mu V_k}}{\left(1+e^{\mu V_k}\right)^2}$	$\dfrac{1}{(x_k+\gamma_k)}\dfrac{1}{2\sigma\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{-V_k}{\sqrt{2}\sigma}\right)^2}$
First moment of the density function for positive consumption values	$\mu\gamma_k\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)}\right)\left(I_{k1}-\dfrac{1}{\mu}\dfrac{1}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)}\right)}\right)$	$\dfrac{\gamma_k}{\delta_k}\left(e^{(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)+\sigma^2}\delta_{k1}-\delta_k\right)$
Second central moment of the density function for positive consumption values	$\mu\gamma_k^2\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)}\right)\left(\dfrac{1}{\mu}\dfrac{1}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)}\right)}+I_{k2}-2I_{k1}\right)$ $-\left(\mu\gamma_k\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)}\right)\left(I_{k1}-\dfrac{1}{\mu}\dfrac{1}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)}\right)}\right)\right)^2$	$\dfrac{\gamma_k^2}{\delta_k}\left\{e^{2(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)+4\sigma^2}\delta_{k2}+\delta_k-2e^{(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)+\sigma^2}\delta_{k1}\right\}$ $-\left(\dfrac{\gamma_k}{\delta_k}\left(e^{(\boldsymbol{\beta}'\mathbf{z_k}-\ln p_k)+\sigma^2}\delta_{k1}-\delta_k\right)\right)^2$

Notes for Table 3:

The above moments are for conditional distributions $f_{X_k^*|0<X_k<x_k^{\max}}(x_k)$ of optimal consumptions (conditioned on the choice of the good).

In the above expressions, $V_k=\boldsymbol{\beta}'\mathbf{z}_k-\ln\left(\dfrac{x_k}{\gamma_k}+1\right)-\ln p_k$; $I_{k1}=\int_1^\infty\dfrac{t^\mu dt}{\left(t^\mu+e^{\mu(\beta_k z_k-\ln p_k)}\right)^2}$; $I_{k2}=\int_1^\infty\dfrac{t^{\mu+1}dt}{\left(t^\mu+e^{\mu(\beta_k z_k-\ln p_k)}\right)^2}$; $\delta_k=1-\Phi\left(\dfrac{-\boldsymbol{\beta}'\mathbf{z}_k+\ln p_k}{\sigma\sqrt{2}}\right)$; $\delta_{k1}=1-\Phi\left(\dfrac{-\boldsymbol{\beta}'\mathbf{z}_k+\ln p_k-2\sigma^2}{\sigma\sqrt{2}}\right)$; and $\delta_{k2}=1-\Phi\left(\dfrac{-\boldsymbol{\beta}'\mathbf{z}_k+\ln p_k-4\sigma^2}{\sigma\sqrt{2}}\right)$, where $\Phi(.)$ is standard normal CDF function. It is worth noting here that the integrals I_{k1} and I_{k2} do not necessarily have a closed-form for all values of μ, except maybe for a few integer values. But these integrals can be computed easily with numerical integration techniques available in many off-the-shelf programs. Similarly, δ_k, δ_{k1}, and δ_{k2} do not have a closed form, but these terms involve standard normal CDFs that can be computed rather easily.

Table 4. Demonstration of the IOA property of the $L\gamma$-profile MDCEV model with infinite budget

MDCEV choice model estimated on consumptions simulated from MDC choice model with infinite budget

Parameter	ASC_2	ASC_3	ASC_4	β_{2,z_1}	β_{3,z_1}	β_{4,z_1}	β_{2,z_2}	β_{3,z_2}	β_{4,z_2}	w_2	w_3	w_4	σ
True parameter	-1.800	-1.200	-1.500	0.500	0.700	0.800	-1.000	-1.200	0.600	1.000	1.800	2.500	0.400
Estimate	-1.809	-1.202	-1.495	0.502	0.700	0.799	-1.006	-1.201	0.600	0.998	1.799	2.498	0.399
APB (%)	0.500	0.174	0.320	0.460	0.056	0.094	0.654	0.100	0.129	0.189	0.043	0.043	0.250
FSSE	0.02	0.021	0.021	0.005	0.004	0.003	0.021	0.017	0.012	0.026	0.022	0.026	0.002
ASE	0.031	0.02	0.019	0.007	0.006	0.005	0.020	0.016	0.013	0.025	0.025	0.027	0.002

BDCEV choice model estimated for the first inside good (estimated on consumptions simulated from MDCEV choice model with infinite budget)

Parameter	ASC_2	β_{2,z_1}	β_{2,z_2}	w_2	σ
True parameter	-1.800	0.500	-1.000	1.000	0.400
Estimate	-1.799	0.499	-1.003	1.006	0.398
APB (%)	0.004	0.048	0.361	0.631	0.500
FSSE	0.034	0.010	0.029	0.044	0.007
ASE	0.048	0.012	0.030	0.049	0.010

BDCEV choice model estimated for the second inside good (estimated on consumptions simulated from MDCEV choice model with infinite budget)

Parameter	ASC_3	β_{3,z_1}	β_{3,z_2}	w_3	σ
True parameter	-1.200	0.700	-1.200	1.800	0.400
Estimate	-1.202	0.699	-1.202	1.804	0.400
APB (%)	0.223	0.008	0.172	0.235	0.058
FSSE	0.019	0.007	0.021	0.038	0.006
ASE	0.022	0.007	0.018	0.032	0.005

BDCEV choice model estimated for the third inside good (estimated on consumptions simulated from MDCEV choice model with infinite budget)

Parameter	ASC_4	β_{4,z_1}	β_{4,z_2}	w_4	σ
True parameter	-1.500	0.800	0.600	2.500	0.400
Estimate	-1.495	0.799	0.599	2.500	0.399
APB (%)	0.280	0.110	0.144	0.029	0.250
FSSE	0.023	0.004	0.011	0.032	0.003
ASE	0.020	0.005	0.014	0.032	0.004

Table 5. Aggregate expenditure patterns in the empirical data (N = 4,981 trips)

	Participation rate (Percentage of households who expended in the category)	Average expenditure by households who spent in that category (in ₹)
Accommodation	46.9	3200
Food and beverages (F&B)	93.7	1610
Shopping	84.4	1970
Recreation and leisure	37.4	600

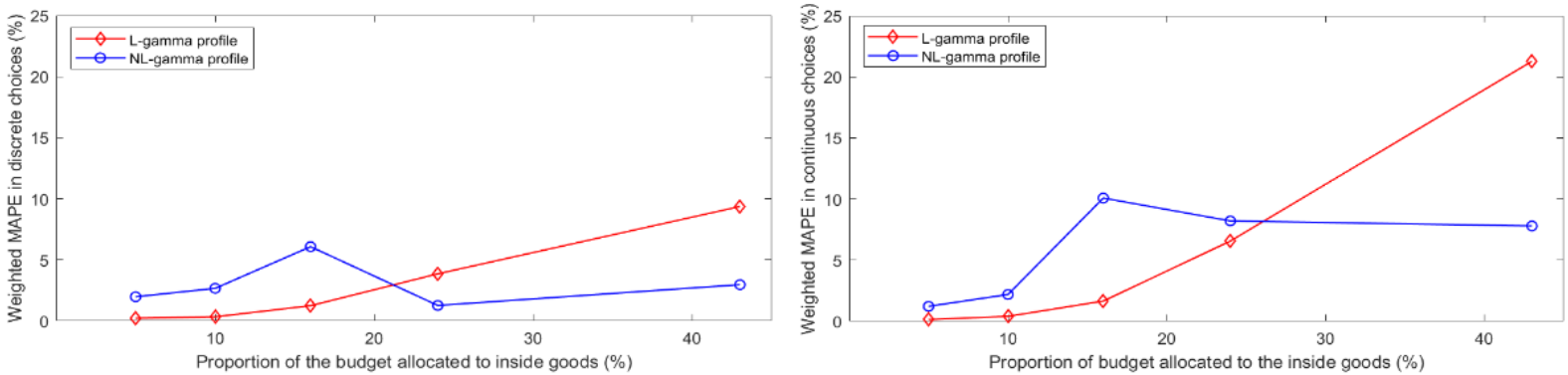
Table 6. Estimation results of the model for household expenditures on leisure trips in India

	Expenditure classes			
	Accommodat -ion	**Food and beverages**	**Shopping**	**Recreation and leisure**
Baseline preference function				
Constants	-1.15 *(-9.85)*	1.43 *(16.17)*	0.44 *(6.67)*	-0.91 *(-10.16)*
Household specific variables				
Urban household (Base category: Rural)	0.11 *(2.80)*	IS	-0.21 *(-4.44)*	IS
Income (Base category: UMCE > ₹20K)				
Low-income (UMCE < ₹10K)	-0.76 *(-12.82)*	-0.53 *(-7.58)*	IS	-0.23 *(-4.33)*
Medium-income (UMCE: ₹10-₹20K)	-0.43 (*-8.21*)	-0.44 (-9.28)	IS	-0.12 *(-2.56)*
Travel group and trip specific variables				
Size of the travel group	-0.01 *(-1.10)*	IS	0.06 *(3.34)*	0.03 *(1.12)*
Proportion of women in the group	-0.30 *(-3.75)*	-0.50 *(-6.00)*	0.08 *(1.30)*	IS
Trip duration (Base: duration > 10 nights)				
Trip duration is 1-3 nights	1.00 *(13.29)*	IS	IS	IS
Trip duration is 4-10 nights	0.53 *(7.70)*	IS	IS	IS
Trip destination (Base: Same district as UPR)				
Same state of UPR (not same district)	0.75 *(9.75)*	0.61 *(10.74)*	0.32 *(4.72)*	0.57 *(8.05)*
Outside the state of UPR	1.33 *(16.67)*	1.21 *(20.34)*	0.61 *(8.42)*	0.97 *(13.44)*
Satiation function				
Constants	3.48 *(17.95)*	-0.23 *(-4.07)*	1.11 *(9.05)*	1.30 *(8.58)*
Household specific variables				
Urban household (Rural is base)	0.19 *(1.70)*	0.22 *(4.63)*	0.31 *(4.50)*	0.20 *(2.04)*
Income (Base category: UMCE > ₹20K)				
Low-income (UMCE < ₹10K)	IS	-0.30 *(-3.28)*	-0.54 *(-9.11)*	-0.38 *(-3.22)*
Medium-income (UMCE: ₹10-₹20K)	IS	IS	-0.28 *(-5.11)*	-0.15 *(-1.32)*
Group and trip specific variables				
Size of the travel group	0.10 *(3.48)*	0.17 *(8.76)*	0.07 *(3.43)*	0.07 *(2.75)*
Proportion of women in travel group	0.49 *(3.09)*	0.42 *(4.19)*	IS	IS
Duration of stay *(Base: Duration>10 nights)*				
If the duration of stay was 1-3 nights	-1.84 (*-10.42*)	*	*	*
If the duration of stay was 4-10 nights	-0.92 *(-5.17)*	*	*	*
Number of nights	*	0.02 (*17.14*)	0.03 (*10.10*)	0.02 (*3.85*)
Trip destination (Base: Same district as UPR)				
Same state of UPR (not same district)	IS	IS	0.12 (*4.73*)	IS
Outside the state of UPR	IS	IS	0.48 (5.*34*)	IS
Scale of error terms of all goods	0.66 *(44.7)*			
Goodness of fit measures				
Number of cases (N)	3500			
Log-likelihood for constant only model	-39756.85			
Log-likelihood of the final specification	-38302.25			
Number of parameters of the final specification	55			

Notes: t-statistics of each estimated parameter are provided in parentheses next to the estimate.
IS: The parameter turned out to be statistically insignificant. The corresponding variable was dropped from the specification.
* No. of nights was specified as categorical variables in the accommodation satiation function and continuous variable in other satiation functions.

Table 7. Predictive performance of the empirical model on estimation and validation samples

	Estimation sample (N=3500)						Validation sample (N=1481)					
	Discrete choice shares (Percentage of households)			Aggregate Expenditures (100s of ₹)			Discrete choice shares (Percentage of households)			Aggregate Expenditures (100s of ₹)		
	Observed	Simulated	Analytic	Observed	Simulated	Analytic	Observed	Simulated	Analytic	Observed	Simulated	Analytic
Accommodation	46.9	46.7	46.7	32.0	37.9	34.2	49.7	47.2	47.2	32.8	39.8	35.3
Food & beverages	93.7	93.1	93.2	16.1	17.7	17.1	93.6	93.4	93.4	18.5	18.0	17.6
Shopping	84.4	81.6	81.7	19.7	21.2	21.3	84.2	81.7	81.2	21.8	22.7	21.7
Recreation & leisure	37.4	37.6	37.7	6.0	7.2	6.5	37.9	38.2	38.2	6.4	7.4	6.6
Weighted MAPE (%)	--	1.44	1.41	--	13.82	7.18	--	2.07	2.26	--	11.82	4.65



Note: Data generation was done using the $L\gamma$-profile for this comparison.

Figure 1. Prediction performance of $L\gamma$-profile and the $NL\gamma$-profile models under different proportional allocations to inside goods

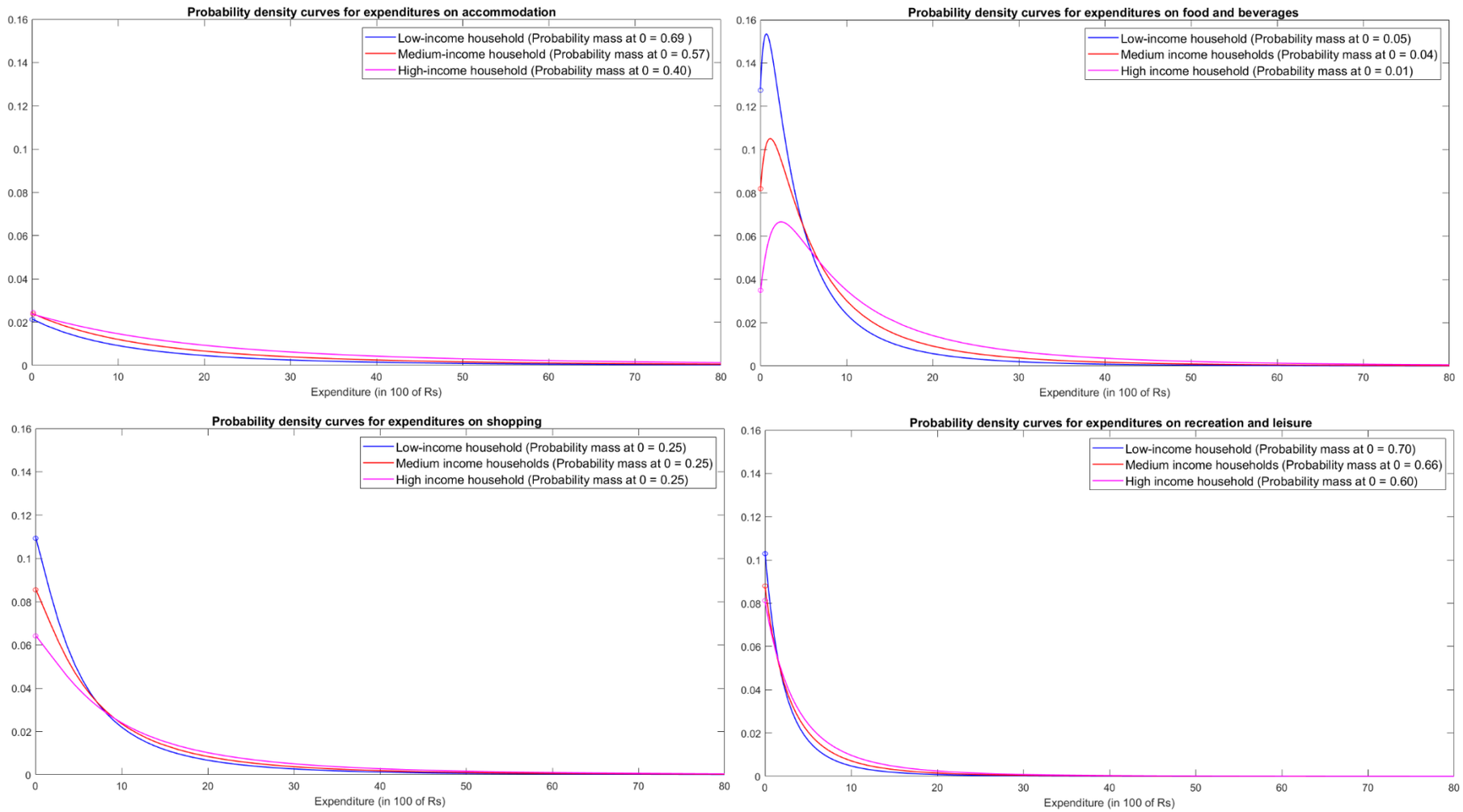


Figure 2. Probability density curves for expenditures on overnight leisure trips by three households belonging to low-, medium-, and high-income categories

APPENDIX A: Derivation of the distribution of demand functions for the $L\gamma$-profile MDCEV and MDCP models with infinite budgets

A.1. Distribution of demand functions for the $L\gamma$-profile *MDCEV model*

From Eq. (15) in the text, the uncensored expression for optimal consumption is:

$$x_k = \left(\frac{\psi_k}{\psi_1 p_k} - 1 \right) \gamma_k \tag{A1}$$

In the above expression, the utility parameters ψ_k and ψ_1 are log-extreme value distributed, and specified as $\psi_k = \exp(\boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_k)$ and $\psi_1 = \exp(\varepsilon_1)$, where ε_k $(k = 1, 2, ..., K)$ are IID type-1 extreme value distributed with scale parameter σ. For ease in notation, we use the inverse of the scale parameter $\mu = \frac{1}{\sigma}$ in this and all other derivations for MDCEV models.

Consider another random variable Y such that $Y = \varepsilon_1$. To derive the density of X_k, a random variable representing x_k from the above equation, we derive the joint density of (X_k, Y) and use it to derive the marginal density of X_k. Using the change of variables technique, the joint density of (X_k, Y) can be written as:

$$g_{X_k,Y}(x_k, y) = h(-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k} + 1\right) + \ln p_k + y, y) \frac{1}{x_k + \gamma_k} \tag{A2}$$

where, $h(\cdot,\cdot)$ is the joint PDF of IID error terms $(\varepsilon_k, \varepsilon_1)$, as below:

$$h(\varepsilon_k, \varepsilon_1) = \mu^2 e^{-\mu\varepsilon_k} e^{-e^{-\mu\varepsilon_k}} \times e^{-\mu\varepsilon_1} e^{-e^{-\mu\varepsilon_1}}$$

Therefore, the expression in Eq. (A2) can be rewritten as:

$$\mu^2 e^{-\mu\left(-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k + y\right)} e^{-e^{-\mu\left(-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k + y\right)}} \times e^{-\mu(y)} e^{-e^{-\mu(y)}} \frac{1}{x_k + \gamma_k}$$

Next, the marginal density of X_k can be written as:

$$\int_{y=-\infty}^{\infty} \mu^2 e^{-\mu\left(-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k + y\right)} e^{-e^{-\mu\left(-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k + y\right)}} \times e^{-\mu(y)} e^{-e^{-\mu(y)}} \frac{1}{x_k + \gamma_k} dy$$

$$= \frac{\mu e^{-\mu\left(-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k\right)}}{x_k + \gamma_k} \int_{y=-\infty}^{\infty} e^{-\mu(y)} e^{-e^{-\mu\left(-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k + y\right)}} \mu e^{-\mu(y)} e^{-e^{-\mu(y)}} dy$$

Let $e^{-\mu y}$ *be* t. Then, $-e^{-\mu y}dy$ is equal to dt. The above integral can now be written as:

$$\frac{\mu e^{-\mu\left(-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k\right)}}{x_k + \gamma_k} \int_{t=0}^{\infty} t e^{-te^{-\mu\left(-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k\right)}} e^{-t} dt$$

The above integral simplifies to the following expression, which is the same as the likelihood of an MDCEV model with one inside good and linear utility profile on outside good:

$$\frac{\mu e^{\mu\left(\boldsymbol{\beta}'\mathbf{z}_k - \ln\left(\frac{x_k}{\gamma_k}+1\right) - \ln p_k\right)}}{\left(x_k + \gamma_k\right)\left(1 + e^{\mu\left(\boldsymbol{\beta}'\mathbf{z}_k - \ln\left(\frac{x_k}{\gamma_k}+1\right) - \ln p_k\right)}\right)^2} \tag{A3}$$

Simplifying the above expression, we write the marginal density of X_k as:

$$f_{X_k}(x_k) = \frac{\mu}{\left(x_k + \gamma_k\right)} \frac{e^{\mu(V_k)}}{\left(1 + e^{\mu(V_k)}\right)^2}, \text{ where } V_k = \boldsymbol{\beta}'\mathbf{z}_k - \ln\left(\frac{x_k^*}{\gamma_k} + 1\right) - \ln(p_k). \tag{A4}$$

A.2. Distribution of the demand function for the $L\gamma -$ *profile MDCP model*

Using the KKT conditions from Eq. (12) in the text, we write

$$(\psi_k)\left(\frac{x_k}{\gamma_k} + 1\right)^{-1} = \lambda p_k \tag{A5}$$

Taking logarithm on both sides and rearranging the terms, we rewrite the above condition as:

$$\varepsilon_k - \varepsilon_1 = -\boldsymbol{\beta}'\mathbf{z}_k + \ln p_k + \ln\left(\frac{x_k}{\gamma_k} + 1\right) \tag{A6}$$

For the MDCP model, ε_k and ε_1 are IID normal distributed with zero mean and scale σ. Let $\varepsilon_k - \varepsilon_1$ be ε_{k1}. Then, $\varepsilon_{k1} \sim N(0, \sqrt{2}\sigma)$. Therefore, using change of variables, the density of X_k can be written as:

$$f_{X_k}(x_k) = h_{\varepsilon_{k1}}(-\boldsymbol{\beta}'\mathbf{z}_k + \ln p_k + \ln\left(\frac{x_k}{\gamma_k}+1\right))\left(\frac{1}{x_k+\gamma_k}\right) \tag{A7}$$

where, $h_{\varepsilon_{k1}}$ is the density function of ε_{k1}. Therefore, the above density function can be written as:

$$f_{X_k}(x_k) = \frac{1}{(x_k+\gamma_k)}\frac{1}{2\sigma\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{-V_k}{\sqrt{2}\sigma}\right)^2} \text{ where } V_k = \boldsymbol{\beta}'\mathbf{z}_k - \ln\left(\frac{x_k}{\gamma_k}+1\right) - \ln p_k\,. \tag{A8}$$

APPENDIX B: Derivation of first and second moments for the demand functions

B.1 Derivations for the $L\gamma-$profile MDCEV model

The moments are derived for the conditional distribution $X_k^* \mid X_k > 0$. The PDF for the conditional distribution is:

$$f_{X_k^*|X_k>0}(x_k) = \frac{f_{X_k}(x_k)}{\left[1-F_{X_k}(0)\right]} = \frac{1}{\left[1-F_{X_k}(0)\right]}\times\frac{\mu}{(x_k+\gamma_k)}\times\frac{e^{\mu(V_k)}}{\left(1+e^{\mu(V_k)}\right)^2} \tag{B1}$$

where, $1-F_{X_k}(0) = 1-\int_{-\gamma_k}^{0}\frac{\mu}{(x_k+\gamma_k)}\times\frac{e^{\mu V_k}}{\left(1+e^{\mu V_k}\right)^2}dx_k$.

Consider $F_{X_k}(0)$ in the above expression. Expanding V_k, $F_{X_k}(0)$ can be written as:

$$\begin{aligned} F_{X_k}(0) &= \int_{-\gamma}^{0}\frac{\mu e^{\mu\left(\boldsymbol{\beta}'\mathbf{z}_k-\ln\left(\frac{x_k}{\gamma_k}+1\right)-\ln p_k\right)}}{(x_k+\gamma_k)\left(1+e^{\mu\left(\boldsymbol{\beta}'\mathbf{z}_k-\ln\left(\frac{x_k}{\gamma_k}+1\right)-\ln p_k\right)}\right)^2}dx_k \\ &= \frac{\mu e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}}{\gamma_k}\int_{-\gamma}^{0}\frac{\left(\frac{x_k}{\gamma_k}+1\right)^{\mu-1}}{\left(\left(\frac{x_k}{\gamma_k}+1\right)^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dx_k \end{aligned} \tag{B2}$$

Let $t = \frac{x_k}{\gamma_k}+1$. Then, $dt = \frac{dx_k}{\gamma_k}$. The above integral can now be simplified as:

$$F_{X_k}(0) = e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\int_0^1\frac{\mu t^{\mu-1}dt}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2} = e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\left(\frac{-1}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}\right)\Bigg|_0^1 = 1-\frac{e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}$$

Therefore, $1-F_{X_k}(0)=\dfrac{e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}$ (B3)

Note that the above expression is the same as $P(\psi_k>\psi_1 p_1)$ which is the discrete choice probability of the inside good being chosen.

Substituting the above expression for $1-F_{X_k}(0)$ into Eq. (B1) and after further simplification, we get the following conditional PDF:

$$f_{X_k^*|X_k>0}(x_k)=\frac{\mu\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}{\gamma_k}\frac{\left(\dfrac{x_k}{\gamma_k}+1\right)^{\mu-1}}{\left(\left(\dfrac{x_k}{\gamma_k}+1\right)^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2} \tag{B4}$$

Next, we derive the moments of the conditional distribution.

B.1.1. First raw moment (Mean)

$$E[X_k^*\mid X_k>0]=\int_0^\infty x_k f_{X_k^*|X_k>0}(x_k)dx_k$$

$$=\frac{\mu\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}{\gamma_k}\int_0^\infty\frac{x_k\left(\dfrac{x_k}{\gamma_k}+1\right)^{\mu-1}}{\left(\left(\dfrac{x_k}{\gamma_k}+1\right)^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dx_k \tag{B5}$$

Let $t=\dfrac{x_k}{\gamma_k}+1$. Then, $\dfrac{dx_k}{\gamma_k}$ is *dt*. The above integral can now be written as:

$$E[X_k^*\mid X_k>0]=\mu\gamma_k\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)\int_1^\infty\frac{(t-1)t^{\mu-1}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dt \tag{B6}$$

Rewriting the integrand:

$$\int_1^\infty\left(\frac{t^{\mu}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}-\frac{t^{\mu-1}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}\right)dt \tag{B7}$$

The second term in the above expression integrates to the following:

$$\frac{1}{\mu}\frac{-1}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}\Bigg|_1^{\infty}=\frac{1}{\mu}\frac{1}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)} \tag{B8}$$

However, integration of the first term, *i.e.*, $\int_1^{\infty}\frac{t^{\mu}dt}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}$ depends on the value of μ.

Since further simplification of this integral is not possible without knowing the value of μ, the first moment can be written as:

$$E[X_k^*\mid X_k>0]=\mu\gamma_k\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)\left(I_{k1}-\frac{1}{\mu}\frac{1}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}\right) \tag{B9}$$

where, $I_{k1}=\int_1^{\infty}\frac{t^{\mu}dt}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}$.

One can verify that for $\mu\le 1$, the integral $\int_1^{\infty}\frac{t^{\mu}dt}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}$ diverges to infinity, thus implying that the first moment does not exist when $\mu\le 1$. However, we delve into this aspect in more detail in Appendix C.

B.1.2. Second central moment (variance)

$$Variance=\int_{x_k=0}^{\infty}\left(x_k-\overline{X_k}\right)^2 f_{X_k^*|X_k>0}(x_k)dx_k$$

Expanding the above expression, we get:

$$\begin{aligned}Variance&=\int_{x_k=0}^{\infty}\left(x_k^2-2x_k\overline{X_k}+\overline{X_k}^2\right)f_{X_k^*|X_k>0}(x_k)dx_k\\&=\int_{x_k=0}^{\infty}\left(x_k^2\right)f_{X_k^*|X_k>0}(x_k)dx_k-\left(\overline{X_k}\right)^2\end{aligned} \tag{B10}$$

The first term in the above expression is the second raw moment and the second term is the square of the first moment. Considering the first term in the above expression, feeding the density, and simplifying the expression, we get:

$$\int_{x_k=0}^{\infty}\left(x_k^2\right) f_{X_k^*|X_k>0}(x_k)dx_k = \frac{\mu\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}{\gamma_k}\int_0^{\infty}\frac{x_k^2\left(\frac{x_k}{\gamma_k}+1\right)^{\mu-1}}{\left(\left(\frac{x_k}{\gamma_k}+1\right)^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dx_k \tag{B11}$$

Again, let $t=\frac{x_k}{\gamma_k}+1$. Then, $dt=\frac{dx_k}{\gamma_k}$. The above integral is now written as:

$$\int_{x_k=0}^{\infty}\left(x_k^2\right) f_{X_k^*|X_k>0}(x_k)dx_k = \mu\gamma_k^2\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)\int_1^{\infty}\frac{(t-1)^2t^{\mu-1}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dt$$

Expanding the integrand in the above expression,

$$\int_1^{\infty}\frac{(t-1)^2t^{\mu-1}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dt=\int_1^{\infty}\frac{(t^2+1-2t)t^{\mu-1}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dt=\int_1^{\infty}\frac{t^{\mu+1}+t^{\mu-1}-2t^{\mu}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dt$$

Expanding the above expression further,

$$\int_1^{\infty}\frac{t^{\mu+1}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dt+\int_1^{\infty}\frac{t^{\mu-1}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dt-2\int_1^{\infty}\frac{t^{\mu}}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}dt \tag{B12}$$

The second term in the above expression integrates to $\frac{1}{\mu}\frac{1}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}$, while the integration of the other two terms depends on μ, which we will discuss more in Appendix C.

For ease in notation, the second central moment can be written as:

$$Variance\left[X_k^*\mid X_k>0\right]=$$

$$\mu\gamma_k^2\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)\left(I_{k2}+\frac{1}{\mu}\frac{1}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}-2I_{k1}\right)-\left(\mu\gamma_k\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)\left(I_{k1}-\frac{1}{\mu}\frac{1}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}\right)\right)^2 \tag{B13}$$

In the above expression, $I_{k1}=\int_1^{\infty}\frac{t^{\mu}dt}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}$ and $I_{k2}=\int_1^{\infty}\frac{t^{\mu+1}dt}{\left(t^{\mu}+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}$.

B.2 Derivations for the $L\gamma$-profile *MDCP model*

Using Eq. (22) from the text (without upper bounds, *i.e.*, $x_k^{\max} = \infty$), the conditional density is written as:

$$f_{X_k^*|X_k>0}(x_k) = \frac{1}{\left[1 - F_{X_k}(0)\right]} \times \frac{1}{(x_k + \gamma_k)} \times \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{-V_k}{\sqrt{2}\sigma}\right)^2} \tag{B14}$$

where, $F_{X_k}(0) = \int_{-\gamma_k}^{0} \frac{1}{(x_k + \gamma_k)} \times \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{-V_k}{\sqrt{2}\sigma}\right)^2} dx_k$

Expanding the above expression for $F_{X_k}(0)$, we can write,

$$F_{X_k}(0) = \int_{-\gamma_k}^{0} \frac{1}{(x_k + \gamma_k)} \times \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k}{\sqrt{2}\sigma}\right)^2} dx_k \tag{B15}$$

Let $\ln\left(\frac{x_k}{\gamma_k}+1\right)$ be t. Then, $dt = \frac{dx_k}{x_k + \gamma_k}$. The above probability expression can be written as:

$$F_{X_k}(0) = \int_{-\infty}^{0} \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{t - \boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sqrt{2}\sigma}\right)^2} dx_k = \Phi\left(\frac{-\boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sigma\sqrt{2}}\right) \tag{B16}$$

Let $1 - F_{X_k}(0)$ be δ_k, where $\delta_k = 1 - \Phi\left(\frac{-\boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sigma\sqrt{2}}\right)$

Therefore, the PDF for the conditional distribution $X_k^* \mid X_k > 0$ is written as:

$$f_{X_k^*|X_k>0}(x_k) = \frac{1}{\delta_k} \times \frac{1}{(x_k + \gamma_k)} \times \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{-V_k}{\sqrt{2}\sigma}\right)^2} \tag{B17}$$

B.2.1. First raw moment (Mean)

$$E[X_k^* \mid X_k > 0] = \int_{x_k=0}^{\infty} x_k f_{X_k^* \mid X_k > 0}(x_k) dx_k$$

$$= \int_{x_k=0}^{\infty} x_k \frac{1}{\delta_k} \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k}{\sqrt{2}\sigma}\right)^2} \left(\frac{1}{x_k + \gamma_k}\right) dx_k \tag{B18}$$

Let $t = \ln\left(\frac{x_k}{\gamma_k} + 1\right)$. Then, $dt = \frac{dx_k}{x_k + \gamma_k}$ and $x_k = (e^t - 1)\gamma_k$. Rewrite the above integral as:

$$E[X_k^* \mid X_k > 0] = \int_{t=0}^{\infty} (e^t - 1)\gamma_k \frac{1}{\delta_k} \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{t - \boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sigma\sqrt{2}}\right)^2} dt \tag{B19}$$

Solving the above integral:

$$\frac{\gamma_k}{\delta_k}\left(\int_{t=0}^{\infty} e^t \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{t - \boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sigma\sqrt{2}}\right)^2} dt - \int_{t=0}^{\infty} \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{t - \boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sigma\sqrt{2}}\right)^2} dt\right)$$

Replace the second integral in the above expression with δ_k to rewrite the above expression as:

$$\frac{\gamma_k}{\delta_k}\left(\int_{t=0}^{\infty} e^t \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{t - \boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sigma\sqrt{2}}\right)^2} dt - \delta_k\right)$$

Now, consider the first integral,

$$\int_{t=0}^{\infty} e^t \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{t - \boldsymbol{\beta}'\mathbf{z}_k + \ln p_k}{\sigma\sqrt{2}}\right)^2} dt$$

For brevity, let $\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k$ be c_k. Rewriting the above integral,

$$\int_{t=0}^{\infty} e^t \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{(t - c_k)}{\sqrt{2}\sigma}\right)^2} dt$$

Let $\frac{t - c_k}{\sqrt{2}\sigma}$ *be u.* Then, the above expression becomes:

$$\int_{u=-c_k/\sqrt{2}\sigma}^{\infty} e^{c_k+\sqrt{2}\sigma u} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(u)^2} du$$

Further solving the above expression,

$$e^{c_k} \int_{u=-c_k/\sqrt{2}\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(u^2-2\sqrt{2}\sigma u+2\sigma^2\right)} e^{\sigma^2} du = e^{c_k+\sigma^2} \int_{u=-c_k/\sqrt{2}\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(u-\sqrt{2}\sigma\right)^2} du$$

Feeding the expression for c_k in the above equation, the expected value can be written as:

$$E[X_k^* \mid X_k > 0] = \frac{\gamma_k}{\delta_k}\left(e^{\boldsymbol{\beta}'\mathbf{z_k} - \ln p_k + \sigma^2} \int_{u=-c_k/\sqrt{2}\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(u-\sqrt{2}\sigma\right)^2} du - \delta_k \right)$$

Rewriting the above expression, the expected value can be written as:

$$E[X_k^* \mid X_k > 0] = \frac{\gamma_k}{\delta_k}\left(e^{\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k + \sigma^2} \delta_{k1} - \delta_k \right) \tag{B20}$$

where, $\delta_{k1} = 1 - \Phi\left(-\frac{c_k + 2\sigma^2}{\sqrt{2}\sigma}\right)$, $\delta_k = 1 - \Phi\left(-\frac{c_k}{\sqrt{2}\sigma}\right)$, and $\Phi(.)$ is the operator for standard normal CDF.

B.2.2. Second central moment (Variance)

Using the expression for variance as used in *B.1.2*, and expanding it, we get,

$$Variance = \int_{x_k=0}^{\infty} x_k^2 \frac{1}{\delta_k} \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{-\boldsymbol{\beta}'\mathbf{z}_k + \ln\left(\frac{x_k}{\gamma_k}+1\right) + \ln p_k}{\sqrt{2}\sigma}\right)^2} \left(\frac{1}{x_k+\gamma_k}\right) dx_k - \left(\frac{\gamma_k}{\delta_k}\left(e^{\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k + \sigma^2}\delta_{k1} - \delta_k\right)\right)^2 \tag{B21}$$

We focus on the first term on the RHS of the above expression. Using the transformation $\ln\left(\frac{x_k}{\gamma_k}+1\right) = t$, the above expression can be rewritten as:

$$\int_{t=0}^{\infty} (e^t - 1)^2 \gamma_k^2 \frac{1}{\delta_k} \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{\left(t - \boldsymbol{\beta}'\mathbf{z}_k + \ln p_k\right)}{\sqrt{2}\sigma}\right)^2} dt$$

Again, for brevity, let $\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k$ be c_k . Expanding the above expression, we get,

$$\frac{\gamma_k^2}{\delta_k}\left\{\int_{t=0}^{\infty} e^{2t}\frac{1}{2\sigma\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{(t-c_k)}{\sqrt{2}\sigma}\right)^2}dt + \int_{t=0}^{\infty}\frac{1}{2\sigma\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{(t-c_k)}{\sqrt{2}\sigma}\right)^2}dt - 2\int_{t=0}^{\infty} e^{t}\frac{1}{2\sigma\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{(t-c_k)}{\sqrt{2}\sigma}\right)^2}dt\right\}$$

The above expression can be further simplified to

$$\frac{\gamma_k^2}{\delta_k}\left\{\int_{t=0}^{\infty} e^{2t}\frac{1}{2\sigma\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{(t-c_k)}{\sqrt{2}\sigma}\right)^2}dt + \delta_k - 2e^{c_k+\sigma^2}\delta_{k1}\right\}$$

Simplifying the first term in the above expression,

$$\int_{t=0}^{\infty} e^{2t}\frac{1}{2\sigma\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{(t-c_k)}{\sqrt{2}\sigma}\right)^2}dt = e^{2c_k+4\sigma^2}\int_{u=-\frac{c_k}{\sqrt{2}\sigma}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(u-2\sqrt{2}\sigma\right)^2}du = e^{2c_k+4\sigma^2}\delta_{k2}$$

where, $\delta_{k2} = 1-\Phi\left(-\frac{c_k+4\sigma^2}{\sqrt{2}\sigma}\right)$.

Collecting all the terms together, the variance of the conditional distribution $X_k^* \mid X_k > 0$ is:

$$Var[X_k^* \mid X_k > 0] = \frac{\gamma_k^2}{\delta_k}\left\{e^{2(\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k)+4\sigma^2}\delta_{k2} + \delta_k - 2e^{(\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k)+\sigma^2}\delta_{k1}\right\} - \left(\frac{\gamma_k}{\delta_k}\left(e^{(\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k)+\sigma^2}\delta_{k1} - \delta_k\right)\right)^2 \tag{B22}$$

where, $\delta_k = 1-\Phi\left(-\frac{c_k}{\sqrt{2}\sigma}\right)$, $\delta_{k1} = 1-\Phi\left(-\frac{c_k+2\sigma^2}{\sqrt{2}\sigma}\right)$, $\delta_{k2} = 1-\Phi\left(-\frac{c_k+4\sigma^2}{\sqrt{2}\sigma}\right)$, and $\Phi(\cdot)$ is the operator for standard normal CDF.

APPENDIX C: Existence and verification of moments of optimal consumptions from $L\gamma$-profile MDC models with infinite budgets

C.1. Existence of finite moments for consumptions implied by the $L\gamma-$profile *MDCEV model*

C.1.1 Existence of the first moment

The expression for first moment for the $L\gamma-$profile *MDCEV* model is:

$$\mu\gamma_k\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)\left(\int_1^\infty \frac{t^\mu dt}{\left(t^\mu+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}-\frac{1}{\mu}\frac{1}{\left(1+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)}\right) \tag{C1}$$

Existence of the first moment depends upon the convergence of the integral in the above equation. Consider the integrand as below:

$$\int_1^\infty \frac{t^\mu dt}{\left(t^\mu+e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}\right)^2}$$

For the sake of brevity, let $e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k-\ln p_k)}$ *be* Q_k. Therefore, the integral is written as:

$$I_{k1}=\int_1^\infty \frac{t^\mu dt}{\left(t^\mu+Q_k\right)^2};\ \mu>0,\ Q_k>0 \tag{C2}$$

The antiderivative for the above integral can be written as

$$\int \frac{t^\mu dt}{\left(t^\mu+Q_k\right)^2}dt=\frac{t^{\mu+1}\,{}_2F_1\left(2,1+{}^1\!/\!_\mu;2+{}^1\!/\!_\mu;-\frac{t^\mu}{Q_k}\right)}{(\mu+1)Q_k{}^2}+\ constant \tag{C3}$$

where, ${}_2F_1(a,b;c;z)$ is a hypergeometric function.

The hypergeometric function can be expanded as an integral as:

$${}_2F_1(a,b;c;z)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 \frac{u^{b-1}(1-u)^{c-b-1}}{(1-uz)^a}du \tag{C4}$$

Using the above expansion and imposing limits on the integral, we can rewrite the integral in Eq. (C3) as:

$$\int_1^\infty \frac{t^\mu dt}{\left(t^\mu + Q_k\right)^2} = \frac{t^{\mu+1}}{(\mu+1)Q_k^{\,2}} \frac{\Gamma\left(2+ {}^{1}\!/_{\mu}\right)}{\Gamma\left(1+ {}^{1}\!/_{\mu}\right)\Gamma(1)} \int_0^1 \frac{u^{\frac{1}{\mu}}}{\left(1+u\dfrac{t^\mu}{Q_k}\right)^2} du \Bigg|_1^\infty \tag{C5}$$

Rewriting the above expression,

$$\begin{aligned}\int_1^\infty \frac{t^\mu dt}{\left(t^\mu + Q_k\right)^2} = \lim_{t\to\infty} \frac{t^{\mu+1}}{(\mu+1)Q_k^{\,2}} \frac{\Gamma\left(2+ {}^{1}\!/_{\mu}\right)}{\Gamma\left(1+ {}^{1}\!/_{\mu}\right)\Gamma(1)} \int_0^1 \frac{u^{\frac{1}{\mu}}}{\left(1+u\dfrac{t^\mu}{Q_k}\right)^2} du \\ - \frac{1}{(\mu+1)Q_k^{\,2}} \frac{\Gamma\left(2+ {}^{1}\!/_{\mu}\right)}{\Gamma\left(1+ {}^{1}\!/_{\mu}\right)\Gamma(1)} \int_0^1 \frac{u^{\frac{1}{\mu}}}{\left(1+\dfrac{u}{Q_k}\right)^2} du\end{aligned} \tag{C6}$$

The condition for convergence requires the two integrals on RHS of the above expression to exist in limit. It is easy to verify that the second integral converges. Therefore, for the integral to converge, the first integral must converge. That is,

$$\lim_{t\to\infty} \frac{t^{\mu+1}}{(\mu+1)Q_k^{\,2}} \frac{\Gamma\left(2+ {}^{1}\!/_{\mu}\right)}{\Gamma\left(1+ {}^{1}\!/_{\mu}\right)\Gamma(1)} \int_0^1 \frac{u^{\frac{1}{\mu}}}{\left(1+u\dfrac{t^\mu}{Q_k}\right)^2} du \text{ must be finite.} \tag{C7}$$

For the above limit to exist, the degree of *t* in the numerator must be less than the degree of *t* in the denominator in the above expression. Since, degree of *t* in the numerator is $\mu+1$ while that in the denominator is 2μ. Therefore, for the limit to exist, $2\mu > \mu+1$ or $\mu > 1$.

In summary, the inverse of the scale parameter μ of the error terms in the utility function must be greater than 1 (*i.e.*, the scale parameter $\sigma < 1$) to yield finite first moments for the optimal consumptions from $L\gamma-$profile MDCEV models with infinite budgets.

C.1.2 Existence of finite variance (Second central moment)

The expression for the second moment of $X_k^* \mid X_k > 0$ given by Eq. (B12) is:

$$\int_1^\infty \frac{t^{\mu+1}}{\left(t^\mu + e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k)}\right)^2} dt + \int_1^\infty \frac{t^{\mu-1}}{\left(t^\mu + e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k)}\right)^2} dt - 2\int_1^\infty \frac{t^{\mu}}{\left(t^\mu + e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k)}\right)^2} dt \tag{C8}$$

In the above expression, the second integral converges for all μ, while the third integral converges when $\mu > 1$ (verified in the earlier section). Therefore, the criterion for the existence of finite variance depends on the convergence of the first integral in the above expression (with $\mu > 1$). Again, let $e^{\mu(\boldsymbol{\beta}'\mathbf{z}_k - \ln p_k)}$ be Q_k. Expanding the first integral in the above expression using hypergeometric functions, we get the following expression:

$$\int_1^{\infty} \frac{t^{\mu+1}}{\left(t^{\mu}+Q_k\right)^2} dt = \lim_{t\to\infty} \frac{t^{\mu+2}}{(\mu+2)Q_k^{\,2}} \frac{\Gamma\left(2+2/\mu\right)}{\Gamma\left(1+2/\mu\right)\Gamma(1)} \int_0^1 \frac{u^{\frac{1}{\mu}}}{\left(1+u\dfrac{t^{\mu}}{Q_k}\right)^2} du - \frac{1}{(\mu+2)Q_k^{\,2}} \frac{\Gamma\left(2+2/\mu\right)}{\Gamma\left(1+2/\mu\right)\Gamma(1)} \int_0^1 \frac{u^{\frac{1}{\mu}}}{\left(1+\dfrac{u}{Q_k}\right)^2} \tag{C9}$$

The second term in RHS of the above expression converges to a finite value. However, for the above expression to converge to a finite value, the first term must also converge. For this to happen as $t \to \infty$, the degree of t in the numerator must be less than the degree of t in the denominator (or, the numerator must rise slower than the denominator). That is, $2\mu > \mu + 2$ or $\mu > 2$. Therefore, $\mu > 2$ or $\sigma < 0.5$ is the criterion for the second moment to exist.

C.2. Simulation-based verification of the conditions for existence of first and second moments

To verify the existence of moments and the accuracy of the analytic formulae for computing the moments, we simulated consumption data for a choice setting with four alternatives – one outside good and three inside goods – for both the $L\gamma-$profile *MDCEV* and the $L\gamma-$profile *MDCP* models. The utility function considered for simulating data from both models is:

$$U = \psi_1 x_1 + \psi_2 \gamma_2 \ln\left(\frac{x_2}{\gamma_2}+1\right) + \psi_3 \gamma_3 \ln\left(\frac{x_3}{\gamma_3}+1\right) + \psi_4 \gamma_4 \ln\left(\frac{x_4}{\gamma_4}+1\right) \tag{C10}$$

where, $\psi_1 = \exp(\varepsilon_1)$, $\psi_k = \exp(ASC_k + \varepsilon_k), k = \{2,3,4\}$, and $\gamma_k = \exp(w_k), k = \{2,3,4\}$. And the parameters assumed for this simulation experiment are provided below:

Parameters	**Alternative 2**	**Alternative 3**	**Alternative 4**
Baseline preference constants (ASC$_k$)	-1.5	-1.0	-0.5
Satiation function constants (γ_k)	0.5	1.8	2.5
Scale (σ) for $L\gamma-$profile *MDCEV*		1.00	
Scale (σ) for $L\gamma-$profile *MDCP*		1.28	

To simulate consumptions for the MDCEV model, we considered standard IID Gumbel distribution with a scale parameter $\sigma = 1 (\text{or } \mu = 1)$. To simulate consumptions for the MDCP model, we considered a scale parameter $\sigma = 1.28$, which results in the same variance as that from a standard Gumbel distribution. For both cases, we simulated 15 datasets of 10,000 individuals. From each of these datasets, we computed the simulated mean and standard deviation of consumptions for each of the three inside goods. We also used the analytic expressions derived in this paper to compute the means and standard deviations. Table C.1 reports all these values – both simulated and those calculated using analytic expressions.

In the case of the $L\gamma$-profile MDCEV model, as can be observed from Table C.1, the simulated average consumption values and the standard deviations for all the three alternatives fluctuate considerably across the 15 datasets, implying that the moments do not exist when $\sigma = 1$. We carried out similar simulations and observed that the simulated average consumption values stabilize for $\sigma < 1$ and the variances of the distributions stabilize for $\sigma < 0.5$. These results corroborate the non-existence of finite first moment of the distributions for optimal consumptions from $L\gamma$-profile MDCEV model when the scale parameter $\sigma \geq 1$.

For the $L\gamma$-profile MDCP model, on the other hand, note from Table C.1 that the simulated average consumptions (and the standard deviations of consumptions) do not fluctuate (so much as they did in the case of the MDCEV model) across the different simulated datasets. These results corroborate that finite moments exist for the distributions of optimal consumptions from the $L\gamma$-profile MDCP model with infinite budgets regardless of the value of the model's scale parameter. Further, the simulated averages of the moments across the 15 simulated datasets are close to the values computed using the analytic expressions derived in this paper. These results help verify the correctness of the analytic expressions we derived in this paper for the moments of the distributions of optimal consumptions from the $L\gamma$-profile MDCP model.

Table C.1. First and second moments of the consumptions implied by the $L\gamma$-profile *MDCEV* and *MDCP* models

	Alternative 2				**Alternative 3**				**Alternative 4**			
	Mean		Standard deviation		Mean		Standard deviation		Mean		Standard deviation	
Sample	$L\gamma-$profile *MDCEV*	$L\gamma-$profile *MDCP*	$L\gamma-$profile *MDCEV*	$L\gamma-$profile *MDCP*	$L\gamma-$profile *MDCEV*	$L\gamma-$profile *MDCP*	$L\gamma-$profile *MDCEV*	$L\gamma-$profile *MDCP*	$L\gamma-$profile *MDCEV*	$L\gamma-$profile *MDCP*	$L\gamma-$profile *MDCEV*	$L\gamma-$profile *MDCP*
1	82.76	6.21	2786.94	17.38	73.67	30.54	963.86	108.81	149.67	81.72	2214.80	214.04
2	15.73	6.07	130.53	20.10	66.90	26.47	583.07	91.97	194.57	80.55	3860.53	284.91
3	22.59	5.75	300.37	22.48	56.51	29.89	460.30	88.68	241.77	73.75	7824.20	256.06
4	23.93	6.21	408.88	18.42	71.25	28.66	1169.61	82.53	205.69	76.96	2842.90	204.95
5	16.39	5.95	138.64	14.94	129.56	29.43	2325.98	85.33	179.26	85.97	2026.86	376.61
6	17.58	6.55	110.65	21.23	334.26	28.45	11808.95	96.08	247.28	70.69	3755.80	279.99
7	17.90	5.76	266.10	18.58	80.96	35.10	1652.33	116.31	116.43	77.67	863.56	363.54
8	74.83	6.08	2744.41	20.74	89.19	28.62	1737.60	85.56	161.63	78.91	2333.03	289.84
9	11.62	6.51	48.82	12.45	65.78	28.51	654.90	121.54	142.19	79.11	1055.82	327.88
10	19.30	5.79	289.96	13.90	142.93	26.73	3444.78	100.99	182.79	73.85	1697.56	295.06
11	699.72	5.71	29461.78	15.84	278.66	28.16	9904.84	104.23	133.67	84.04	1007.78	266.50
12	16.84	5.92	175.88	14.48	62.29	29.39	745.35	88.35	347.83	80.36	11661.82	227.05
13	86.02	6.23	2895.36	18.11	134.72	26.25	2760.77	97.56	171.47	78.20	1712.83	310.80
14	12.33	6.29	74.42	16.61	2143.07	29.76	106897.78	87.89	246.27	81.07	4059.21	245.74
15	35.62	5.91	948.25	15.71	76.13	30.74	846.02	88.55	187.29	82.16	2109.61	382.40
Simulated average across 15 samples	76.88	6.06	2718.73	17.40	253.73	29.11	9730.41	96.29	193.85	79.00	3268.42	288.36
Analytic value	Not defined	6.13	Not defined	20.11	Not defined	29.29	Not defined	103.55	Not defined	78.95	Not defined	299.33

To verify the accuracy of the moments of the distributions of optimal consumptions from the $L\gamma$-profile MDEV model with $\sigma < 1$, we compared the simulated and analytic predictions for the simulated data used in Section 3 (*i.e.*, consumptions that correspond to infinite budget). Table C.2 shows these comparisons. It can be observed from this table that the analytic predictions of both the discrete choice shares and the moments of the continuous consumption values match closely with the simulated values.

Table C.2. Comparison of simulated and analytic predictions for the data simulated in Section 3 for very large (infinite) budget.

	Participation rates		Average consumption (first moment)		Standard deviation (second central moment)	
	Simulated	Analytical	Simulated	Analytical	Simulated	Analytical
Alternative 2	40.4	40.5	7.6	7.2	17.7	18.5
Alternative 3	71.8	71.8	84.7	82.6	101.3	104.1
Alternative 4	87.0	87.0	483.0	481.4	623.2	626.5